1. Question Details

Factor the trigonometric expression. There is more than one correct form of the answer.

$$5\sin^2 x - 9\sin x - 2$$

$5u^2 - 9u - 2$

$a=5, b=-9, c=-2$

$b^2-4ac = (-9)^2 - 4(5)(-2)$

$= 81 + 40$

$= 121 = 11^2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{9 \pm \sqrt{121}}{2(5)} = \dfrac{9\pm 11}{10}$ → $\frac{20}{10} = 2$, $\frac{-2}{10} = -\frac{1}{5}$

FACTORS

$5(x-2)(x-(-\frac{1}{5}))$

$= (x-2)(5)(x+\frac{1}{5})$

$= (x-2)(5x+1)$

$5u^2 - 9u - 2$

$(5)(-2) = -10$

$-9 = -10+1$ $\quad -10$

$5u^2 - 10u + 1u - 2$

$= 5u(u-2) + 1(u-2)$

$= (u-2)(5u+1)$

$= (\sin x - 2)(5\sin x + 1)$

2. Question Details

Factor the trigonometric expression. There is more than one correct form of the answer.

$$9\cos^2 x + 12\cos x - 5$$

$= (3\cos x + 5)(3\cos x - 1)$

$2\,\underline{|324}$
$2\,\underline{|162}$
$\;\;81$

$a=9, b=12, c=-5$

$b^2-4ac = 12^2 - 4(9)(-5)$

$= 144 + 180$

$= 324 = 18^2$

$x = \dfrac{-12 \pm 18}{18} = \dfrac{-2\pm 3}{3}$ → $\frac{1}{3}$, $\frac{-5}{3}$

$9(u-\frac{1}{3})(u+\frac{5}{3})$

$= (3u-1)(3u+5)$

$(9)(-5) = -45$

$12 = 13-1$ $\quad -13$

$= 14-2$ $\quad -28$

$= 15-3$ $\quad -45$ Sweet!

$9u^2 + 15u - 3u - 5$

$= 3u(3u+5) - 1(3u+5)$

$= (3u+5)(3u-1)$

3. **Question Details**

Factor the trigonometric expression. There is more than one correct form of the answer.

$\cot^2 x + \csc x - 11$

$\csc^2(x) - 1 + \csc(x) - 11$

$= \csc^2(x) + \csc(x) - 12$

$(\csc(x) - 3)(\csc(x) + 4)$

4. **Question Details** LarTrig9 2.1.034. [2446883]

Perform the multiplication and use the fundamental identities to simplify. There is more than one correct form of the answer.

$(4 \csc x + 4)(4 \csc x - 4)$

$16\csc^2(x) - 16 = 16(\csc^2(x) - 1) = 16\tan^2(x)$.

5. Question Details LarTrig9

Use the fundamental identities to simplify the expression. There is more than one correct form of the answer.

$3\tan(-x)\cos x$

$$= -3\tan(x)\cos(x) = -3\frac{\sin(x)}{\cancel{\cos(x)}}\cancel{\cos(x)} = -3\sin(x)$$

6. Question Details LarTrig9 2

Use the fundamental identities to simplify the expression. There is more than one correct form of the answer.

$$\frac{1-\cos^2 x}{\sec^2 x - 1} = \frac{\sin^2 x}{\tan^2 x} = \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} = \left(\cancel{\sin^2 x}\right)\left(\frac{\cos^2 x}{\cancel{\sin^2 x}}\right) = \cos^2 x$$

HARD WAY

$$\frac{(1-\cos x)(1+\cos x)}{(\sec x-1)(\sec x+1)} = \frac{(1-\cos x)(1+\cos x)}{\left(\frac{1}{\cos x}-1\right)\left(\frac{1}{\cos x}+1\right)} = \frac{(1-\cos x)(1+\cos x)}{\left(\frac{1-\cos x}{\cos x}\right)\left(\frac{1+\cos x}{\cos x}\right)}$$

$$= \frac{(1-\cos x)(1+\cos x)}{\frac{(1-\cos x)(1+\cos x)}{\cos^2 x}} = \frac{(1-\cancel{\cos x})\cancel{(1+\cos x)}}{1}\cdot\frac{\cos^2 x}{(1-\cancel{\cos x})\cancel{(1+\cos x)}} = \cos^2 x$$

