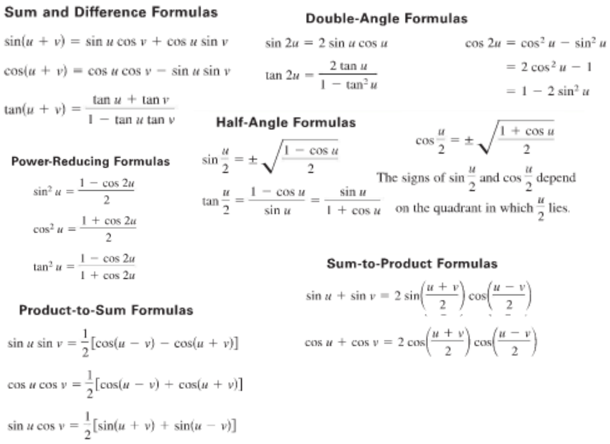


Sum and Difference Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\begin{aligned} \cos 2u &= \cos^2 u - \sin^2 u \\ &= 2\cos^2 u - 1 \\ &= 1 - 2\sin^2 u \end{aligned}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

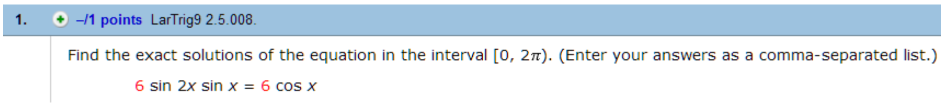
$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$



1. –/1 points LarTrig9 2.5.008.

Find the exact solutions of the equation in the interval [0, 2π). (Enter your answers as a comma-separated list.)

$6\sin 2x\sin x = 6\cos x$

$\cancel{6}(2\sin x\cos x)\sin y = \cancel{6}\cos x$

$\Rightarrow 2\sin^2 x\cos x - \cos x = 0$

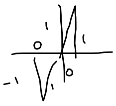
$\Rightarrow (\cos x)(2\sin^2 x - 1) = 0$

$\cos x = 0$ $\quad 2\sin^2 x - 1 = 0$

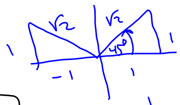
$2\sin^2 x = 1$

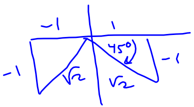
$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$



$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$





$\boxed{x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}}$

$\sin x = \frac{1}{\sqrt{2}}$

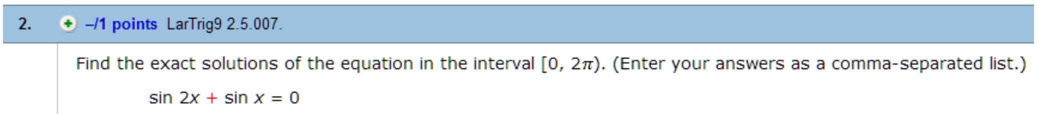
$\frac{\pi}{4}, \frac{3\pi}{4},$

$\sin x = -\frac{1}{\sqrt{2}}$

$\frac{5\pi}{4}, \frac{7\pi}{4}$

↑

$2\pi - \frac{\pi}{4} = \frac{(8-1)\pi}{4} = \frac{7\pi}{4}$



2. –/1 points LarTrig9 2.5.007.

Find the exact solutions of the equation in the interval [0, 2π). (Enter your answers as a comma-separated list.)

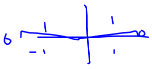
$\sin 2x + \sin x = 0$

$2\sin x\cos x + \sin x = 0$

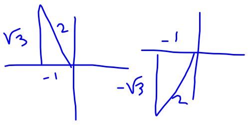
$(\sin x)(2\cos x + 1) = 0$

$\boxed{x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}}$

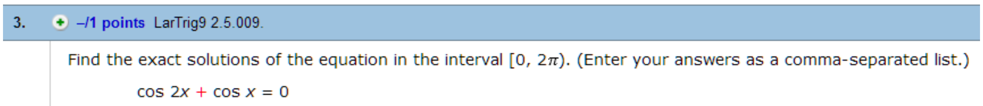
$\sin x = 0$ or $\cos x = -\frac{1}{2}$



$0, \pi$



$\frac{2\pi}{3}, \frac{4\pi}{3}$



3. –/1 points LarTrig9 2.5.009.

