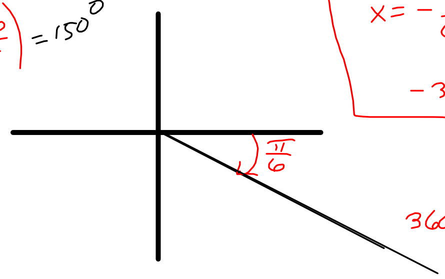
You don't have to write out the questions on a test. The questions are already included in the product you're turning in (the "cover sheets")

1. (10 pts) Find two angles, between -2π and 2π (i.e., 0^0 and 360^0) that are coterminal with $\frac{35\pi}{6}$. Give exact answers in degrees and radians.

$\frac{35}{6} = 5 + \frac{5}{6}$

$\frac{35\pi}{6} = 5\pi + \frac{5\pi}{6}$

$\left(\frac{5\cancel{\pi}}{\cancel{6}}\right)\left(\frac{\overset{30}{\cancel{180}}}{\cancel{\pi}}\right) = 150^0$



$\boxed{x = -\frac{\pi}{6}, \ \frac{11\pi}{6} \quad \text{OR} \quad -30^0, \ 330^0}$

$360^0 - 30^0 = 330^0$

2. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 6$ cm
   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 2344^0. Round to 3 decimal places.

The *sweet* relationship between angles and arc length (and area, too) is *only* sweet if you're using radian measure.

arc length $= s = r\theta = (6)(2344^0)\left(\frac{\pi \text{ radians}}{180^0}\right) = \frac{1172}{15}\pi$ cm