Scratch

$$\frac{1}{\cos x} - 1 = \frac{1}{\cos x} - \frac{\cos x}{\cos x} = \frac{1-\cos x}{\cos x}$$

7. Question Details LarTrig9 2

Use the fundamental identities to simplify the expression. There is more than one correct form of the answer.

$\cos\left(\frac{\pi}{2} - x\right)\sec x = \sin x \sec x$ cofunction identity.

$= \sin x\left(\frac{1}{\cos x}\right) = \tan x$ reciprocal identity

8. Question Details LarTrig9 2.1.047. [254820

Perform the subtraction and use the fundamental identities to simplify. There is more than one correct form of the answer.

$\cot x - \frac{\csc^2 x}{\cot x} = (\cot x)\left(\frac{\cot x}{\cot x}\right) - \frac{\csc^2 x}{\cot x} = \frac{\cot^2 x - \csc^2 x}{\cot x}$

$= \frac{\cot^2 x - (\cot^2 x + 1)}{\cot x} = \frac{\cot^2 x - \cot^2 x - 1}{\cot x} = -\frac{1}{\cot x} = \boxed{-\tan x}$

9. Question Details LarTrig9 2.1.048. [254

Perform the addition and use the fundamental identities to simplify. There is more than one correct form of the answer.

$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

$\left(\frac{\cos x}{1+\sin x}\right)\left(\frac{\cos x}{\cos x}\right) + \left(\frac{1+\sin x}{\cos x}\right)\left(\frac{1+\sin x}{1+\sin x}\right) = \frac{(\cos^2 x) + (1+\sin x)^2}{(\cos x)(1+\sin x)}$

$= \frac{1 - \sin^2 x + \sin^2 x + 2\sin x + 1}{LCD} = \frac{2\sin x + 2}{LCD} = \frac{2\cancel{(\sin x + 1)}}{(\cos x)\cancel{(1+\sin x)}} = \boxed{\frac{2}{\cos x} = 2\sec x}$

$(1+\sin x)^2 = (\sin x + 1)^2 = \sin^2 x + 2\sin x + 1$

10. Question Details LarTrig9

Rewrite the expression so that it is not fractional form. There is more than one correct form of the answer.

$$\frac{\sin^2 y}{1-\cos y} = \frac{1-\cos^2(y)}{1-\cos(y)} = \frac{\cancel{(1-\cos(y))}(1+\cos(y))}{\cancel{1-\cos(y)}} = 1+\cos(y) = \cos(y)+1$$

11. Question Details LarTrig9

Rewrite the expression so that it is not in fractional form. There is more than one correct form of the answer.

METHOD 1

$$\frac{9}{\tan x + \sec x} = \frac{9}{\frac{\sin x}{\cos x}+\frac{1}{\cos x}} = \frac{9}{\frac{\sin x + 1}{\cos x}} = \frac{9\cos x}{\sin x + 1}$$

$$= \left(\frac{9\cos x}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{(9\cos x)(1-\sin x)}{1-\sin^2 x} = \frac{(9\cos x)\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}(1+\sin x)}$$

$$= \cancel{\frac{9\cos x}{1+\sin x}} = \frac{(9\cos x)(1-\sin x)}{\cos^2 x} = \frac{9(1-\sin x)}{\cos x}$$

$$= 9\left[\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right] = \boxed{9[\sec x - \tan x]}$$

$$= 9\sec x - 9\tan x$$

METHOD 2

$$\frac{9}{\tan x + \sec x} = \left(\frac{9}{\tan x + \sec x}\right)\left(\frac{\tan x - \sec x}{\tan - \sec x}\right) = \frac{9(\tan x - \sec x)}{\tan^2 x - \sec^2 x}$$

