1. **Question Details**

Fill in the blank.

An equation that is true for all real values in its domain is called a(n) ---

IDENTITY!

2. **Question Details**

Fill in the blank.

An equation that is true for only some values in its domain is called a(n)

CONDITIONAL EQ'N.

3. **Question Details**

Fill in the blank to complete the fundamental trigonometric identity.

$$\frac{1}{\cot u} = \boxed{\tan(u)}$$

$$\frac{1}{\frac{\cos u}{\sin u}} = (1)\left(\frac{\sin u}{\cos u}\right) = \tan u$$

4. **Question Details**

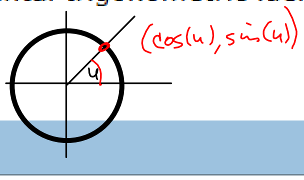
Fill in the blank to complete the fundamental trigonometric identity.

$$\frac{\cos u}{\sin u} = \boxed{\cot(u)}$$

5. Question Details

Fill in the blank to complete the fundamental trigonometric identity.

$\sin^2 u + \boxed{\cos^2 u} = 1$



6. Question Details

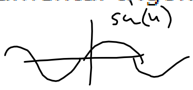
Fill in the blank to complete the fundamental trigonometric identity.

$\cos\left(\frac{\pi}{2} - u\right) = \boxed{\sin(u)}$

7. Question Details

Fill in the blank to complete the fundamental trigonometric identity.

$\csc(-u) = \boxed{-\csc(u)}$



$\sin(-u) = -\sin(u)$

$\csc(-u) = \frac{1}{\sin(-u)} = -\frac{1}{\sin(u)}$

$= -\csc(u)$

8. Question Details

Verify the identity. (Simplify at each step.)

$2\cot^2 y(\sec^2 y - 1) = 2$

$2\cot^2 y(\sec^2 y - 1) = 2\cot^2 y\left(\boxed{}\right) = 2$

$2\cot^2 y(\sec^2 y - 1) = 2\cot^2 y(\tan^2 y) = 2((\cot y)(\tan y))^2$

$= 2(1)^2 = 2$

9. Question Details

Verify the identity. (Simplify at each step.)

$8\cos^2\beta - 8\sin^2\beta = 8 - 16\sin^2\beta$

$8\cos^2\beta - 8\sin^2\beta = \left(8 - \boxed{\text{UGH!}}\right) - 8\sin^2\beta = 8 - 16\sin^2\beta$

$= 8(1-\sin^2\beta) - 8\sin^2\beta = 8 - 8\sin^2\beta - 8\sin^2\beta = \boxed{8 - 16\sin^2\beta}$

10. Question Details

Verify the identity. (Simplify at each step.)

$\dfrac{\tan^2\theta}{\sec\theta} = \sin\theta\tan\theta$

$= \dfrac{\frac{\sin^2\theta}{\cos^2\theta}}{\left(\frac{1}{\cos\theta}\right)} = \left(\dfrac{\sin^2\theta}{\cancel{\cos^2}\theta}\right)\left(\dfrac{\cancel{\cos\theta}}{1}\right) = \dfrac{\sin^2\theta}{\cos\theta} = \sin\theta \cdot \dfrac{\sin\theta}{\cos\theta} = \sin\theta\tan\theta$

11. Question Details

Verify the identity. (Simplify at each step.)

$\dfrac{\cot^2 t}{\csc t} = \dfrac{1 - \sin^2 t}{\sin t}$

$= \dfrac{\frac{\cos^2 t}{\sin^2 t}}{\left(\frac{1}{\sin t}\right)} = \left(\dfrac{\cos^2 t}{\sin^{\cancel{2}} t}\right)\left(\dfrac{\cancel{\sin(t)}}{1}\right) = \dfrac{\cos^2(t)}{\sin(t)} = \dfrac{1-\sin^2 t}{\sin t}$

12. Question Details

Verify the identity. (Simplify at each step.)

