§1.7 Inverse Trig Functions.

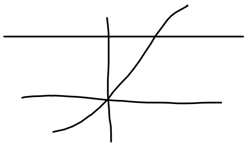
RECALL : 1-to-1 : $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

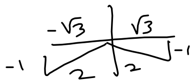
$\sin x = -\frac{1}{2}$

$\sin^{-1}(\sin x) = \sin^{-1}(-\frac{1}{2})$

The answer!



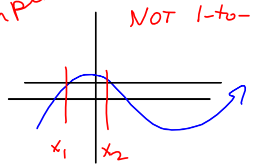
Horizontal Line only touches it once.



$\frac{7\pi}{6}, \frac{11\pi}{6}, -\frac{\pi}{x}$

Non-example:

NOT 1-to-1



But $f(x_1) = f(x_2)$

why do we care?

we want $f^{-1}(x)$ to be a function, too!

That requires 1-to-1 property!

for $f^{-1}(x)$ to pass vertical line test, $f(x)$ must pass the horizontal line test.

Inverse function : $f^{-1}(x)$ = "f-inverse-of-x" satisfies

$f^{-1} \circ f = \text{identity}$

$f(x) = x^3 \Rightarrow f^{-1}(x) = \sqrt[3]{x}$

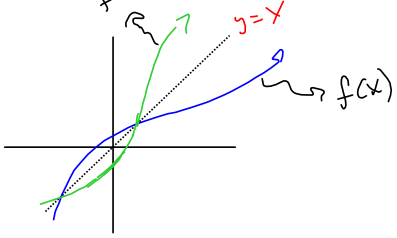
$f^{-1} \circ f = f^{-1}(f(x)) = f^{-1}(x^3) = \sqrt[3]{x^3} = (x^3)^{\frac{1}{3}} = x^1 = \text{identity}$

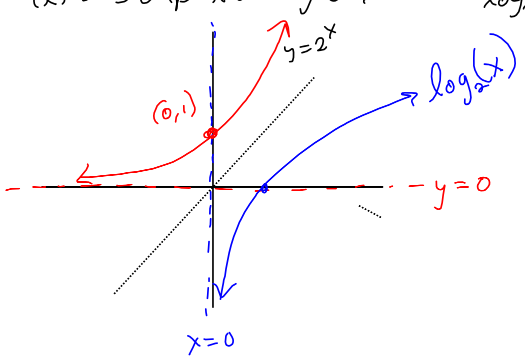
GRAPHS of inverses: Symmetric about $y = x$.

$2^x = 7$

$f^{-1}(x)$: Swap x's & y's!

$\log_2(2^x) = x = \log_2(7)$



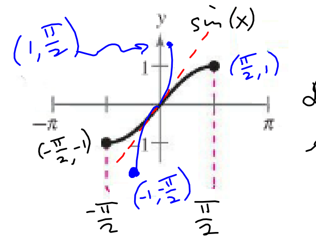


Graph of $\sin^{-1}(x) = \arcsin(x)$

I prefer "$\arcsin(x)$" b/c $\sin^{-1}(x)$ is ambiguous.

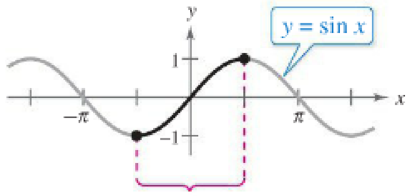
calc. key for arcsine!

Restricted $\sin(x)$



$\mathcal{D} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\mathcal{R} = [-1, 1]$



sin x has an inverse function on this interval.

Figure 1.51

$\frac{\pi}{2} \approx 1.57$, FYI

Definition of Inverse Sine Function

The **inverse sine function** is defined by

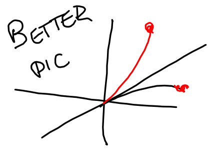
$$y = \arcsin x \quad \text{if and only if} \quad \sin y = x$$

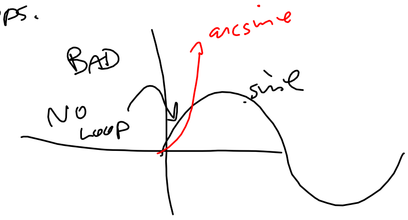
where $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$. The domain of $y = \arcsin x$ is $[-1, 1]$, and the range is $[-\pi/2, \pi/2]$.

