Sum and Difference Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$\left.\begin{aligned} \sin(-v) &= -\sin(v) \\ \cos(-v) &= \cos(v) \\ \tan(-v) &= -\tan(v) \end{aligned}\right\} \text{From symmetry (odd/even)}$$

$\sin(u-v) = \sin(u+(-v))$

$= \sin(u)\cos(-v) + \sin(-v)\cos(u)$

$= \sin u \cos v - \sin v \cos u$

1. Question Details

Fill in the blank.

$\cos(u + v) =$ $\cos u \cos v - \sin u \sin v$

2. Question Details

Fill in the blank.

$\tan(u + v) =$ $\dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

3. Question Details

Fill in the blank.

$\sin(u + v) =$ $\sin u \cos v + \sin v \cos u$

4. Question Details

Fill in the blank.

$\cos(u - v) =$ $\cos u \cos(-v) - \sin u \sin(-v) = \cos u \cos v + \sin u \sin v$

5. Question Details

Fill in the blank.

$\tan(u - v) =$ $\tan(u+(-v))$ $= \dfrac{\tan u + \tan(-v)}{1 - \tan u \tan(-v)} = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

6. Question Details

Find the exact value of each expression.

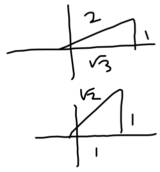
(a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

(b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{6} = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{3}}{2}}$

Totally Different!

$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

Rationalized denominator.



7. Question Details

Find the exact value of each expression.

(a) $\sin\left(\frac{19\pi}{6} - \frac{31\pi}{3}\right)$

(b) $\sin\left(\frac{19\pi}{6}\right) - \sin\left(\frac{31\pi}{3}\right)$

STUDENTS NEED TO KNOW THESE ARE DIFFERENT

(a) $\frac{19\pi}{6} = \frac{18\pi}{6} + \frac{1\pi}{6} = 3\pi + \frac{\pi}{6}$

$\frac{31\pi}{6} = \frac{30\pi}{6} + \frac{1\pi}{6} = 5\pi + \frac{\pi}{6}$

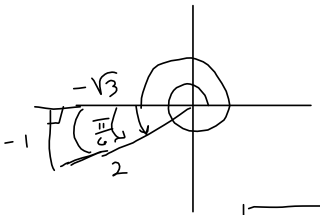
~~Going to be coterminal!~~ No!

So $\sin\left(\frac{19\pi}{6} - \frac{31\pi}{3}\right)$

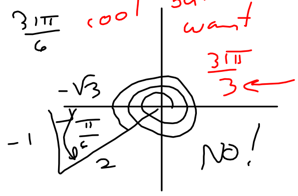
$\sin\frac{19\pi}{6}\cos\frac{31\pi}{3} - \sin\frac{31\pi}{3}\cos\frac{19\pi}{6}$

$= \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = \frac{-1+3}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

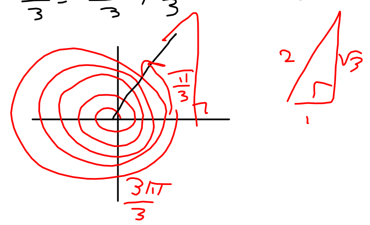
$\frac{19\pi}{6}$



$\frac{31\pi}{6}$ cool but want $\frac{31\pi}{3}$



$\frac{31\pi}{3} = \frac{30\pi}{3} + \frac{\pi}{3} = 10\pi + \frac{\pi}{3}$



(b) $-\frac{1}{2} - \frac{\sqrt{3}}{2} = \boxed{\frac{1-\sqrt{3}}{2}}$

8. Question Details

Find the exact value of each expression.

(a) $\sin(135° - 60°)$ $= \sin 135° \cos(-60°) + \sin(-60°)\cos(135°)$

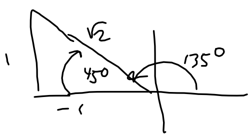
(b) $\sin 135° - \cos 60°$

$= \sin 135° \cos 60° - \sin 60° \cos 135°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{2}}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$

(b) $= \frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{2-\sqrt{2}}{2\sqrt{2}} = \frac{2\sqrt{2}-2}{4}$

$= \frac{\sqrt{2}-1}{2}$

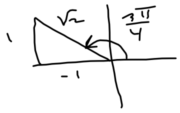


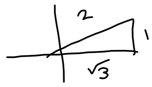
$180° - 135° = 45°$

9. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$\frac{11\pi}{12} = \frac{\pi}{4} + \frac{2\pi}{3}$ $\qquad \frac{11\pi}{12} = \frac{10\pi}{12} + \frac{1\pi}{12} = \frac{9\pi}{12} + \frac{2\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$