$\sin^{3/2} x\cos x - \sin^{7/2} x\cos x = \cos^3 x\sqrt{\sin^3 x}$

$\left(\sin^{\frac{3}{2}} x\right)(\cos x)\left[1 - \sin^2 x\right] = \left(\sin^{\frac{3}{2}} x\right)(\cos x)(\cos^2 x)$

$= (\cos^3 x)\left(\sqrt{\sin^3 x}\right)$

13. Question Details

Verify the identity. (Simplify at each step.)

$$\frac{3\sec\theta - 3}{1-\cos\theta} = 3\sec\theta$$

$(1-\cos\theta)(1+\cos\theta) = 1-\cos^2\theta = \sin^2\theta$

$(\sec\theta-1)(\sec\theta+1) = \sec^2\theta - 1 = \tan^2\theta$

$$\left(\frac{3(\sec\theta-1)}{(1-\cos\theta)}\right)\left(\frac{1+\cos\theta}{1+\cos\theta}\right) = \frac{3(\sec\theta-1)(\cos\theta+1)}{\sin^2\theta} = \frac{3(\sec\theta-1)(\sec\theta+1)(\cos\theta+1)}{(\sin^2\theta)(\sec\theta+1)}$$

$$= \frac{(3\tan^2\theta)(\cos\theta+1)}{(\sin^2\theta)(\sec\theta+1)} = \frac{\left(3\frac{\cancel{\sin^2\theta}}{\cos^2\theta}\right)(\cos\theta+1)}{(\cancel{\sin^2\theta})(\sec\theta+1)} = \frac{\frac{3}{\cos^2\theta}(\cos\theta+1)}{\sec\theta+1}$$

$$\frac{\frac{3}{\cos\theta}+3\sec^2\theta}{\sec\theta+1}$$

Meh. Try again!

WRITE MUCH.
THINK LITTLE

$$\frac{3(\sec\theta-1)(\cos\theta+1)}{\sin^2\theta} = \frac{3(1+\sec\theta-\cos\theta-1)}{\sin^2\theta} = \frac{3(\sec\theta-\cos\theta)}{\sin^2\theta}$$

$$= \frac{3(\sec\theta(1-\cos^2\theta))}{1-\cos^2\theta} = 3\sec\theta$$

Factored out $\sec\theta$

Factoring-out $\sec\theta$:

$$\sec\theta - \cos\theta = \sec\theta\left(\frac{\sec\theta}{\sec\theta} - \frac{\cos\theta}{\sec\theta}\right)$$

$$= \sec\theta\left(1 - \frac{\cos\theta}{\frac{1}{\cos\theta}}\right)$$

$$= \sec\theta\left(1-(\cos\theta)\left(\frac{\cos\theta}{1}\right)\right)$$

$$= \sec\theta(1-\cos^2\theta)$$

14. Question Details

Verify the identity. (Simplify at each step.)

$$\frac{8\cos\theta\cot\theta}{1-\sin\theta} - 8 = 8\csc\theta$$

$$= \frac{(8\cos\theta)\left(\frac{\cos\theta}{\sin\theta}\right)}{1-\sin\theta} - 8 \quad \frac{8(\cos^2\theta)(\csc\theta)}{1-\sin\theta} - 8 = \left(\frac{8(\cos^2\theta)(\csc\theta)}{1-\sin\theta}\right)\left(\frac{1+\sin\theta}{1+\sin\theta}\right) - 8$$

$$= \left(\frac{8\cos^2\theta\csc\theta}{\cos^2\theta}\right)(1+\sin\theta) - 8$$

$$= 8\csc\theta(1+\sin\theta) - 8 = 8\csc\theta + 8 - 8 = 8\csc\theta$$

15. Question Details

Verify the identity. (Simplify at each step.)

$$9\cos x - \frac{9\cos x}{1-\tan x} = \frac{9\sin x\cos x}{\sin x - \cos x}$$

$$\frac{9\cos x - 9\cos x\tan x - 9\cos x}{1-\tan x} = \frac{-9\cos x\left(\frac{\sin x}{\cos x}\right)}{1-\tan x} = \frac{9\sin x}{1-\tan x}$$

$$= \frac{9\sin x}{1-\frac{\sin x}{\cos x}} = \frac{9\sin x}{\frac{\cos x - \sin x}{\cos x}} = (9\sin x)\left(\frac{\cos x}{\cos x - \sin x}\right) = \frac{9\sin x\cos x}{\cos x - \sin x}$$