$\mathcal{D} = [-1, 1]$

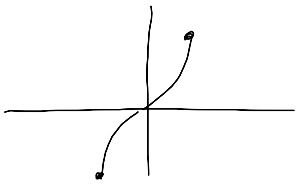
$\mathcal{R} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

when graphing sine & arcsine together, avoid loops.



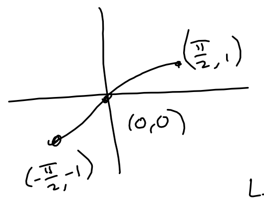


Know how to restrict sine to keep it 1-to-1, then $\mathcal{D}$ & $\mathcal{R}$ of arcsine are cake (easy).

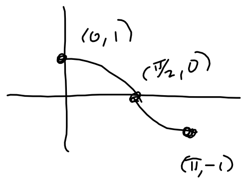


Restricted ...

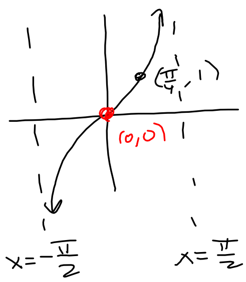
... sin(x)



... cos(x)

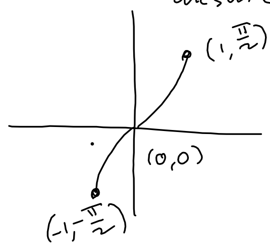


... tan (x)

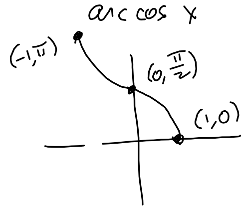


Lead us to
the inverse funcsi

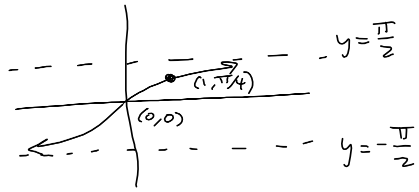
arcsin (x)



arc cos x



arctan (x)



Classic Domain Restriction

$f(x) = x^2$ is NOT 1-to-1.

But we pick a domain where it IS!

$\{x \mid x \geq 0\}$

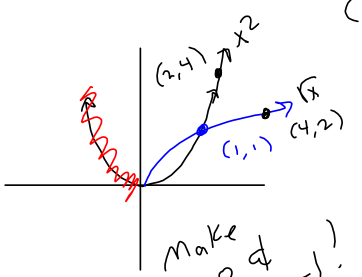
$y = x^2$

$x = y^2$

$y^2 = x$

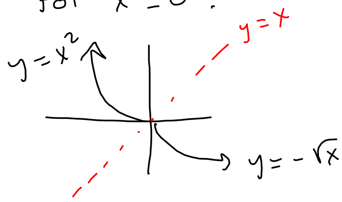
$y = \pm\sqrt{x}$ → $y = +\sqrt{x}$ for $x \geq 0$

→ $y =$



Make $x \geq 0$ & it's 1-to-1!

For $x \leq 0$:



1. –/2 points LarTrig9 1.7.001.

Fill in the blanks. (Enter the domain in interval notation.)

Function	Alternative Notation	Domain	Range
$y = \arcsin(x)$	$y = \sin^{-1}(x)$	$\{x \mid -1 \le x \le 1\}$ $= [-1, 1]$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\{y \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$

$= [-\frac{\pi}{2}, \frac{\pi}{2}]$

Restricted to domain make sine 1-to-1.

2. –/2 points LarTrig9 1.7.002.

Fill in the blanks. (Enter the range in interval notation.)

Function	Alternative Notation	Domain	Range
$y = \arccos(x)$	$y = \cos^{-1} x$	$\{x \mid -1 \le x \le 1\}$	$[0, \pi]$ $= \{y \mid 0 \le y \le \pi\}$

Inverse wrt function

composition, not multiplication

cos(x) restricted

3. –/3 points LarTrig9 1.7.003.

Fill in the blanks. (Enter the domain and the range in interval notation.)

Function	Alternative Notation	Domain	Range
$y = \arctan x$	$y = \tan^{-1}(x)$	$(-\infty, \infty)$	$\{y \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$ $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

restricted tangent

4. –/1 points LarTrig9 1.7.004.

Fill in the blank.

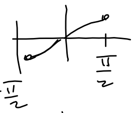
Without restrictions, no trigonometric function has a(n) ---Select--- function.

inverse

Always an inverse RELATION which will be a function only if f(x) is 1-to-1.

There are infinitely many solutions to any sin(x) = y, if there are any at all.

