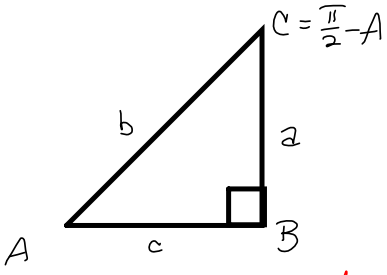
Section 2.1 -Trigonometric Identities

1. Cofunction Identities
2. Pythagorean Identities
3. (Reciprocal Identities)

Trig Substitution & Trig Integrals in Calculus.



$\sin A = \frac{a}{b} = \cos C = \cos\left(\frac{\pi}{2} - A\right)$

The source of Cofunction identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ → Remember!

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\sec\left(\frac{\pi}{2} - \theta\right) = \text{cosecant}\,\theta = \csc\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

Remember the two & Remember the idea.

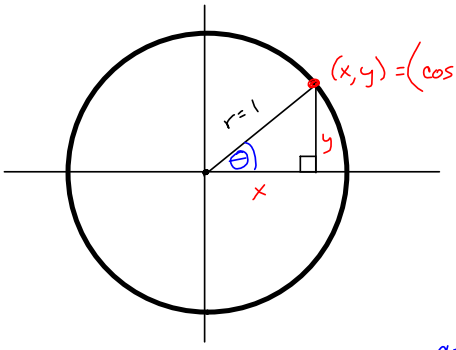
Section 2.1 -Trigonometric Identities

1. Cofunction Identities ✓
2. Pythagorean Identities
3. (Reciprocal Identities)

$\cos^2\theta$ means $(\cos\theta)^2$ (which is why I prefer $\arccos(x)$ over $\cos^{-1}(x)$)

$x^2+y^2=1$

$\sin^2\theta+\cos^2\theta=1$



$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta} = \boxed{\sec^2\theta - 1 = \tan^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

by same kinds of moves.

$\int \cot^4\theta \csc^2\theta \, d\theta$ in Calc II

Section 2.1 -Trigonometric Identities

1. Cofunction Identities ✓
2. Pythagorean Identities ✓
3. (Reciprocal Identities)

$\sin\theta = \frac{1}{\csc\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

1. –/1 points LarTrig9 2.1.001.

Fill in the blank to complete the trigonometric identity.

$$\frac{\sin(u)}{\cos(u)} = \boxed{\tan(u)}$$

$\tan u = \frac{y}{x} = \frac{y}{r} \cdot \frac{r}{x} = \sin(u) \cdot \sec(u)$

$= \frac{\sin(u)}{\cos(u)}$

2. –/1 points LarTrig9 2.1.002.

Fill in the blank to complete the trigonometric identity.

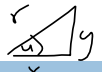
$$\frac{1}{\sec u} = \boxed{\cos(u)}$$

3. –/1 points LarTrig9 2.1.003.

Fill in the blank to complete the trigonometric identity.

$$\frac{1}{\cot u} = \boxed{\tan(u)}$$

$\cot(u) = \frac{x}{y} = \frac{1}{\tan(u)}$



4. –/1 points LarTrig9 2.1.004.

Fill in the blank to complete the trigonometric identity.

$$\sec\left(\frac{\pi}{2} - u\right) = \boxed{\csc(u)}$$

5. –/1 points LarTrig9 2.1.005.

Fill in the blank to complete the trigonometric identity.

$$1 + \boxed{\cot^2\theta} = \csc^2 u$$

Cheat-sheet-probable

KNOW

$\sin^2\theta + \cos^2\theta = 1$

6. **–/1 points** LarTrig9 2.1.006.

Fill in the blank to complete the trigonometric identity.

$\cot(-u) =$ $-\cot(u)$

Sine is odd — $\sin(-x) = -\sin(x)$

Cosine is even — $\cos(-x) = \cos(x)$

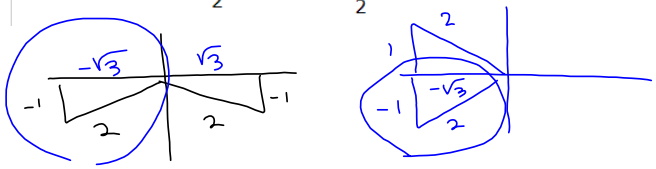
$\frac{\sin(-u)}{\cos(-u)} = \frac{-\sin(u)}{\cos(u)} =$

$\frac{(\text{odd})(\text{even})(\text{odd})(\text{odd})}{(\text{even})(\text{odd})(\text{odd})(\text{even})} \rightarrow \frac{(-)(+)(-)(-)}{(+)(-)(-)(+)} = \frac{-}{+} = -$ is odd.