16. **Question Details**

Verify the identity.

$$\frac{3\tan x + 9\tan y}{1 - \tan x\tan y} = \frac{9\cot x + 3\cot y}{\cot x\cot y - 1}$$

$$= \frac{3\tan x + 9\tan y}{\frac{\cos x\cos y}{\cos x\cos y} - \frac{\sin x\sin y}{\cos x\cos y}} = \frac{3\tan x + 9\tan y}{\frac{\cos x\cos y - \sin x\sin y}{\cos x\cos y}}$$

$$= \left(3\frac{\sin x}{\cos x} + 9\frac{\sin y}{\cos y}\right)\left(\frac{\cos x\cos y}{\cos x\cos y - \sin x\sin y}\right)$$

$$\frac{(3\sin x\cos y + 9\sin y\cos x)}{(\cos x\cos y - \sin x\sin y)}\frac{\left(\frac{1}{\sin x\sin y}\right)}{\left(\frac{1}{\sin x\sin y}\right)} = \frac{3\cot y + 9\cot x}{\cot x\cot y - 1}$$

17. **Question Details**

Verify the identity. (Simplify at each step.)

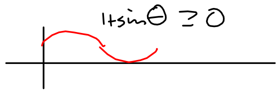
$$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} = \frac{1 + \sin\theta}{|\cos\theta|}$$

THINK: $\sqrt{x^2} = |x|$

$$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} = \frac{\sqrt{(1+\sin\theta)^2}}{\sqrt{(1-\sin^2\theta)}} = \frac{|1+\sin\theta|}{\sqrt{\cos^2\theta}} = \frac{|1+\sin\theta|}{|\cos\theta|}$$

$$= \frac{1+\sin\theta}{|\cos\theta|}$$

$1+\sin\theta \geq 0$



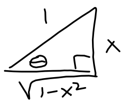
19. Question Details

Verify the identity.

$$5\cos(\sin^{-1} x) = \sqrt{25 - 25x^2}$$

Let $\theta = \sin^{-1} x \Rightarrow \sin\theta = x = \frac{x}{1}$. Thus,

$\theta = \sin^{-1}(x)$



$5\cos(\sin^{-1}(x))$

$= 5\left(\sqrt{1-x^2}\right)$

$= \sqrt{5^2}\sqrt{1-x^2}$

$= \sqrt{25(1-x^2)}$

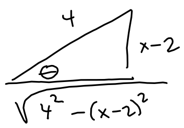
$= \sqrt{25-25x^2}$

20. Question Details

Verify the identity. (Simplify at each step.)

$$\tan\left(\sin^{-1}\frac{x-2}{4}\right) = \frac{x-2}{\sqrt{16-(x-2)^2}}$$

Let $\theta = \sin^{-1}\frac{x-2}{4} \Rightarrow \sin\theta = \frac{x-2}{4}$. Thus,



$\Rightarrow \tan\theta = \dfrac{x-2}{\sqrt{16-(x-2)^2}}$

21. Question Details

Consider the following equation.

$$(1 + \tan^2 x)(\sin^2 x) = \tan^2 x$$

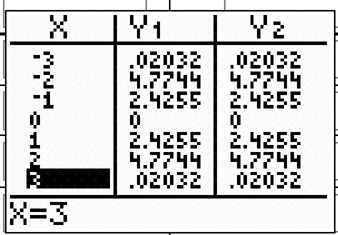
(a) Use a graphing utility to graph each side of the equation.

Determine whether the equation is an identity.

- ◯ Based on the graph, the equation is an identity.
- ◯ Based on the graph, the equation is not an identity.

(b) Use the *table* feature of a graphing utility. (Round each answer to three decimal places. If an answer does not exis enter DNE.)

x	Left-Hand Side	Right-Hand Side
−3		
−2		
−1		
0		
1		
2		
3		



X	Y1	Y2
-3	.02032	.02032
-2	4.7744	4.7744
-1	2.4255	2.4255
0	0	0
1	2.4255	2.4255
2	4.7744	4.7744
3	.02032	.02032

X=3

Determine whether the equation is an identity.

