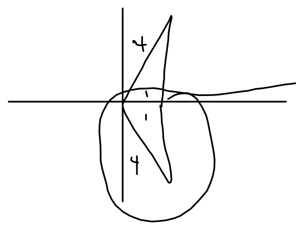
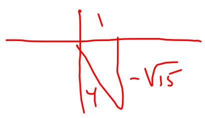
1. (10 pts) Find the values of all six trigonometric functions, given $\sec(\theta)=4$ and $\sin(\theta)<0$.

$\cos(\theta)=\frac{1}{4}$



$4^2-1^2=15$



$\sin\theta=-\frac{\sqrt{15}}{4}$ $\csc\theta=-\frac{4}{\sqrt{15}}$

$\cos\theta=\frac{1}{4}$ $\sec\theta=4$

$\tan\theta=-\sqrt{15}$ $\cot\theta=-\frac{1}{\sqrt{15}}$

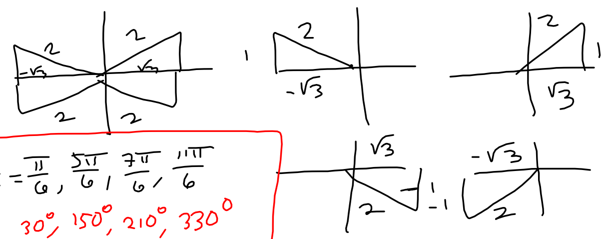
2. Consider the equation $4\cos^2(x)-3=0$.

a. (10 pts) Find all solutions x, in radians *and* degrees, to the equation in the interval $[0,2\pi]$.)

$4\cos^2 x=3$

$\cos^2 x=\frac{3}{4}$

$\cos x=\pm\sqrt{\frac{3}{4}}=\pm\frac{\sqrt{3}}{2}$



$x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

$30^\circ, 150^\circ, 210^\circ, 330^\circ$

$\mathbb{N}=\{1,2,3,\ldots\}$

$\mathbb{Z}=\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

b. (10 pts) Find all real solutions x, in radians *and* degrees.

$\frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi, \frac{7\pi}{6}+2n\pi, \frac{11\pi}{6}+2n\pi, \forall n\in\mathbb{N}\to\mathbb{Z}$

$\frac{\pi}{6}+n\pi, \frac{5\pi}{6}+n\pi, \forall n\in\mathbb{N}\to\mathbb{Z}$

$30^\circ+360^\circ n, 150^\circ+360^\circ n, 210^\circ+360^\circ n, 330^\circ+360^\circ n, \forall n\in\mathbb{N}\to\mathbb{Z}$

Shorten: $30^\circ+180^\circ n, 150^\circ+180^\circ n, \forall n\in\mathbb{N}\to\mathbb{Z}$

3. Consider the equation $4\cos^2(2x) - 3 = 0$. (Use your answer from #2, right or wrong.)

   a. (10 pts) Find all solutions x to the equation in the interval $[0, 2\pi]$. (Do degrees and radians in fin answer.)

Looking for all sol'ns $x \in [0, 2\pi)$

open interval @ 2π for our test

For our test, open @ 2π

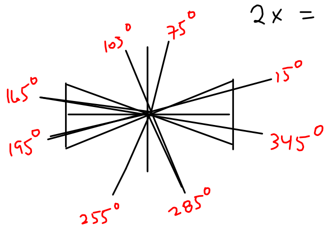
Last one:

$4\cos^2(x) - 3 = 0$

Now $4\cos^2(2x) - 3 = 0$

$x \in [0, 2\pi) \Rightarrow 2x \in [0, 4\pi)$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6} \Rightarrow \boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{6}}$



$3\pi - \frac{\pi}{6} = \frac{(18-1)\pi}{6} = \frac{17\pi}{6}$

$3\pi + \frac{\pi}{6} = \frac{19\pi}{6}$

$4\pi - \frac{\pi}{6} = \frac{23\pi}{6}$

$4\pi - \frac{\pi}{6} = \frac{23\pi}{6}$

$\frac{11\pi}{6} + 2\pi = \frac{23\pi}{6}$

$\boxed{x = 15^\circ, 75^\circ, 105^\circ, 165^\circ, 195^\circ, 255^\circ, 285^\circ, 345^\circ}$

b. (5 pts) Find all real solutions x, in degrees *and* radians.

OK

$\frac{\pi}{12} + 2n\pi, n \in \mathbb{Z}$

$\frac{5\pi}{12}$ ..

$\frac{7\pi}{12}$ ..

$\frac{11\pi}{12}$ ..

$\frac{13\pi}{12}$ ..

$\frac{17\pi}{12}$ ..

$\frac{19\pi}{12}$ ..

$\frac{23\pi}{12}$ ..

MORE ELEGANT

$\frac{\pi}{12} + n\pi, n \in \mathbb{Z}$

$\frac{5\pi}{12}$ ..

$\frac{7\pi}{12}$ ..