So be careful about domains and ranges, for these "restricted" trig functions and their corresponding inverses!!!

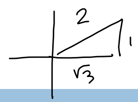


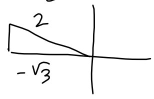
5. –/1 points LarTrig9 1.7.005.

Evaluate the expression without using a calculator. (Enter your answer in radians.)

$\arcsin \frac{1}{2}$ The angle whose sine is $\frac{1}{2}$.

$= \frac{\pi}{6}$

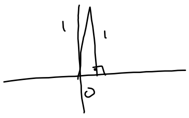




6. –/1 points LarTrig9 1.7.006.

Evaluate the expression without using a calculator. (Enter your answer in radians.)

$\arcsin 1$ "My" $\sin x = 1$ picture.



$x = \frac{\pi}{2}$ or $90°$

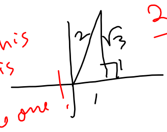
7. –/1 points LarTrig9 1.7.007.

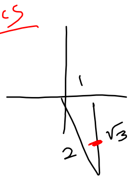
Evaluate the expression without using a calculator. (Enter your answer in radians.)

$\arccos \frac{1}{2} = \frac{\pi}{3}$

2 Pics

This is the one!





Not between 0 & π, so not what $\arccos(\frac{1}{2})$ will spit out!

8. –/1 points LarTrig9 1.7.009.

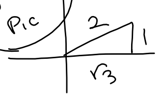
Evaluate the expression without using a calculator. (Enter your answer in radians.)

$\arctan \frac{\sqrt{3}}{3} = \arctan\left(\frac{1}{\sqrt{3}}\right)$ is something I can "see"

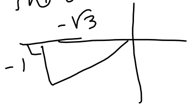
$\left(\frac{\sqrt{3}}{3}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

60-30 right triangle, when $\sqrt{3}$ floating around.

THE Pic



The other pic!



$\frac{7\pi}{6}$ also satisfies $\tan x = \frac{\sqrt{3}}{3}$

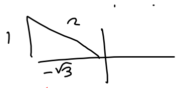
9. –/1 points LarTrig9 1.7.011.

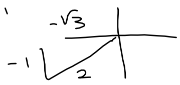
Evaluate the expression without using a calculator. (Enter your answer in radians.)

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \arccos\left(-\frac{\sqrt{3}}{2}\right)$, NOT $\frac{1}{\cos\left(-\frac{\sqrt{3}}{2}\right)}$!!!!!!

Beware NOTATION.

Sometimes $\cos^{-1}(x) = \arccos(x)$



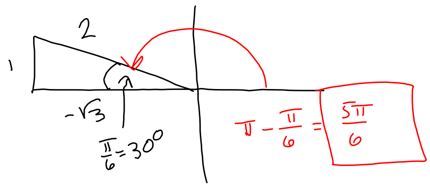


Sometimes $\cos^{-1}(x) = \sec(x)$!!

Depends on the context! It's like "homonyms."

C, sea, see?

$\arccos\left(-\frac{\sqrt{3}}{2}\right)$

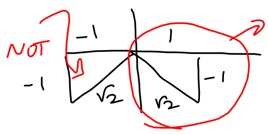


$\pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

10. –/1 points LarTrig9 1.7.012.

Evaluate the expression without using a calculator. (Enter your answer in radians.)

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \arcsin\left(-\frac{1}{\sqrt{2}}\right)$$



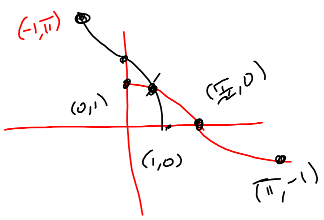
Yes, bc $-\frac{\pi}{4} \in \mathcal{R}(\arcsin(x)) = \mathcal{D}(\sin(x))$ after restriction!

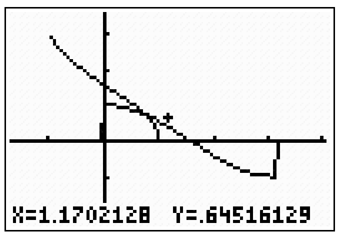
11. –/2 points LarTrig9 1.7.019.

Use a graphing utility to graph f, g, and $y = x$ in the same viewing window. (Be sure to restrict the domain of f properly.)

$f(x) = \cos x$, $g(x) = \arccos x$

$\cos^{-1}x$ is the key on your calculator.