$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{3\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{2\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

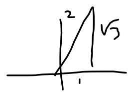
$\cos\left(\frac{11\pi}{12}\right) =$

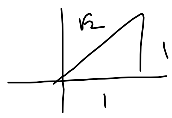
$\tan\left(\frac{11\pi}{12}\right) =$

10. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$ $\qquad 6+1, \; 5+2, \; 4+3$ each! $\qquad \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$





$\sin\left(\frac{7\pi}{12}\right) =$

$\cos\left(\frac{7\pi}{12}\right) =$

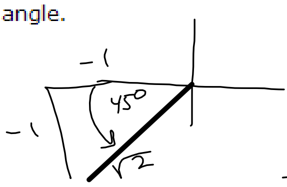
$\tan\left(\frac{7\pi}{12}\right) =$

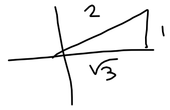
11. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$$\frac{17\pi}{12} = \frac{17\pi}{4} - \frac{17\pi}{6}$$

$\frac{15\pi}{12} + \frac{2\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$



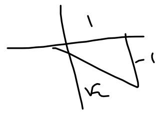


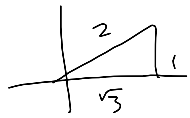
$\sin\left(\frac{17\pi}{12}\right) =$

$\cos\left(\frac{17\pi}{12}\right) =$

$\tan\left(\frac{17\pi}{12}\right) =$

12. Question Details





Find the exact values of the sine, cosine, and tangent of the angle.

$$-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$$

Tricking $\frac{\pi}{12}$ times...

$-1 = -2+1 = -3+2$

$-\frac{\pi}{12} = -\frac{3\pi}{12} + \frac{2\pi}{12} = -\frac{\pi}{4} + \frac{\pi}{6}$

$\sin\left(-\frac{\pi}{12}\right) =$

$\cos\left(-\frac{\pi}{12}\right) =$

$\tan\left(-\frac{\pi}{12}\right) =$

13. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$$105° = 60° + 45°$$

$105 = 100+5,\ 95+10,\ 90+15,\ 85+20,\ 80+25,\ 75+30,$
$70+35,\ 65+40,\ 60+45$ cool.

$\sin(105°) =$

$\cos(105°) =$

$\tan(105°) =$

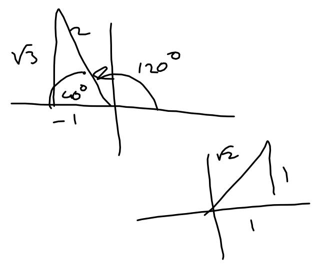
14. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$165° = 135° + 30°$

$165 - 45 = 120$

$165 = 120 + 45$



$\sin(165°) =$

$\cos(165°) =$

$\tan(165°) =$

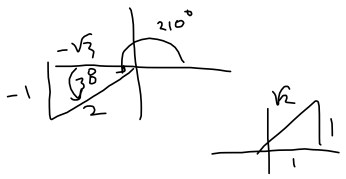
15. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$255° = 300° - 45°$

$255 - 45 = 210$

$255 = 210 + 45$



$\sin(255°) =$

$\cos(255°) =$

$\tan(255°) =$

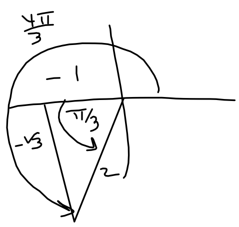
16. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$\frac{19\pi}{12}$