$\frac{11\pi}{12}$ ..

workable (OK)

$15^\circ + 360^\circ n, n \in \mathbb{Z}$

75° ..

105° ..

165° ..

195° ..

255° ..

285° ..

345° ..

More Elegant

$15^\circ + 180^\circ n, n \in \mathbb{Z}$

75° ..

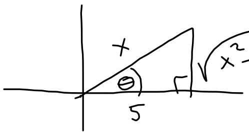
105° ..

165° ..

4. (10 pts) Re-write $\tan\left(\sec^{-1}\left(\frac{x}{5}\right)\right)$ as an algebraic expression.

$\tan\left(\cos^{-1}\left(\frac{5}{x}\right)\right)$

$= \tan(\theta)$



$\sqrt{x^2-5^2} = \sqrt{x^2-25}$

$\tan\theta = \boxed{\dfrac{\sqrt{x^2-25}}{5}}$

5. (5 pts) Square both sides of $\cos(x)-1=\sin(x)$ and solve. Find all solutions in $[0,2\pi]$. Give answer in degrees and radians.

Squaring both sides is casting a net, but sometimes you have to throw back some of the fishies.

$A=B \Rightarrow A^2=B^2$ $\quad 3=3 \Rightarrow 3^2=3^2$

$A^2=B^2 \Rightarrow A=\pm B$ $\quad (-3)^2=3^2$, but $-3\neq 3$

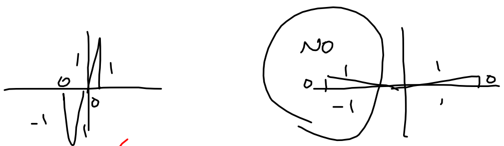
CHECK WORK ON THIS.

$(\cos(x)-1)^2 = (\sin(x))^2$

$\cos^2 x - 2\cos x + 1 = \sin^2 x = 1-\cos^2 x$

$2\cos^2 x - 2\cos x = (2\cos x)(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x = 1$



$x = \cancel{\frac{\pi}{2}}, \frac{3\pi}{2}$

$= 90^\circ, 270^\circ$

$x=0 = 0^\circ$

$x=2\pi = 360^\circ$ won't be included on test, b/c I'll say $[0,2\pi)$, not $[0,2\pi]$

$\cos\frac{\pi}{2} - 1 = -1 \overset{?}{=} \sin\frac{\pi}{2} = 1$ No

$\cos\left(\frac{3\pi}{2}\right) - 1 = -1 \overset{?}{=} \sin\frac{3\pi}{2} = -1$ Yes

$\cos(0) - 1 = 0 \overset{?}{=} \sin(0) = 0$ Yes

Sol'n: $\boxed{x = 0, \frac{3\pi}{2}}$

$x=\frac{\pi}{2}$ is extraneous.

6. Find the *exact* value of $\sin\left(\frac{19\pi}{12}\right)$ in two ways: (Hint: If degrees are easier for you, *use degrees*.)

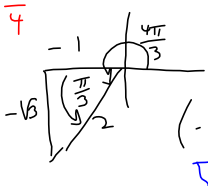
   a. (10 pts) Use a Sum identity.

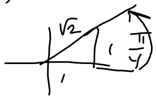
$19 = 18 + 1 = 17 + 2 = 16 + 3$

$\frac{16\pi}{12} = \frac{4\pi}{3}, \quad \frac{3\pi}{12} = \frac{\pi}{4}$

$= \sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)$





$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right)$

$= \boxed{\frac{-\sqrt{3} - 1}{2\sqrt{2}}} = \frac{-\sqrt{6} - \sqrt{2}}{4}$

   b. (10 pts) Use a Half-Angle identity

$\sin \frac{19\pi}{12} =$

Trying to recognize $\frac{19\pi}{12}$ as one-half of something.

$\frac{19\pi}{12} = \frac{19\pi}{2 \cdot 6} = \frac{1}{2}\left(\frac{19\pi}{6}\right) \Rightarrow u = \frac{19\pi}{6}$

$\frac{18\pi}{12} = \frac{3\pi}{2}$

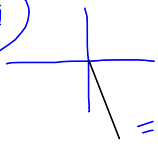
$\frac{19\pi}{12} = \frac{3\pi}{2} + \frac{\pi}{12}$

IN QIV

Sine is negative

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1 - \cos(u)}{2}}$

$\sin\left(\frac{19\pi}{12}\right) = -\sqrt{\frac{1 - \cos\left(\frac{19\pi}{6}\right)}{2}}$



$\frac{19\pi}{12} = \frac{u}{2} \Rightarrow$

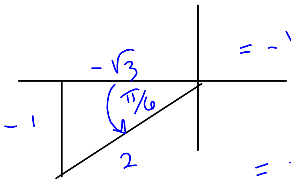
$\left(\frac{19\pi}{12}\right)(2) = u$

$\frac{19\pi}{6} = u$

$= \frac{18\pi}{6} + \frac{\pi}{6} =$

$= 3\pi + \frac{\pi}{6}$

$= -\sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}}$



$= -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{2+\sqrt{3}}{4}}$

$= \boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}} \quad = \frac{-\sqrt{3} - 1}{2\sqrt{2}} \; ?!$