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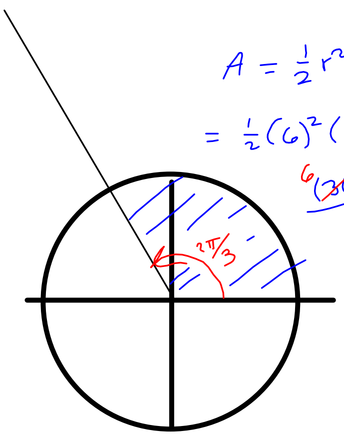
            4688/5
6*2344/180
        78.13333333
Ans▸Frac
           1172/15
Ans*π
         245.463106
```

$\approx \boxed{245.463 \text{ cm}}$

b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \frac{2\pi}{3}$

$r = 6$ cm



$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(6)^2\left(\frac{2\pi}{3}\right)$

$\frac{(\overset{6}{\cancel{36}})(2)(\pi)}{\underset{1}{\cancel{6}}}$

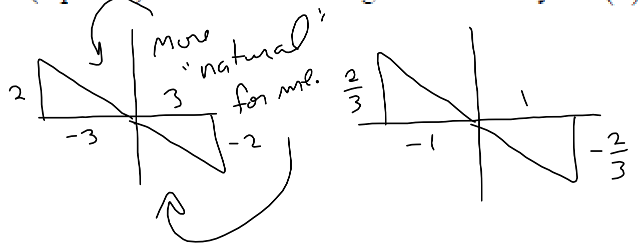
$= \boxed{12\pi \text{ cm}^2}$

πr^2

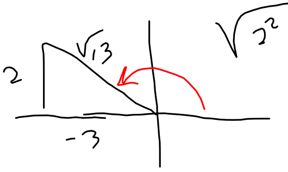
$2\pi r$

3. Answer the questions about the equation $\tan(\theta) = \frac{-2}{3}$.

a. (5 points) Sketch two triangles that satisfy $\tan(\theta) = \frac{-2}{3}$. $= \frac{y}{x}$



b. (5 pts) Assume the terminal side of the angle θ lies in the 2nd quadrant. Find the other five trigonometric functions of θ.



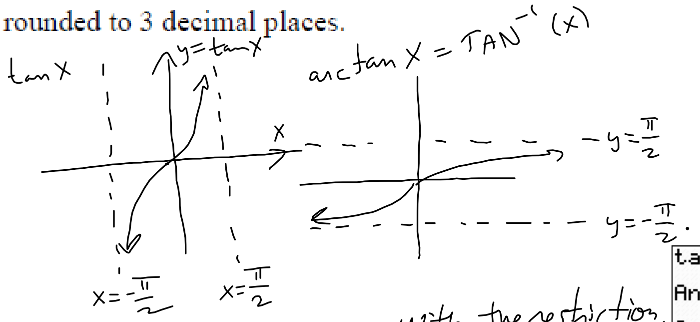
$\sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

$\sin\theta = \frac{2}{\sqrt{13}} = \frac{y}{r}$ $\qquad \csc\theta = \frac{\sqrt{13}}{2}$

$\cos\theta = -\frac{3}{\sqrt{13}} = \frac{x}{r}$ $\qquad \sec\theta = -\frac{\sqrt{13}}{3}$

$\tan\theta = -\frac{2}{3} = \frac{y}{x}$ $\qquad \cot\theta = -\frac{3}{2}$

c. (5 pts) Again, assuming θ's terminal side lies in Q II, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

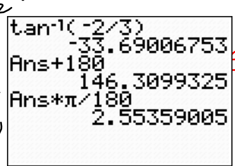


$\tan x = -\frac{2}{3}$

$\arctan(\tan x) = \arctan(-\frac{2}{3})$ is not exactly true.

Restrict to $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ to keep it 1-to-1

with the restriction, arctangent is now a function!

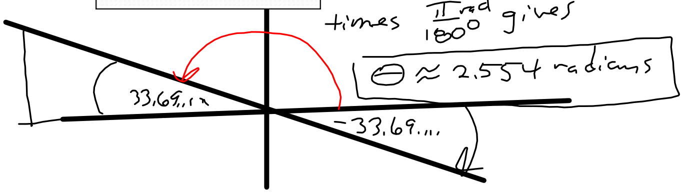


$\theta = 180^\circ - 33.69\ldots^\circ$

$\approx \boxed{146.310^\circ}$

times $\frac{\pi \text{ rad}}{180^\circ}$ gives

$\boxed{\theta \approx 2.554 \text{ radians}}$



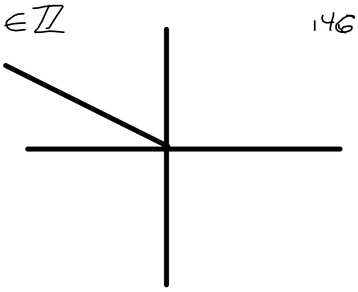
d. (5 pts) Give *all* solutions to the equation $\tan(\theta) = -\frac{2}{3}$, in degrees *and* radians, rounded to three (3) decimal places.

$146.310^\circ + 180^\circ n \quad \forall n \in \mathbb{Z}$

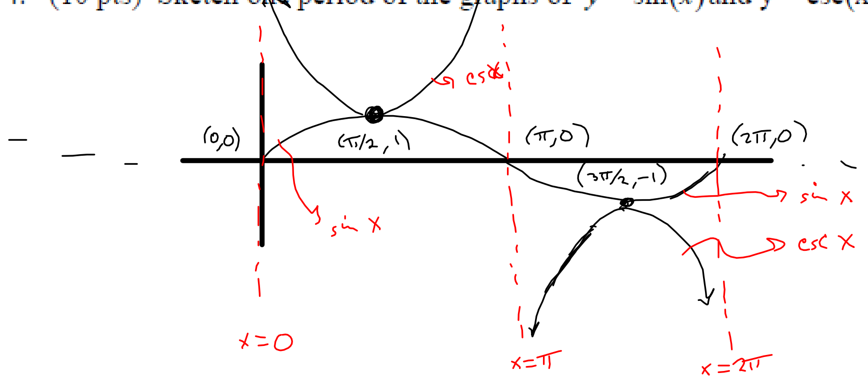
$2.554 + \pi n \quad \forall n \in \mathbb{Z}$

$146.310^\circ \pm 180^\circ n$, $n = 0, 1, 2, \ldots$

$146.310^\circ + 180^\circ n$, $n = 0, \pm 1, \pm 2, \ldots$

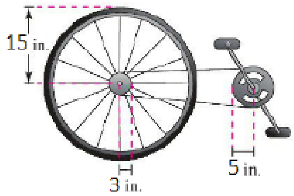