7. **–/6 points** LarTrig9 2.1.007.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.

$\sin x = -\frac{1}{2}, \cos x = -\frac{\sqrt{3}}{2}$



$\tan(x) = \frac{1}{\sqrt{3}}$

$\cot(x) = \sqrt{3}$

$\sec(x) = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

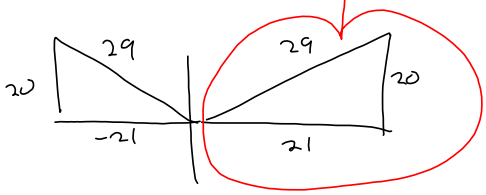
$\csc(x) = -2$

8. **–/6 points** LarTrig9 2.1.008.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.)

$\csc\theta = \frac{29}{20}, \tan\theta = \frac{20}{21}$

$\sin\theta = \frac{20}{29}$



$29^2 - 20^2 = 841 - 400 = 441$

$\sqrt{441} = 21$

3 | 441
3 | 147
7 | 49
7

$\sin\theta = \frac{20}{29}$

$\cos\theta = \frac{21}{29}$ $\sec\theta = \frac{29}{21}$

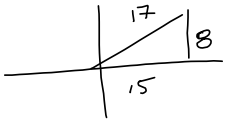
$\cot\theta = \frac{21}{20}$

9. –/6 points LarTrig9 2.1.009.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.

$\cos\left(\frac{\pi}{2} - x\right) = \frac{8}{17}$, $\cos x = \frac{15}{17}$

$\sin(x) = \frac{8}{17}$



$\tan(x) = \frac{8}{15}$

$\cot(x) = \frac{15}{8}$

$\sec(x) = \frac{17}{15}$

$\csc(x) = \frac{17}{8}$

10. –/6 points LarTrig9 2.1.010.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINE

$\sin(-x) = -\frac{1}{3}$, $\tan x = -\frac{\sqrt{2}}{4}$ $= -\frac{\sqrt{2}}{4} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2}{4\sqrt{2}} = -\frac{1}{2\sqrt{2}}$

$\sin(x) = \frac{1}{3}$

$\cos(x) = -\frac{2\sqrt{2}}{3}$

$\cot(x) = -2\sqrt{2}$

$\sec(x) = -\frac{3}{2\sqrt{2}} = -\frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$

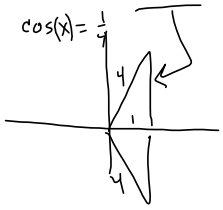
GOOD BOOK

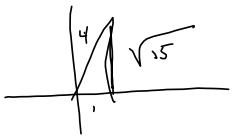
$\csc(x) = -3$

11. –/6 points LarTrig9 2.1.011.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.)

sec $x = 4$, sin $x > 0$





$\sin x = \frac{\sqrt{15}}{4}$ $\quad \csc x = \frac{4}{\sqrt{15}}$ $\quad \left(\frac{4\sqrt{15}}{15}\right)$

$\cos x = \frac{1}{4}$ $\quad \sec x = 4$

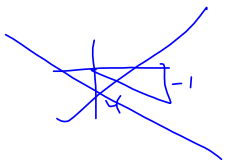
$\tan x = \frac{\sqrt{15}}{1}$ $\quad \cot x = \frac{1}{\sqrt{15}}$

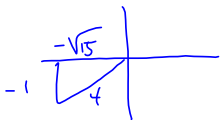
12. –/6 points LarTrig9 2.1.012.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.

csc $\theta = -4$, cos $\theta < 0$ → Left half → Bottom Left!

$\sin\theta = -\frac{1}{4}$ → Bottom half

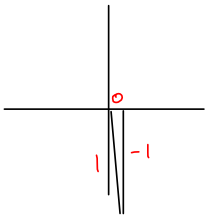




13. –/6 points LarTrig9 2.1.013.

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.)

sin $\theta = -1$, cot $\theta = 0$ → Duh.



$\sin\theta = -1$ $\quad \csc\theta = -1$

$\cos\theta = \frac{0}{1} = 0$ $\quad \sec\theta = \frac{1}{0}$ ∄

$\tan\theta = \frac{1}{0}$ ∄ $\quad \cos\theta = 0 = \frac{0}{1}$

14. –/1 points LarTrig9 2.1.015.

Match the trigonometric expression with one of the following.

$\sec x \cos x = \frac{1}{\cos x} \cdot \cos x = 1$

oops! Didn't copy the choices!

15. –/1 points LarTrig9 2.1.016.

Match the trigonometric expression with one of the following.

$\cot^2 x - \csc^2 x = \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} = \frac{\cos^2 x - 1}{\sin^2 x} = -\frac{1 - \cos^2 x}{\sin^2 x}$

$= -\frac{\sin^2 x}{\sin^2 x} = -1$.