Find the exact solutions of the equation in the interval [0, 2π). (Enter your answers as a comma-separated list.)

$\cos 2x + \cos x = 0$

$\Rightarrow 2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1)$

$u = \frac{1}{2}$ or $u = -1$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

$a = 2, b = 1, c = -1$

$b^2 - 4ac = 1^2 - 4(2)(-1)$

$= 1 + 8 = 9$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{9}}{4}$

$= \frac{-1 \pm 3}{4} \nearrow \frac{2}{4} = \frac{1}{2}$, $\searrow \frac{-4}{4} = -1$

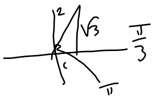
$u = \frac{1}{2}, -1$

$\cos x = \frac{1}{2}$ or $\cos x = 1$

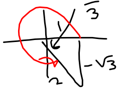
$2u^2 + u - 1$

$= 2(u - \frac{1}{2})(u+1)$

$= (2u-1)(u+1)$

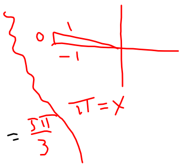


$\frac{\pi}{3}$



$\frac{\pi}{3}$

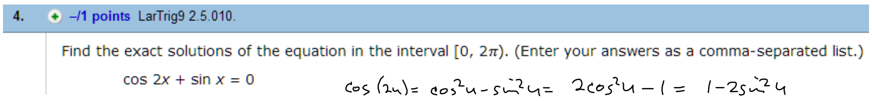
$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$



$\pi = x$

$\frac{\pi}{3}, \frac{5\pi}{3}$

$\boxed{x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}}$



4. –/1 points LarTrig9 2.5.010.

Find the exact solutions of the equation in the interval [0, 2π). (Enter your answers as a comma-separated list.)

$\cos 2x + \sin x = 0$

$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\Rightarrow 1 - 2\sin^2 x + \sin x = 0$

$-2\sin^2 x + \sin x + 1 = 0$

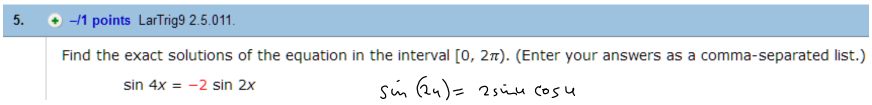
$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\quad \sin x = 1$

$\frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad \frac{\pi}{2}$

$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$



5. –/1 points LarTrig9 2.5.011.

Find the exact solutions of the equation in the interval [0, 2π). (Enter your answers as a comma-separated list.)

$\sin 4x = -2 \sin 2x$

$\sin(2u) = 2\sin u \cos u$

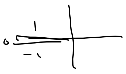
$2\sin(2x)\cos(2x) = -2\sin(2x)$

$\sin(2x)\cos(2x) + \sin(2x) = 0$

$\sin(2x)\left[\cos(2x) + 1\right] = 0$

$\sin 2x = 0$ or $\quad \cos(2x) + 1 = 0$

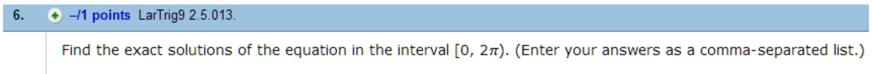
$\cos(2x) = -1$



$\boxed{x = 0, \frac{\pi}{2}}$

$2x = 0, 2x = \pi$ $\quad 2x = \pi$

$x = 0, \ x = \frac{\pi}{2}$ $\quad x = \frac{\pi}{2}$



6. –/1 points LarTrig9 2.5.013.

Find the exact solutions of the equation in the interval [0, 2π). (Enter your answers as a comma-separated list.)