$17+2, \quad 16+3$

$\frac{16\pi}{12} + \frac{3\pi}{12} = \frac{4\pi}{3} + \frac{\pi}{4}$



$\sin\left(\frac{19\pi}{12}\right) =$

$\cos\left(\frac{19\pi}{12}\right) =$

$\tan\left(\frac{19\pi}{12}\right) =$

17. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$$-\frac{7\pi}{12}$$

$\sin\left(-\frac{7\pi}{12}\right) =$

$\cos\left(-\frac{7\pi}{12}\right) =$

$\tan\left(-\frac{7\pi}{12}\right) =$

18. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$$-\frac{13\pi}{12}$$

$\sin\left(-\frac{13\pi}{12}\right) =$

$\cos\left(-\frac{13\pi}{12}\right) =$

$\tan\left(-\frac{13\pi}{12}\right) =$

19. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

$$\frac{5\pi}{12}$$

$\sin\left(\frac{5\pi}{12}\right) =$

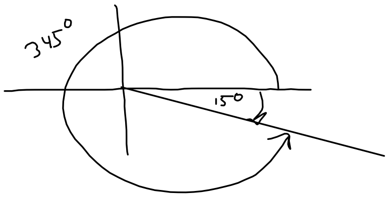
$\cos\left(\frac{5\pi}{12}\right) =$

$\tan\left(\frac{5\pi}{12}\right) =$

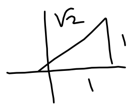
20. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

345°



$300° + 45°$



sin(345°) =

cos(345°) =

tan(345°) =

21. Question Details

Find the exact values of the sine, cosine, and tangent of the angle.

15° $= 45° - 30°$

sin(15°) =

cos(15°) =

tan(15°) =

22. Question Details

Write the expression as the sine, cosine, or tangent of an angle.

$\sin 3 \cos 1.3 - \cos 3 \sin 1.3 = \sin 3 \cos(-1.3) + \cos(3) \sin(-1.3) =$

$\sin(3 - 1.3) = \sin(1.7)$

23. Question Details

Write the expression as the sine, cosine, or tangent of an angle.

$\cos\left(\frac{5\pi}{7}\right) \cos\left(\frac{7\pi}{5}\right) - \sin\left(\frac{5\pi}{7}\right) \sin\left(\frac{7\pi}{5}\right) = \cos\left(\frac{5\pi}{7} + \frac{7\pi}{5}\right) = \cos\left(\frac{25\pi + 49\pi}{35}\right)$

$= \cos\left(\frac{74\pi}{35}\right)$

24. Question Details

Write the expression as the sine, cosine, or tangent of an angle.

$\cos 110° \cos 20° - \sin 110° \sin 20°$

25. Question Details

Write the expression as the sine, cosine, or tangent of an angle.

$$\frac{\tan 50° - \tan 15°}{1 + \tan 50° \tan 15°} = \tan(50° - 15°)$$

$$= \tan(35°)$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

26. Question Details

Find the exact value of the expression.

$$\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos\left(\frac{4\pi}{16}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

27. Question Details

Find the exact value of the expression.

sin 90° cos 45° − cos 90° sin 45°

28. Question Details

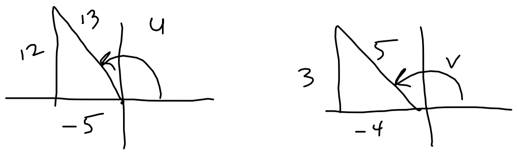
Find the exact value of the expression.

$$\frac{\tan(5\pi/6) - \tan(\pi/6)}{1 + \tan(5\pi/6)\tan(\pi/6)} = \tan\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \tan \frac{4\pi}{6} = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

29. Question Details LarTrig9 2.4.042.MI. [2447077]

Find the exact value of the trigonometric function given that sin u = 12/13 and cos v = -4/5. (Both u and v are in Quadrant II.)

$$\cos(u - v) = \cos u \cos v + \sin u \sin v = \left(-\frac{5}{13}\right)\left(-\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$$



$$= \frac{20 + 36}{65} = \boxed{\frac{56}{65}}$$

$169 - 144 = 25$

30. Question Details LarTrig9 2.4.043. [2446596]

Find the exact value of the trigonometric expression given that $\sin u = \frac{3}{5}$ and $\cos v = -\frac{8}{17}$. (Both u and v are in Quadrant II.)

tan(u + v)

31. Question Details LarTrig9 2.4.044. [2446573]

Find the exact value of the trigonometric function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II.)

$\csc(u - v)$

32. Question Details LarTrig9 2.4.046. [2446225]

Find the exact value of the trigonometric function given that $\sin u = \frac{3}{5}$ and $\cos v = -\frac{8}{17}$. (Both u and v are in Quadrant II.)

$\cot(u + v)$

33. Question Details LarTrig9 2.4.048. [2446406]

Find the exact value of the trigonometric function given that $\sin u = -\frac{3}{5}$ and $\cos v = -\frac{8}{17}$. (Both u and v are in Quadrant III.)