- ◯ Based on the table, the equation is an identity.
- ◯ Based on the table, the equation is not an identity.

(c) Confirm the results of parts (a) and (b) algebraically.

$(1 + \tan^2 x)(\sin^2 x)$ [?] $\tan^2 x$

$$(\sec^2 x)(\sin^2 x)$$

$$= \frac{1}{\cos^2 x}\sin^2 x = \frac{\sin^2 x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2 = \tan^2 x$$

22. Question Details

Consider the following equation.

$$\frac{3+3\cos x}{3\sin x} = \frac{3\sin x}{3-3\cos x} \quad = \quad 3$$

Factored & cancelled the redundant '3.'

```
Plot1 Plot2 Plot3
\Y1=(1+cos(X))/s
in(X)
\Y2=sin(X)/(1-co
s(X))
\Y3=
\Y4=
\Y5=
```

(a) Use a graphing utility to graph each side of the equation.

X	Y1	Y2
-3	-.0709	-.0709
-2	-.6421	-.6421
-1	-1.83	-1.83
0	ERR:	ERR:
1	1.8305	1.8305
2	.64209	.64209
3	.07091	.07091

X= -3

Determine whether the equation is an identity.

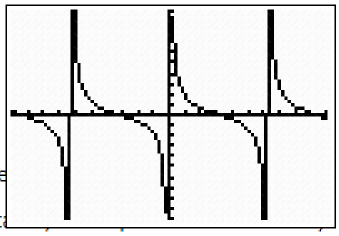
○ Based on the graph, the equation is an identity.

○ Based on the graph, the equation is not an identity.

) Use the *table* feature of a graphing utility. (Round each answer to three decimal places. If an answer does not exist, ter DNE.)

x	Left-Hand Side	Right-Hand Side
−3		
−2		
−1		
0		
1		
2		
3		

Looks like same graph



Determine whether the

○ Based on the t

○ Based on the table, the equation is not an identity.

(c) Confirm the results of parts (a) and (b) algebraically.

$$\frac{3+3\cos x}{3\sin x} \quad ? \quad \frac{3\sin x}{3-3\cos x}$$

$$= \left(\frac{3+3\cos x}{3\sin x}\right)\left(\frac{3-3\cos x}{3-3\cos x}\right) = \frac{9-9\cos^2 x}{(3\sin x)(3-3\cos x)} = \frac{9(1-\cos^2 x)}{9\cdot(\sin x)(1-\cos x)} = \frac{\sin^{\cancel{2}} x}{\cancel{(\sin x)}(1-\cos x)}$$

Conjugate thing

$$= \frac{\sin x}{1-\cos x} = \frac{3\sin x}{3-3\cos x}$$

23. **Question Details**

Verify the identity. (Simplify at each step.)

$$\tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$$

$$(\tan^2 x)(\tan^2 x) = (\tan^2 x)(\sec^2 x - 1)$$

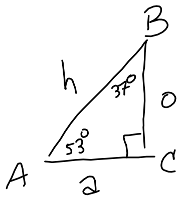
$$= (\tan^2 x)(\sec^2 x) - \tan^2 x$$

24. **Question Details**

Use the cofunction identities to evaluate the expression without the aid of a calculator.

$$\sin^2 53^\circ + \sin^2 37^\circ$$

complementary angles



$$\frac{o}{h} = \sin 53^\circ = \cos 37^\circ = \frac{o}{h} \;!$$

$$= \sin^2 53^\circ + \cos^2 53^\circ$$

$$= \cos^2 37^\circ + \sin^2 37^\circ$$

$$= 1$$

25. **Question Details**

Use the cofunction identities to evaluate the expression without using a calculator.

$$\tan^2 76^\circ + \cot^2 65^\circ - \sec^2 25^\circ - \csc^2 14^\circ$$

$$\cot^2 14^\circ - \csc^2 14^\circ + \cot^2 65^\circ - \csc^2 65^\circ$$

$$= \csc^2 14^\circ - 1 - \csc^2 14^\circ + \csc^2 65^\circ - 1 - \csc^2 65^\circ$$

$$= -2$$