$6 \tan 2x - 6 \cot x = 0$

$\tan(2x) - \cot(x) = 0$

$\Rightarrow \frac{2\tan x}{1 - \tan^2 x} - \cot x = 0$

$\frac{2\tan x}{1 - \tan^2 x} = \cot x$

$\frac{2\tan x}{\cot x} = 1 - \tan^2 x$

$2\tan^2 x = 1 - \tan^2 x$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

$\frac{2\frac{\sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}} - \frac{\cos x}{\sin x} = 0$

$\frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} - \frac{\cos x}{\sin x} = 0$

$\left(\frac{2\sin x}{\cos x}\right)\left(\frac{\cos^2 x}{\cos^2 x - \sin^2 x}\right) - \frac{\cos x}{\sin x} = 0$

$\Rightarrow \frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} - \frac{\cos x}{\sin x} = 0$ $\quad$ LCD $= \sin x \ (\cos^2 x - \sin^2 x)$

$\frac{(2\sin^2 x \cos x) - (\cos^3 x - \sin^2 x \cos x)}{\text{LCD}} = 0$

$\Rightarrow 2\sin^2 x \cos x - \cos^3 x + \sin^2 x \cos x = 0$

$3\sin^2 x \cos x - \cos^3 x = 0$

$(\cos x)(3\sin^2 x - \cos^2 x) = 0$

$3\sin^2 x - (1 - \sin^2 x) = 0$

$4\sin^2 x - 1 = 0$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

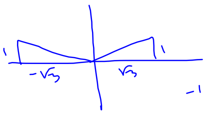
$\sin x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

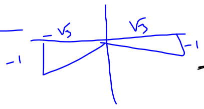
$\frac{2\tan x}{1 - \frac{\sin^2 x}{\cos^2 x}} - \cot x = 0$

$\frac{2\tan x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} - \cot x = 0$

$\left(2\frac{\sin x}{\cancel{\cos x}}\right)\left(\frac{\cos^2 x}{\cos^2 x - \sin^2 x}\right) - \cot x = 0$

$\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} - \frac{\cos x}{\sin x} = 0$





$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

METHOD ON LEFT is like:

$x^2 - 2x = 0$

No!

$x^2 = 2x$

$\boxed{x = 2}$

$x(x-2) = 0$ Yes

$\boxed{x = 0 \text{ OR } x = 2}$

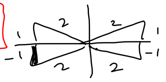
$\cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$

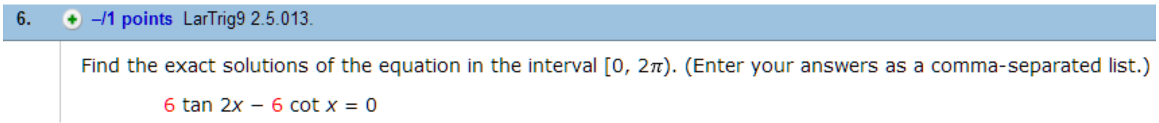
$\frac{3\pi}{2}$ $\quad \frac{11\pi}{6}$ $\quad \frac{7\pi}{6}$

$\frac{1}{2}\pi, -\frac{1}{2}\pi, \frac{1}{6}\pi, \frac{5}{6}\pi, -\frac{1}{6}\pi, -\frac{5}{6}\pi$

$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$



$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$



6. –/1 points LarTrig9 2.5.013.

Find the exact solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$6 \tan 2x - 6 \cot x = 0$

$$\tan 2x - \cot x = 0$$

$$\frac{2\tan x}{1-\tan^2 x} - \cot x = 0$$

$$\frac{2\tan x}{1-\tan^2 x} - \frac{(\cot x)(1-\tan^2 x)}{1-\tan^2 x} = 0$$

$$2\tan x - \cot x + \cot x \tan^2 x = 0$$

$$2\tan x - \cot x + \tan x = 0$$

$$3\tan x - \cot x = 0$$

$$\Rightarrow 3\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}$$

$$= \frac{3\sin^2 x - \cos^2 x}{\sin x \cos x} = 0$$

$$3\sin^2 x - (1-\sin^2 x) = 0$$

$$4\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\frac{1}{2}$$

Here's another stab at it. Not exactly sure why I lost the solutions corresponding to cos(*x*) = 0. But definitely confirmed at least *part* of the work done on the previous page.