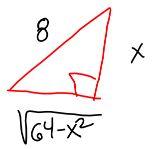
$$= \frac{9(\tan x - \sec x)}{\sec^2 x - 1 - \sec^2 x} = \frac{9(\tan x - \sec x)}{-1} = \boxed{9\sec x - 9\tan x}$$

12. Question Details LarTrig9 2.1.054. [25481

Use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

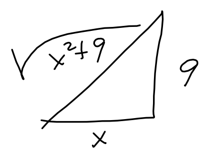
$\sqrt{64 - x^2}$, $x = 8 \sin\theta$ → so, i.e., $\frac{x}{8} = \sin\theta$

won't tell you this in Calculus, maybe



$$\sqrt{64-x^2} = \sqrt{64 - (8\sin\theta)^2}$$
$$= \sqrt{64 - 64\sin^2\theta}$$
$$= \sqrt{64(1-\sin^2\theta)}$$
$$= \sqrt{64}\sqrt{1-\sin^2\theta}$$
$$= 8\sqrt{\cos^2\theta}$$
$$= 8|\cos\theta|$$

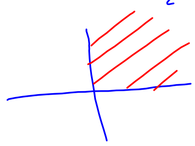
$\sqrt{x^2+9}$



$\sqrt{u^2} = |u|$

$(\sqrt{u})^2 = u$, because $u \geq 0$ for $\sqrt{u}$ to be defined

Now, $0 < \theta < \frac{\pi}{2}$ QI



where $\cos\theta > 0$, so

$|\cos\theta| = +\cos\theta$

$= \boxed{8\cos\theta}$, b/c $0 < \theta < \frac{\pi}{2}$ makes $\cos\theta$ positive.

Piecewise Definition of $|w|$ is

$$|w| = \begin{cases} w & \text{if } w \geq 0 \\ -w & \text{if } w < 0 \end{cases}$$

13. Question Details — LarTrig9 2.1.055. [25480

Use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

$\sqrt{x^2 - 16}$, $x = 4\sec\theta$

$$\sqrt{(4\sec\theta)^2 - 16} = \sqrt{16\sec^2\theta - 16} = \sqrt{16(\sec^2\theta - 1)}$$

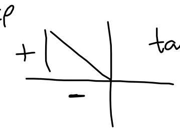
$$= 4\sqrt{\tan^2\theta} = 4|\tan\theta|$$

$\&\ 0 < \theta < \frac{\pi}{2} \Rightarrow \tan\theta > 0$

$\Rightarrow 4|\tan\theta| = 4\tan\theta$

If $\frac{\pi}{2} < \theta < \pi$?

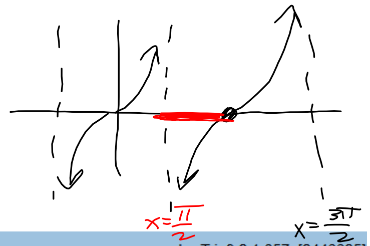
Reference Triangle



$\tan\theta$ negative!

Then $|\tan\theta| = -\tan\theta$

OR



14. Question Details — LarTrig9 2.1.057. [2446625]

Use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$.

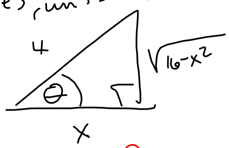
$4 = \sqrt{16 - x^2}$, $x = 4\sin\theta$

2 choices, until this makes it one choice.