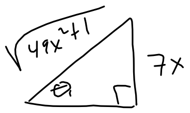
$\sin(u + v)$

34. Question Details

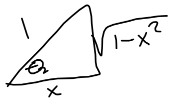
Write the trigonometric expression as an algebraic expression.

sin(arctan $7x$ − arccos x)

$$= \sin(\arctan(7x))\cos(\arccos(x)) - \sin(\arccos(x))\cos(\arctan(7x))$$



arctan (7x)



arccos(x)

$$= \frac{7x}{\sqrt{49x^2+1}} \cdot x - \sqrt{1-x^2} \cdot \frac{1}{\sqrt{49x^2+1}}$$

$$= \boxed{\frac{7x^2 - \sqrt{1-x^2}}{\sqrt{49x^2+1}}}$$

35. Question Details

Write the trigonometric expression as an algebraic expression.

cos(arccos x + arcsin x)

36. Question Details

Write the trigonometric expression as an algebraic expression.

cos(arccos $2x$ − arctan $4x$)

37. Question Details

Prove the identity. (Simplify at each step.)

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

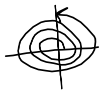
$$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(\boxed{\frac{\pi}{2}}\right)\cos(x) - \cos\left(\frac{\pi}{2}\right)\left(\boxed{\sin x}\right)$$

$$= \left(\boxed{1}\right)\cos(x) - (0)\left(\boxed{\sin x}\right)$$

$$= \cos x$$

38. Question Details

Prove the identity.



$\frac{13\pi}{2}$

$$\sin\left(\frac{13\pi}{2} + x\right) = \cos x$$

$$\sin\left(\frac{13\pi}{2} + x\right) = \sin\left(\frac{13\pi}{2}\right)\left(\boxed{}\right) + \cos\left(\frac{13\pi}{2}\right)\left(\boxed{}\right)$$

$$= (1)\left(\boxed{}\right) + (0)\left(\boxed{}\right)$$

$$= \cos x$$

39. Question Details

Prove the identity. (Simplify at each step.)

$$\sin\left(\frac{\pi}{4} + x\right) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$$

$$\sin\left(\frac{\pi}{4} + x\right) = \sin\left(\frac{\pi}{4}\right)\left(\boxed{}\right) + \cos\left(\frac{\pi}{4}\right)\sin(x)$$

$$= \frac{\sqrt{2}}{2}\left(\boxed{}\right) + \left(\boxed{}\right)\sin(x)$$

$$= \frac{\sqrt{2}}{2}(\cos x + \sin x)$$

40. Question Details LarT

Prove the identity. (Simplify at each step.)

$$\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$$

$$\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = \cos(\pi)\cos(\theta) + \sin(\pi)\sin(\theta) + \sin\left(\frac{\pi}{2}\right)\left(\boxed{}\right) + \cos\left(\frac{\pi}{2}\right)\sin(\theta)$$

$$= (-1)(\cos\theta) + \left(\boxed{}\right)(\sin\theta) + (1)\left(\boxed{}\right) + (\sin\theta)(0)$$

$$= -\cos\theta + \boxed{}$$

$$= 0$$

41. Question Details

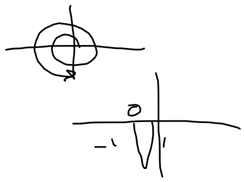
Simplify the expression algebraically and use a graphing utility to confirm your answer graphically.

$\sin\left(\frac{7\pi}{2} + \theta\right) = \sin\left(\frac{7\pi}{2}\right)\cos\theta + \sin\theta\cos\left(\frac{7\pi}{2}\right)$ $\quad \frac{7\pi}{2} = \frac{6\pi}{2} + \frac{\pi}{2} = 3\pi + \frac{\pi}{2}$

$\sin\left(\theta + \frac{7\pi}{2}\right) \quad = -\cos\theta + (\sin\theta)(0)$

$= -\cos\theta$

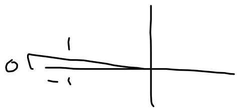
See video for graphed results.