$$x^2 + 5x = 0$$

$$x^2 = -5x \quad \text{No!}$$

$$x = -5$$

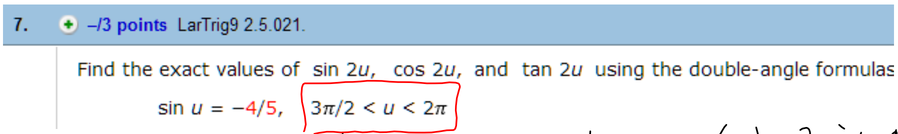
$$3\tan x = \cot x$$

$$\frac{3\tan x}{\cot x} = 1$$

$$3\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm\frac{1}{\sqrt{3}}$$

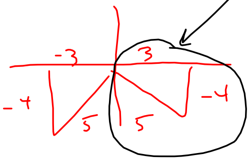
$$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$



7. –/3 points LarTrig9 2.5.021.

Find the exact values of sin 2u, cos 2u, and tan 2u using the double-angle formulas

sin u = −4/5, 3π/2 < u < 2π



$\sin u = -\frac{4}{5} \Rightarrow \sin(2u) = 2\sin u \cos u = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) =$

$= \boxed{\frac{-24}{25} = \sin(2u)}$

$\cos u = \frac{3}{5}$

$\tan u = -\frac{4}{3}$

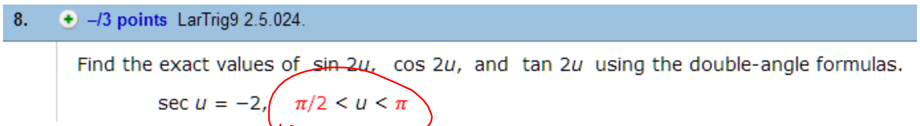
$\Rightarrow \cos(2u) = 2\cos^2 u - 1$

$= 2\left(\frac{3}{5}\right)^2 - 1 = \frac{18}{25} - \frac{25}{25} = \boxed{\frac{-7}{25} = \cos(2u)}$

$\Rightarrow \tan(2u) = \frac{2\tan u}{1-\tan^2 u}$

$= \frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2} = \frac{-\frac{8}{3}}{1-\frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}}$

$= -\frac{8}{3} \cdot -\frac{9}{7} = \boxed{\frac{24}{7} = \tan(2u)}$

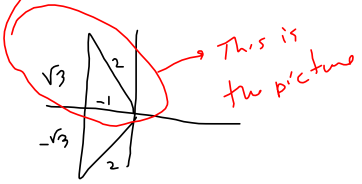


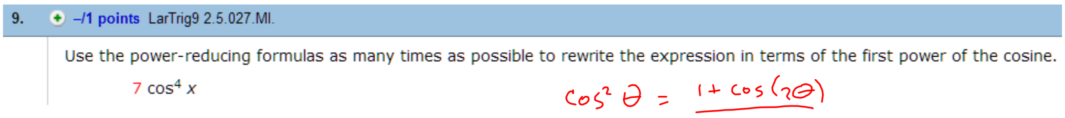
8. –/3 points LarTrig9 2.5.024.

Find the exact values of sin 2u, cos 2u, and tan 2u using the double-angle formulas.

sec u = −2, π/2 < u < π

$\sec u = -2$





9. –/1 points LarTrig9 2.5.027.MI.

Use the power-reducing formulas as many times as possible to rewrite the expression in terms of the first power of the cosine.