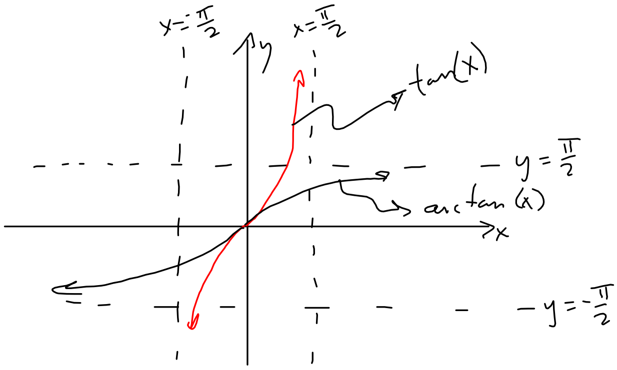


12. –/2 points LarTrig9 1.7.020.

Use a graphing utility to graph f, g, and $y = x$ in the same viewing window. (Be sure to restrict the domain of f properly.)

$f(x) = \tan x, \quad g(x) = \arctan x$

I left $y = x$ out of #11



13. –/1 points LarTrig9 1.7.022.

Use a calculator to evaluate the expression. Round your result to two decimal places. (Enter your answer in radians.)

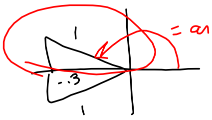
arcsin 0.95

$\boxed{\approx 1.25}$

14. –/1 points LarTrig9 1.7.024.

Use a calculator to evaluate the expression. Round your result to two decimal places. (Enter your answer in radians.)

arccos(−0.3)



$= \arccos(-.3)$

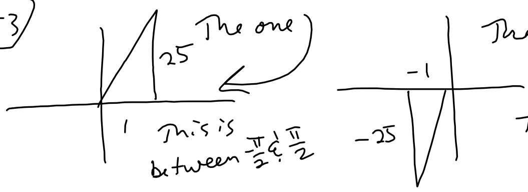
$\approx 1.7855 \approx \boxed{1.79}$

15. –/1 points LarTrig9 1.7.026.

Use a calculator to evaluate the expression. Round your result to two decimal places. (Enter your answer in radians.)

arctan 25 ≈ 1.53

1.53

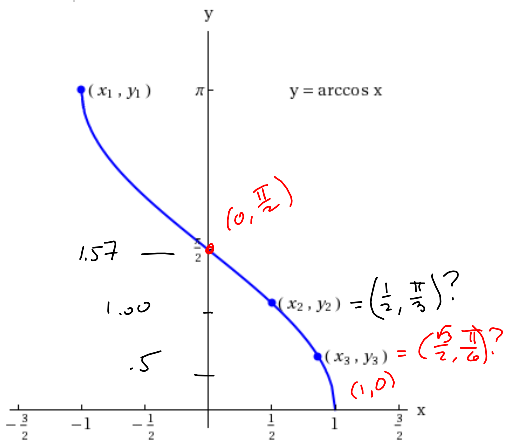


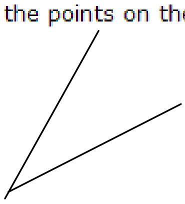
The one — This is between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$

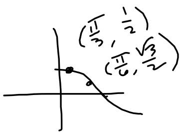
The other — This isn't in $(-\frac{\pi}{2}, \frac{\pi}{2})$

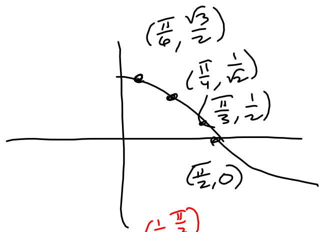
16. –/3 points LarTrig9 1.7.040.

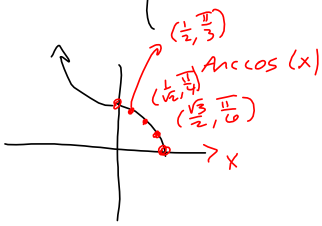
Determine the missing coordinates of the points on the graph of the function.





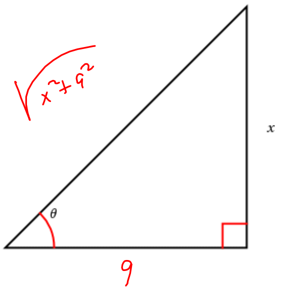






17. –/1 points LarTrig9 1.7.041.