4. (10 pts) Sketch one period of the graphs of $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.



$\csc x = \dfrac{1}{\sin x}$

5. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 15 inches, respectively. A cyclist is pedaling at a rate of 1.4 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.



$$\left(\frac{1.4\ \cancel{\text{rev front}}}{1\ \text{s}}\right)\left(\frac{\frac{5}{3}\ \cancel{\text{rev back}}\ \text{sprocket}}{1\ \cancel{\text{rev front}}}\right)\left(\frac{1\ \text{rev}\ \cancel{\text{back tire}}}{1\ \text{rev}\ \cancel{\text{back sprocket}}}\right)\left(\frac{2\pi \cdot 15\ \text{in}}{1\ \text{rev}\ \cancel{\text{back tire}}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$(1.4)\left(\tfrac{5}{3}\right)(2\pi)(15)\left(\tfrac{1}{12}\right) \approx \boxed{18.3\ \frac{\text{ft}}{\text{s}}}$$

$88\ \text{ft/s} = 60\ \text{mi/hr}$

$$\left(18.3\ldots\ \frac{\text{ft}}{\text{s}}\right)\left(\frac{60\ \text{mi/hr}}{88\ \text{ft/s}}\right) \approx \boxed{12.5\ \frac{\text{mi}}{\text{hr}}}$$

$$\left(18.3\ldots\ \text{ft/s}\right)\left(\frac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\frac{3600\ \text{s}}{1\ \text{hr}}\right) \approx 12.5\ \frac{\text{mi}}{\text{hr}}$$

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1.4*5/3*2π*15/12
          18.32595715
Ans*60/88
          12.49497078
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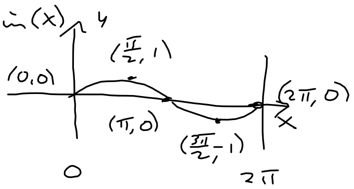
6. (10 pts) Sketch the graph of $f(x) = 8\sin\left(\frac{\pi}{14}x - \frac{5\pi}{14}\right) + 13$.

$y = \sin(x)$



Amplitude = height above/below... = 8

... Midline = vertical shift : $y = 13$

Phase (horizontal) shift : 5 units

Period = $T = 28$

Starting Point : $x = 5$

$$\frac{\pi}{14}x - \frac{5\pi}{14} = \frac{\pi}{14}(x-5)$$

Period: When does $\frac{\pi}{14}x = 2\pi$

$$\Rightarrow x = (2\pi)\left(\frac{14}{\pi}\right)$$

$$= \boxed{28 = T}$$

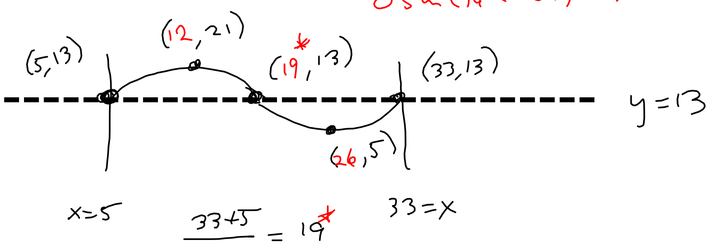
$$\frac{\pi}{14}\left(\frac{\frac{\pi}{14}x}{\frac{\pi}{14}} - \frac{\frac{5\pi}{14}}{\frac{\pi}{14}}\right)$$

$$\frac{\frac{\pi}{14}x}{\frac{\pi}{14}} = \left(\frac{\pi}{14}x\right)\left(\frac{14}{\pi}\right) = x$$

$$\frac{\frac{5\pi}{14}}{\frac{\pi}{14}} = \frac{5\pi}{14} \cdot \frac{14}{\pi} = 5$$

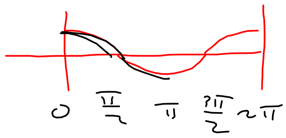
$$\frac{19+5}{2} = \frac{24}{2} = 12$$

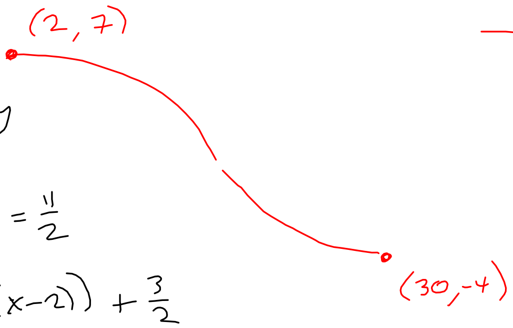
$$8\sin\left(\frac{\pi}{14}(x-5)\right) + 13$$