7 cos^4 x

$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$= 7(\cos^2 x)^2 = 7\left(\frac{1+\cos(2\theta)}{2}\right)^2$

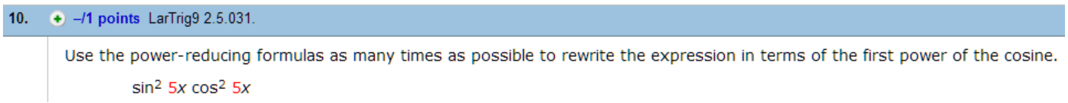
$(a+b)^2 = a^2+2ab+b^2, \quad (a-b)^2 = a^2-2ab+b^2$

$= \frac{7}{4}(\cos(2\theta)+1)^2 = \frac{7}{4}(\cos^2(2\theta)+2\cos(2\theta)+1)$

$= \frac{7}{4}\left(\frac{\cos(4\theta)+1}{2} + 2\cos(2\theta) + 1\right)$

$= \frac{7}{8}\cos(4\theta) + \frac{7}{8} + \frac{7}{2}\cos(2\theta) + \frac{7}{4})$

$= \boxed{\frac{7}{8}\cos(4\theta) + \frac{7}{2}\cos(2\theta) + \frac{21}{8}}$



10. –/1 points LarTrig9 2.5.031.

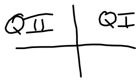
Use the power-reducing formulas as many times as possible to rewrite the expression in terms of the first power of the cosine.

$\sin^2 5x \cos^2 5x$

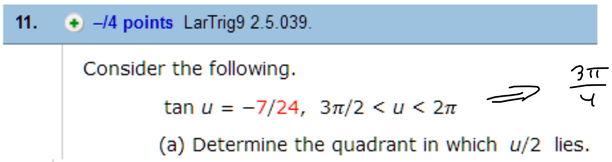
$$\left(\frac{1-\cos(10x)}{2}\right)\left(\frac{1+\cos(10x)}{2}\right) = \frac{1}{4}\left(1-\cos^2(10x)\right) = \frac{1}{4}-\frac{1}{4}\cos^2(10x)$$

$$= \frac{1}{4}-\frac{1}{4}\left(\frac{1+\cos(20x)}{2}\right) = \frac{1}{4}-\frac{1}{8}-\frac{1}{8}\cos(20x)$$

$$= \frac{1}{8}-\frac{1}{8}\cos(20x)$$



$\tan\frac{u}{2} < 0$



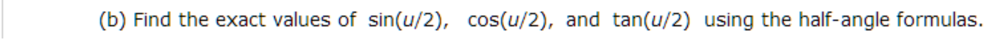
11. –/4 points LarTrig9 2.5.039.

Consider the following.

$\tan u = -7/24, \quad 3\pi/2 < u < 2\pi$ $\Rightarrow \frac{3\pi}{4} < \frac{u}{2} < \pi$

(a) Determine the quadrant in which $u/2$ lies. $\boxed{Q\ II}$ $\Rightarrow$ so, $\sin\frac{u}{2} > 0$

$\cos\frac{u}{2} < 0$



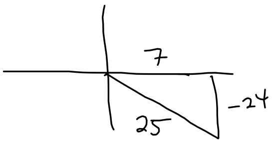
(b) Find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2} \Rightarrow \sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$$

$$= \sqrt{\frac{1-\cos u}{2}} \text{ in QII}$$

$$= \sqrt{\frac{1-\frac{7}{25}}{2}}$$

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$$



$49+576$

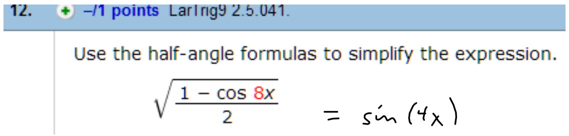
$= 625$

$\sqrt{625} = 25$

$$= \sqrt{\frac{\frac{25-7}{25}}{2}} = \sqrt{\frac{\frac{18}{25}}{2}} = \sqrt{\frac{18}{2\cdot 25}} = \sqrt{\frac{9}{25}} = \boxed{\frac{3}{5} = \sin\left(\frac{u}{2}\right)}$$

$$\cos\frac{u}{2} = \ldots = -\sqrt{\frac{\frac{25+7}{25}}{2}} = -\sqrt{\frac{32}{2\cdot 25}} = -\sqrt{\frac{16}{25}} = \boxed{-\frac{4}{5} = \cos\left(\frac{u}{2}\right)}$$