Use an inverse trigonometric function to write θ as a function of x.



$$\tan\theta = \frac{x}{9}$$

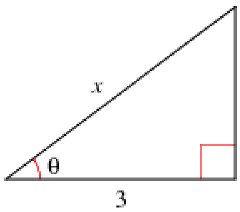
$$\arctan(\tan\theta) = \theta = \arctan\left(\frac{x}{9}\right)$$

$$\theta = \arcsin\left(\frac{x}{\sqrt{x^2+81}}\right)$$

$$\theta = \arccos\left(\frac{9}{\sqrt{x^2+81}}\right)$$

18. –/1 points LarTrig9 1.7.042.

Use an inverse trigonometric function to write θ as a function of x.

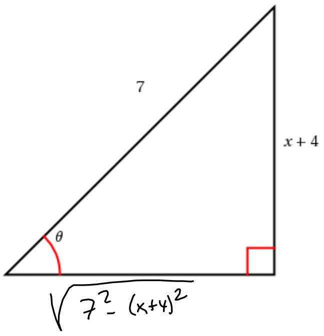


$$\cos\theta = \frac{3}{x}$$

$$\boxed{\theta = \arccos\left(\frac{3}{x}\right)}$$

19. ⊕ –/1 points LarTrig9 1.7.043.

Use an inverse trigonometric function to write θ as a function of x.

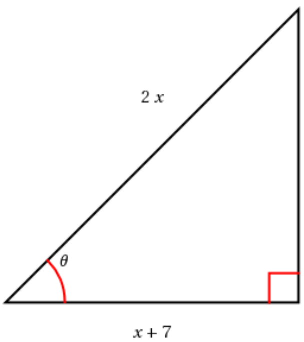


$\sin\theta = \frac{x+4}{7}$

$\theta = \arcsin\left(\frac{x+4}{7}\right)$

20. ⊕ –/1 points LarTrig9 1.7.045.

Use an inverse trigonometric function to write θ as a function of x.



You do this one!
I've done enough to cover the "big 3"
(sine, tangent, cosine)

21. –/1 points LarTrig9 1.7.047.

Use the properties of inverse trigonometric functions to evaluate the expression.

sin(arcsin 0.8) = .8

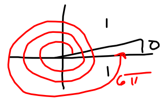
22. –/1 points LarTrig9 1.7.051.

Use the properties of inverse trigonometric functions to evaluate the expression.

arcsin(sin 6π) $\neq 6\pi$!

$\sin(6\pi) = 0$

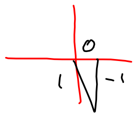
$\arcsin(0) = 0$



23. –/1 points LarTrig9 1.7.052.

Use the properties of inverse trigonometric functions to evaluate the expression. (Enter your answer in radians.)

$\arccos\left(\cos \frac{11\pi}{2}\right) = \frac{\pi}{2}$

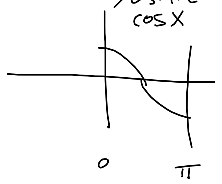


$\cos \frac{11\pi}{2} = 0$

$\arccos(0) = \frac{\pi}{2}$

restrict!

cos X

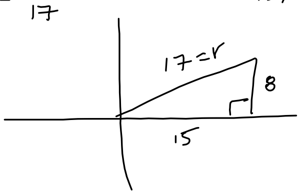


24. –/1 points LarTrig9 1.7.053.

Find the exact value of the expression. (*Hint:* Sketch a right triangle.)

$\sin\left(\arctan \frac{8}{15}\right) = \frac{8}{17}$

$\arctan\left(\frac{8}{15}\right)$



$8^2 + 15^2$

$= 64 + 225$

$= 289 = 17^2$

$r = \sqrt{a^2+b^2}$

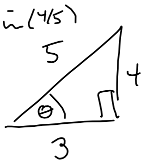
$= \sqrt{289} = 17$

$\begin{array}{r} {}^{4}17 \\ \times 17 \\ \hline 119 \\ 170 \\ \hline 289 \end{array}$

25. –/1 points LarTrig9 1.7.054.MI.

Find the exact value of the expression. (*Hint:* Sketch a right triangle.)

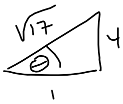
$\sec\left(\arcsin\left(\frac{4}{5}\right)\right) = \frac{5}{3}$ $\arcsin^{-1}(4/5)$



26. –/1 points LarTrig9 1.7.055.

Find the exact value of the expression. (*Hint:* Sketch a right triangle.)