- ○ sin x tan x
- ○ csc x
- ○ $\sec^2 x + \tan^2 x$
- ○ $\sec^2 x$
- ○ 1
- ○ −1

OR

$\cot^2 x - \csc^2 x = (\csc^2 x - 1) - \csc^2 x = -1$

$= \cot^2 x - (\cot^2 x + 1) = -1$

$\tan^2 x$ . . . . $\sec^2 x - 1$

$\cot^2 x$ . . . . $\csc^2 x - 1$

Start with these if you have to derive the "harder" Pythagorean identities.

16. –/1 points LarTrig9 2.1.017.

Match the trigonometric expression with one of the following.

$\sec^4 x - \tan^4 x$

- ○ $\sec^2 x + \tan^2 x$
- ○ $\sin x \tan x$
- ○ 1
- ○ $\sec^2 x$
- ○ -1
- ○ $\csc x$

$a^2 - b^2 = (a-b)(a+b)$

$x^4 = (x^2)^2$!

$= (\sec^2 x)^2 - \tan^4 x = (\tan^2 x + 1)^2 - \tan^4 x$

$\tan^4 x + 2\tan^2 x + 1 - \tan^4 x = 2\tan^2 x + 1$

$= 2(\sec^2 x - 1) + 1 = 2\sec^2 x - 2 + 1 = 2\sec^2 x - 1$

$= \sec^2 x + \sec^2 x - 1 = \sec^2 x + \tan^2 x$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

17. –/1 points LarTrig9 2.1.018.

Match the trigonometric expression with one of the following.

$\cot x \sec x = \frac{\cos(x)}{\sin(x)} \cdot \frac{1}{\cos(x)} = \frac{1}{\sin(x)} = \csc(x)$

- ○ -1
- ○ $\sin x \tan x$
- ○ $\csc x$
- ○ $\sec^2 x + \tan^2 x$
- ○ 1
- ○ $\sec^2 x$

18. –/1 points LarTrig9 2.1.019.

Match the trigonometric expression with one of the following.

$$\frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} =$$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

- ○ -1
- ○ $\sec^2 x + \tan^2 x$
- ○ $\csc x$
- ○ $\sin x \tan x$
- ○ $\sec^2 x$
- ○ 1

19. –/1 points LarTrig9 2.1.020.

Match the trigonometric expression with one of the following.

$$\frac{\cos^2[(\pi/2) - x]}{\cos x} = \frac{\sin^2 x}{\cos x} =$$

Co-func identities

$\left(\frac{\pi}{2} - x\right) + x = \frac{\pi}{2}$, so

$\frac{\pi}{2} - x$ & x are always complementary angles!

- ○ $\sin x \tan x$

$$= \sin x \cdot \frac{\sin x}{\cos x} = \sin(x)\tan(x)$$

- ○ 1
- ○ -1
- ○ $\csc x$
- ○ $\sec^2 x$
- ○ $\sec^2 x + \tan^2 x$

20. –/1 points LarTrig9 2.1.021.

Factor the expression and use the fundamental identities to simplify. There is more than one correct form of the answer.

$6 \tan^2 x - 6 \tan^2 x \sin^2 x$

$6\tan^2 x(1-\sin^2 x) = \tan^2 x \cos^2 x = \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \sin^2 x$

21. –/1 points LarTrig9 2.1.023.

Factor the expression and use the fundamental identities to simplify. There is more than one correct form of the answer.

$\frac{8 \sec^2 x - 8}{\sec x - 1} = \frac{8(\sec^2 x - 1)}{\sec x - 1}$ $\quad a^2 - b^2 = (a-b)(a+b)$

$= \frac{8(\sec(x)-1)(\sec(x)+1)}{\sec(x)-1} = 8(\sec(x)+1)$

22. –/1 points LarTrig9 2.1.024.

Factor the expression and use the fundamental identities to simplify. There is more than one correct form of the answer.

$\frac{\cos x - 4}{\cos^2 x - 16}$

$= \frac{\cos(x)-4}{(\cos(x)-4)(\cos(x)+4)} = \frac{1}{\cos(x)+4}$

23. –/1 points LarTrig9 2.1.026.

Factor the expression and use the fundamental identities to simplify. There is more than one correct form of the answer.

$\sec^4 x - \tan^4 x$

$x^4 - y^4 = (x^2-y^2)(x^2+y^2) = (x-y)(x+y)(x^2+y^2)$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$(\tan^2 x + 1 - \tan^2 x)(\tan^2 x + 1 + \tan^2 x)$

$= 2\tan^2(x) + 1$