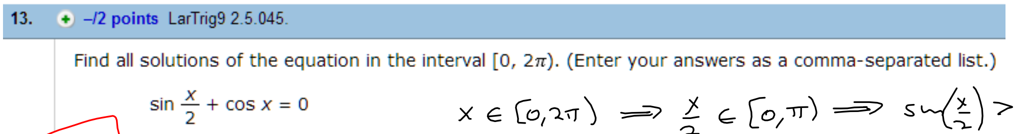
$$\tan\left(\frac{u}{2}\right) = \frac{\sin\frac{u}{2}}{\cos\frac{u}{2}} = \frac{\frac{3}{5}}{\frac{-4}{5}} = \frac{3}{5}\cdot\frac{-5}{4} = \boxed{-\frac{3}{4} = \tan\left(\frac{u}{2}\right)}$$



12. –/1 points LarTrig9 2.5.041.

Use the half-angle formulas to simplify the expression.

$$\sqrt{\frac{1-\cos 8x}{2}} = \sin(4x)$$



13. –/2 points LarTrig9 2.5.045.

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$\sin\frac{x}{2} + \cos x = 0$

$x \in [0,2\pi) \Rightarrow \frac{x}{2} \in [0,\pi) \Rightarrow \sin(\frac{x}{2}) > 0$

M1 $\Rightarrow \sqrt{\frac{1-\cos x}{2}} + \cos x = 0$

$\sqrt{\frac{1-\cos x}{2}} = -\cos x \Rightarrow x < 0$, i.e, QII OR QIII for x

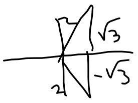
$\frac{1-\cos x}{2} = \cos^2 x$

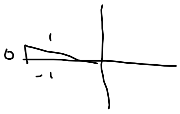
$1-\cos x = 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

$\Rightarrow (2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ OR $\cos x = -1$





Neither is in QII OR QIII

$\boxed{\pi = x}$

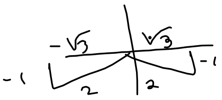
M2 $\sin\frac{x}{2} + (1-2\sin^2\frac{x}{2}) = 0$

$-2\sin^2\frac{x}{2} + \sin\frac{x}{2} + 1 = 0$

$2\sin^2(\frac{x}{2}) - \sin(\frac{x}{2}) - 1 = 0$

$(2\sin(\frac{x}{2})+1)(\sin(\frac{x}{2})-1) = 0$

$\sin(\frac{x}{2}) = -\frac{1}{2}$ OR $\sin(\frac{x}{2}) = 1$

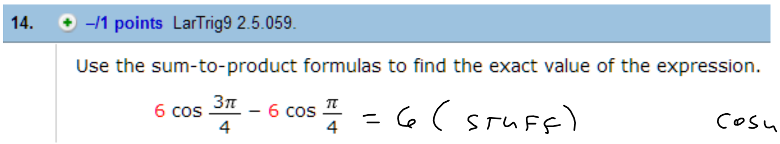


$\sin(\frac{x}{2}) = -\frac{1}{2}$

No way $\frac{x}{2} \in$ QI OR QII which would be true if $x \in [0,2\pi)$

$\frac{x}{2} = \frac{\pi}{2}$

$\boxed{x = \pi}$



14. –/1 points LarTrig9 2.5.059.

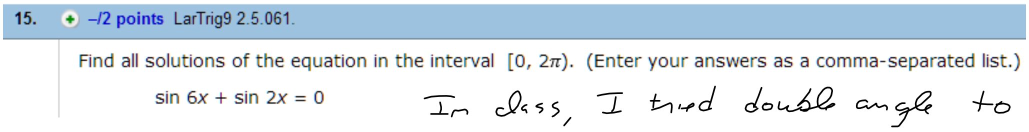
Use the sum-to-product formulas to find the exact value of the expression.

$6\cos\frac{3\pi}{4} - 6\cos\frac{\pi}{4} = 6(\text{STUFF})$

$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

$\Rightarrow \text{STUFF} = -2\sin(\pi)\sin\left(\frac{\pi}{2}\right)$

$\Rightarrow 6\,\text{STUFF} = \boxed{0}.$



15. –/2 points LarTrig9 2.5.061.