$$\frac{33+5}{2} = 19^{*}$$

7. (10 pts) Write the cosine function that achieves its maximum height of $y = 7$ centimeters at time $t = 2$ seconds and its minimum height of $y = -4$ centimeters at $t = 30$ seconds.





midline $\frac{7+-4}{2} = \frac{3}{2} = y$

Amplitude $\frac{7-(-4)}{2} = \frac{11}{2}$

$\frac{11}{2}\cos(b\,(x-2)) + \frac{3}{2}$

$T = 2(30-2) = 2(28) = 56$

$bx = 2\pi$, when $x = 56$

$56b = 2\pi$

$b = \frac{2\pi}{56} = \frac{\pi}{28}$

$$\boxed{f(x) = \frac{11}{2}\cos\left(\frac{\pi}{28}(x-2)\right) + \frac{3}{2}}$$

8. (5 pts) Solve the triangle. That means, find all lengths and angles. Exact answers required.

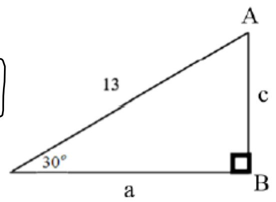


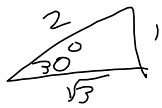
Figure for #8

$\frac{c}{13} = \sin 30^\circ \Rightarrow c = 13\sin 30^\circ = \boxed{\frac{13}{2} = c}$

$\frac{a}{13} = \cos 30^\circ \Rightarrow a = 13\cos 30^\circ = \boxed{\frac{13\sqrt{3}}{2} = a}$

$180^\circ - 90^\circ - 30^\circ = \boxed{60^\circ = A}$

$\boxed{B = 90^\circ}$ by picture



9. Find the exact value of…

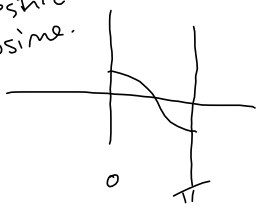
a. … (5 pts) $\tan\left(\arccos\left(\frac{2}{11}\right)\right)$.

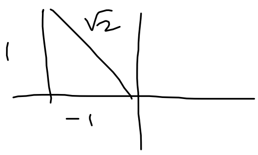
$\boxed{= \frac{\sqrt{117}}{2}}$

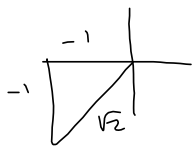
$11^2 - 2^2$
$= 121 - 4 = 117$

b. … (5 pts) $\arccos\left(\sin\left(\frac{7\pi}{4}\right)\right)$ $= \arccos\left(-\frac{1}{\sqrt{2}}\right)$
$= \frac{3\pi}{4}$

Restricted cosine.



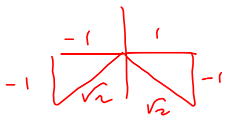




Ought to ask

$\arcsin\left(\sin\left(\frac{7\pi}{4}\right)\right) = -\frac{\pi}{4}$!

$\arcsin\left(-\frac{1}{\sqrt{2}}\right)$



10. (5 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to sin(arccos(3x

Bonus: Answer *two* of the following, for *up to* 10 points:

11. (5 pts) Sketch the pictures corresponding to:
    a. $\sin(x) = 0$
    b. $\sin(x) = 1$
    c. $\sin(x) = \frac{\sqrt{3}}{2}$
    d. $\sin(x) = \frac{1}{\sqrt{2}}$
    e. $\cos(x) = 0$

12. (5 pts) Sketch the graph of one period of $y = \sin(x)$ (restricted to make it 1-to-1) *and* $y = \arcsin(x)$ on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted sine function and its inverse.

13. (5 pts) Sketch the graph of one period of $y = \cos(x)$ (restricted to make it 1-to-1) *and* $y = \arccos(x)$ on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted cosine function and its inverse.

Jan 25-3:01 PM