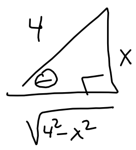
Need this when we get $\sqrt{\cos^2\theta}$

$= |\cos\theta| =$

$\begin{cases} = \cos\theta & \text{if } \cos\theta \geq 0 \\ = -\cos\theta & \text{if } \cos\theta < 0 \end{cases}$



OR



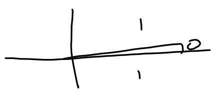
is pic for $x = 4\cos\theta$

is the pic when $x = 4\sin\theta$ is given

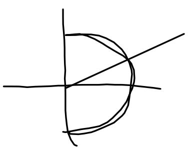
$$4 = \sqrt{16 - (4\sin\theta)^2}$$
$$= \sqrt{16 - 16\sin^2\theta}$$
$$= \sqrt{16(1 - \sin^2\theta)}$$
$$= \sqrt{16}\sqrt{1 - \sin^2\theta}$$
$$= 4\sqrt{\cos^2\theta}$$
$$= 4|\cos\theta| = 4\cos\theta \text{ if } \boxed{-\frac{\pi}{2} < \theta < \frac{\pi}{2}}$$

$4 = 4\cos\theta$

$\boxed{1 = \cos\theta}$



If $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, then $-\cos\theta = 1$



15. Question Details LarTrig9 2.1.058. [2548116

Use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$.

$-3\sqrt{3} = \sqrt{36 - x^2}, \quad x = 6\cos\theta$

Same Game

$$-3\sqrt{3} = \sqrt{36 - 36\cos^2\theta}$$
$$= \sqrt{36}\sqrt{1-\cos^2\theta}$$
$$\vdots$$
$$= \sqrt{36}\,|\sin\theta|$$

which is totally impossible!

$$\underbrace{-3\sqrt{3}}_{<0} = \underbrace{6|\sin\theta|}_{\geq 0}$$

Never agree!

16. **Question Details**

Use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.

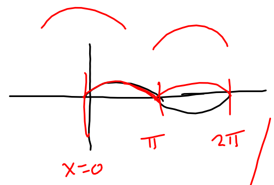
$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

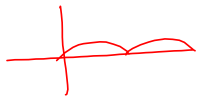
Meh. Use your brain!

$\sqrt{1-\cos^2\theta} = \sqrt{\sin^2\theta} = |\sin\theta|$

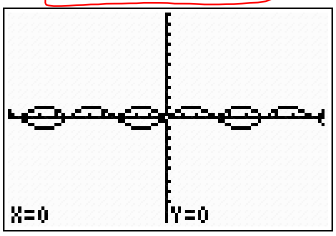
|sin(x)|

- $\frac{\pi}{2} \le \theta \le \pi$
- $0 \le \theta \le \frac{\pi}{2}$
- $\pi \le \theta \le 2\pi$
- $0 \le \theta \le \pi$
- $0 \le \theta \le 2\pi$





$\boxed{x \in [0,\pi]}$



17. **Question Details**

Use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.

$$\sec\theta = \sqrt{1 + \tan^2\theta}$$

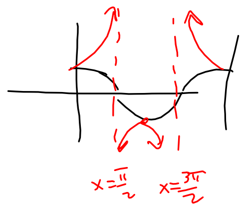
Same as #16, just be aware of sign changes.

- $\frac{3\pi}{2} < \theta < 2\pi$
- $0 \le \theta < \frac{\pi}{2}, \frac{3\pi}{2} < \theta < 2\pi$
- $0 \le \theta < \pi$
- $0 \le \theta < \pi, \pi < \theta < 2\pi$
- $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = |\sec\theta|$

So, when does $\sec\theta = |\sec\theta|$?

Whenever $\sec\theta \ge 0$.



So, looks like: $[0,\frac{\pi}{2})\cup(\frac{3\pi}{2},2\pi)$

18. **Question Details** LarTrig9 2.1.061. [244710

The forces acting on an object weighing W units on an inclined plane positioned at an angle of θ with the horizontal (see figure) are modeled by

$$\mu W \cos\theta = W \sin\theta \implies \mu = \frac{W\sin\theta}{W\cos\theta} = \boxed{\tan\theta}$$

where μ is the coefficient of friction. Solve the equation for μ and simplify the result.