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$\sin 6x + \sin 2x = 0$

$6x = 4x + 2x$

In class, I tried double angle to no avail, after 2/3 pages. Let's try angle sum formula.

$\sin(4x)\underline{\cos(2x)} + \sin(2x)\cos(4x) + \sin(2x) = 0$

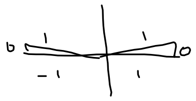
$2\sin(2x)\cos^2(2x) + \sin(2x)\left(2\cos^2(2x) - 1\right) + \sin(2x) = 0$

$\Rightarrow \left(\sin(2x)\right)\left(2\cos^2(2x) + 2\cos^2(2x) - 1 + 1\right) = 0$

$\Rightarrow \left(\sin(2x)\right)\left(4\cos^2(2x)\right) = 0$

$\sin(2x) = 0$ OR $4\cos^2(2x) = 0$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$



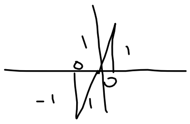
$\cos^2(2x) = 0$

$|\cos(2x)| = 0$

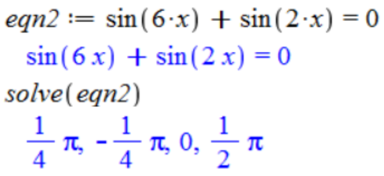
$\cos(2x) = 0$

$2x = 0, \pi, 3\pi$

$x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$



CAS obtains



$eqn2 := \sin(6\cdot x) + \sin(2\cdot x) = 0$

$\sin(6x) + \sin(2x) = 0$

$solve(eqn2)$

$\frac{1}{4}\pi, -\frac{1}{4}\pi, 0, \frac{1}{2}\pi$

$\downarrow$

$\frac{7\pi}{4}$

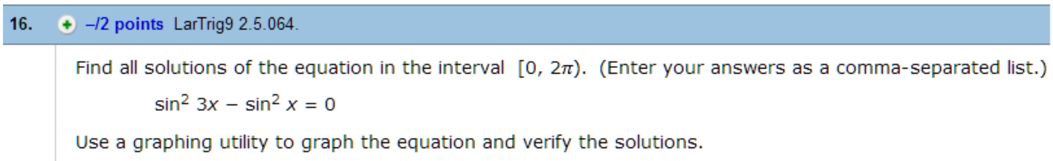
$\frac{3\pi}{2} + 2\pi = \frac{3\pi + 4\pi}{2}$

$= \frac{7\pi}{2}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$$\boxed{x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}}$$



16. ⊕ –/2 points LarTrig9 2.5.064.

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$$\sin^2 3x - \sin^2 x = 0$$

Use a graphing utility to graph the equation and verify the solutions.

$$\frac{1-\cos(6x)}{2} - \frac{1-\cos(2x)}{2} = 0$$

$$1-\cos(6x) - (1-\cos(2x)) = 0$$

$$1-\cos(6x) - 1 + \cos(2x) = 0$$

$$\cos(2x) - \cos(6x) = 0$$

$$\cos(2x) - [\cos(4x)\cos(2x) - \sin(4x)\sin(2x)]$$

$$= \cos(2x) - \cos(4x)\cos(2x) + \sin(4x)\sin(2x)$$

$$= \cos(2x) - \cos(4x)\cos(2x) + 2\cos(2x)\sin^2(2x)$$

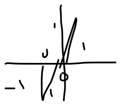
$$= \cos(2x)[1 - \cos(4x) + 2\sin^2(2x)]$$

$$= \cos(2x)[1-(1-2\sin^2(2x)) + 2\sin^2(2x)]$$

$$= \cos(2x)[1 - 1 + 2\sin^2(4x) + 2\sin^2(2x)]$$

$$= \cos(2x)[4\sin^2(2x)] = 0$$

$\cos(2x) = 0$ OR $4\sin^2(2x) = 0$



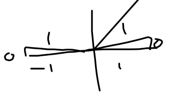
2x between 0 & 4π to capture all x between 0 & 2π

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\sin^2(2x) = 0$

$\sin(2x) = 0$



$2x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$