42. Question Details

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$\sin(x + \pi) - \sin x + 1 = 0$



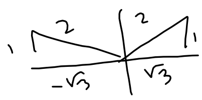
$\sin x \cos\pi + \sin\pi \cos x - \sin x + 1 = 0$

$-\sin x + 0 - \sin x + 1 = 0$

$-2\sin x + 1 = 0$

$-2\sin x = -1$

$\sin x = \frac{-1}{-2} = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

43. Question Details L

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

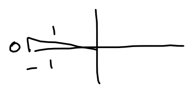
$$\cos\left(x + \frac{3\pi}{4}\right) - \cos\left(x - \frac{3\pi}{4}\right) = 1$$

44. Question Details

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$\tan(x + \pi) + 2\sin(x + \pi) = 0$ $\tan x + 2\sin x = 0$ is equivalent?

$$\frac{\tan x + \tan\pi}{1 - \tan x \tan\pi} + 2[\sin x \cos\pi + \sin\pi \cos x] = 0$$



$$\tan x + 2[-\sin x] = 0.$$

$$\Rightarrow \tan x - 2\sin x = 0$$

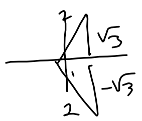
$$\Rightarrow \frac{\sin x}{\cos x} - \frac{2\sin x \cos x}{\cos x} = 0$$

$$\sin x - 2\sin x \cos x = 0$$

$$(\sin x)(1 - 2\cos x) = 0$$

$\sin x = 0$ $\cos x = \frac{1}{2}$

$x = 0, \pi$



$\frac{\pi}{3}, \frac{5\pi}{3}$

$360^\circ - \frac{\pi}{3}$

$= 360^\circ - 60^\circ$

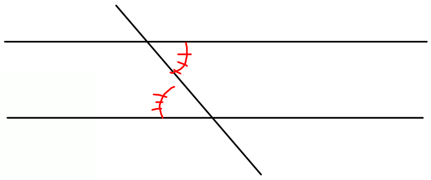
$= (300^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{5\pi}{3}$ rad

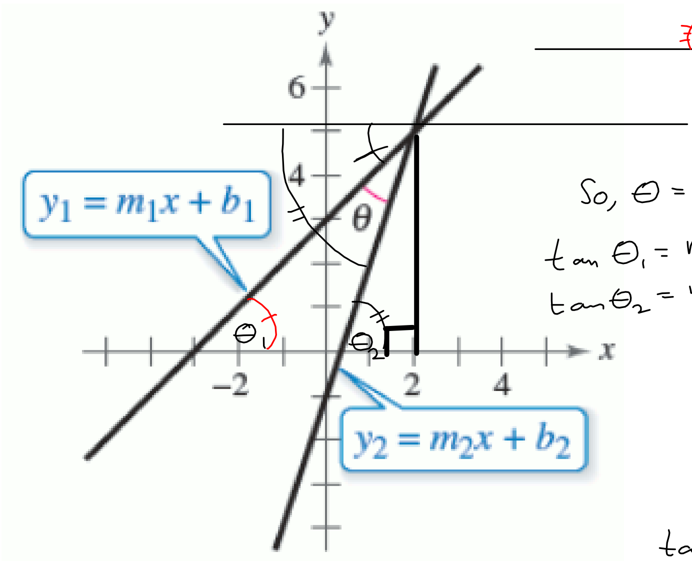
$$\boxed{x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}}$$

45. Question Details LarTrig9 2.4.098. [2595934]

Use the figure, which shows two lines whose equations are $y_1 = m_1x + b_1$ and $y_2 = m_2x + b_2$. Assume that both lines have positive slopes. Derive a formula for the angle between the two lines. Then use your formula to find the angle between the given pair of lines.

$y = x$ and $y = \frac{1}{\sqrt{3}}x$





So, $\theta = \theta_2 - \theta_1$!

$\tan\theta_1 = m_1$

$\tan\theta_2 = m_2$

tangent is ODD

want $\theta = \theta_2 - \theta_1$

$\tan\theta = \tan(\theta_2 - \theta_1) = \tan(\theta_2 + (-\theta_1))$

$$\frac{\tan\theta_2 + \tan(-\theta_1)}{1 - \tan\theta_2 \tan(-\theta_1)} = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1} = \tan\theta$$

$$\boxed{\theta = \arctan\left(\frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2\tan\theta_1}\right)}$$

& $\theta = \arctan(\tan\theta)$ if slopes are both positive, making angle is θ w/ $0 < \theta < \frac{\pi}{2}$

$y = x$ & $y = \frac{1}{\sqrt{3}}x$

$\tan\theta_2 = 1$

$\tan\theta_1 = \frac{1}{\sqrt{3}}$

So $\tan\theta = \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + (1)(\frac{1}{\sqrt{3}})} = \dfrac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \tan\theta$

$$\boxed{\arctan\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)} = \theta = 15^\circ = \frac{\pi}{12}$$