$= -\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2+\sqrt{3}}}{2} = \frac{-\sqrt{2 \cdot 2 + 2\sqrt{3}}}{2\sqrt{2}} = \frac{-\sqrt{4 + 2\sqrt{3}}}{2\sqrt{2}}$

$a := \frac{(-\mathrm{sqrt}(3) - 1)}{2 \cdot \mathrm{sqrt}(2)} = \frac{1}{4}\left(-\sqrt{3} - 1\right)\sqrt{2}$

$b := -\frac{\mathrm{sqrt}(2 + \mathrm{sqrt}(3))}{2} = -\frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}$

=

$a - b = \frac{1}{4}\left(-\sqrt{3} - 1\right)\sqrt{2} + \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}$

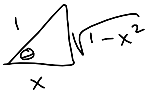
$\mathit{expand}(\%) = -\frac{1}{4}\sqrt{2}\sqrt{3} + \frac{1}{4}\sqrt{6}$

$\mathit{simplify}(\%) = 0$

7. (5 pts) Find the exact value of $\sin(\arcsin(x)+\arccos(x))$. (Hint: Use Sum identity.)

$\sin(u+v) = \sin u \cos v + \sin v \cos u$

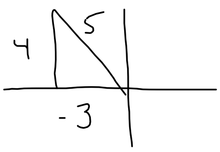
$= \sin(\arcsin(x))\cos(\arccos(x)) + \sin(\arccos(x))\cos(\arcsin(x))$



$(x)(x) + (\sqrt{1-x^2})(\sqrt{1-x^2})$

$= x^2 + (1-x^2) = \boxed{1}$

8. (10 pts) Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\sin(u) = \frac{4}{5}$ and $\cos(u) < 0$.



QI or QII and QII or QIII = QII

$\sin(2u) = 2\sin u \cos u = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \boxed{-\frac{24}{25} = \sin(2u)}$

$\cos(2u) = 2\cos^2 u - 1 = 2\left(-\frac{3}{5}\right)^2 - 1 = \frac{18}{25} - \frac{25}{25} = \boxed{\frac{-7}{25} = \cos(2u)}$

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = \frac{-\frac{24}{25}}{\frac{-7}{25}} = \left(\frac{-24}{25}\right)\left(\frac{25}{-7}\right) = \boxed{\frac{24}{7} = \tan(2u)}$

9. (5 pts) Find the arc length on a circle of radius $r = 9$ that is intercepted by an angle of 900^0.

cm

$$s = r\theta = (9)(900^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{\cancel{9}^{1}\,(\cancel{900}^{45})\pi}{\cancel{180}_{\cancel{20}}} = \boxed{45\pi}$$

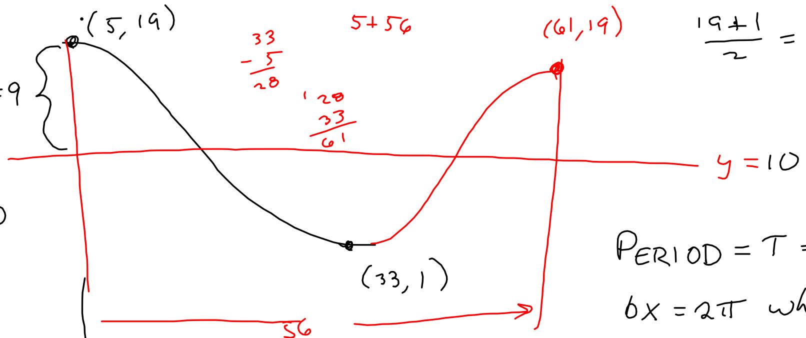
(10 pts) Bonus: Answer *one* of the following, for 10 points:

1. Build a cosine function that achieves its maximum height of $y = 19$ meters at time $x = 5$ seconds and its minimum height of $y = 1$ meter at $x = 33$ seconds.

midline:

$\frac{19+1}{2} = 10$

$19 - 10 = 9$



$9\cos(b(x-a)) + 10$

x=5 is start.

PERIOD = T = 56

$bx = 2\pi$ when $x = 56$

$56b = 2\pi$

$b = \frac{2\pi}{56} = \frac{\pi}{28}$

$$\boxed{9\cos\left(\frac{\pi}{28}(x-5)\right) + 10}$$

2. What is the area of the sector intercepted by an arc of 50^0 in a circle of radius 15? Round to 4 decimal places.

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(15)^2 (50^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{(225)(50)\pi}{180}$$

$$= \frac{\overset{25}{\cancel{\overset{75}{\cancel{(225)}}}}(5)}{\underset{2}{\cancel{\underset{6}{\cancel{18}}}}}\pi = \boxed{\frac{125\pi}{2}}$$