$\cos(\tan^{-1} 4) = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$



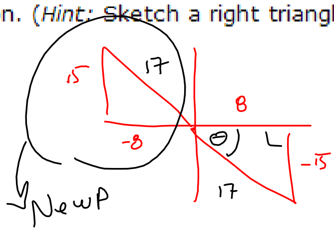
$4^2 + 1^2 = 17$

ok might need this rationalized for webassign to eat it.

27. –/1 points LarTrig9 1.7.058.

Find the exact value of the expression. (*Hint:* Sketch a right triangle.)

$\csc\left[\arctan\left(-\frac{15}{8}\right)\right] = -\frac{17}{15}$

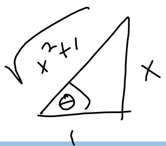


28. –/1 points LarTrig9 1.7.066.

Write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

$\sin(\arctan x)$ Big for calculus – Trig. Substitution CALC II

$= \frac{x}{\sqrt{x^2+1}}$

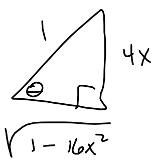


29. –/1 points LarTrig9 1.7.067.

Write an algebraic expression that is equivalent to the given expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

$\cos(\arcsin 4x)$

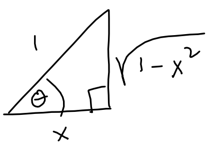
$= \sqrt{1-16x^2}$



30. –/1 points LarTrig9 1.7.069.

Write an algebraic expression that is equivalent to the given expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

$\sin(\arccos x) = \sqrt{1-x^2}$



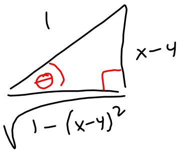
31. –/1 points LarTrig9 1.7.070.

Write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated in Example

sec[arcsin($x - 4$)]

arcsin $(x-4)$ pic

$= \dfrac{1}{\sqrt{1-(x-4)^2}}$



Webassign might not like $\dfrac{1}{\sqrt{1-(x-4)^2}}$ ← Perfect!

$(x-4)^2 = x^2 - 8x + 16$, so

$1 - (x-4)^2 = -x^2 + 8x - 15$

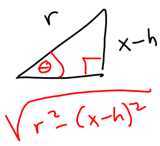
Might want $\dfrac{1}{\sqrt{-x^2+8x-15}}$, but I'm ok with this

32. –/1 points LarTrig9 1.7.074.

Write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated by Example 7.)

$\cos\left[\arcsin\left(\dfrac{x-h}{r}\right)\right]$

$= \dfrac{\sqrt{r^2-(x-h)^2}}{r}$

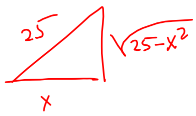


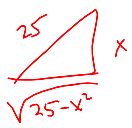
33. –/6 points LarTrig9 1.7.076.

Use a graphing utility to graph f and g in the same viewing window to verify that the two functions are equa

$f(x) = \tan\left(\arccos \dfrac{x}{5}\right)$, $\quad g(x) = \dfrac{\sqrt{25 - x^2}}{x}$

Good to see if you have time, but mainly be able to go from $f(x)$ to $g(x)$. (Maybe from $g(x)$ to $f(x)$, too!)





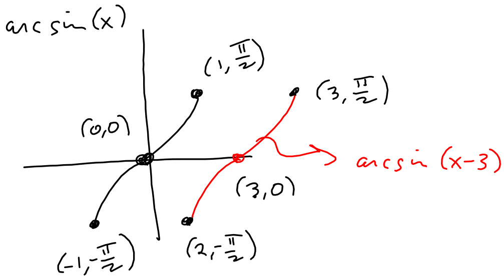
" $=$ " $\cot\left(\arcsin\left(\frac{x}{5}\right)\right)$

34. –/2 points LarTrig9 1.7.081.

Sketch a graph of the function.

$$g(x) = \arcsin(x - 3)$$

Right 3

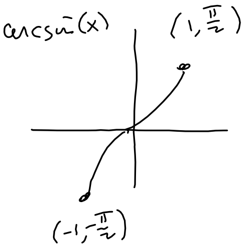


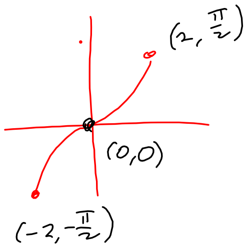
35. –/2 points LarTrig9 1.7.082.

Sketch a graph of the function.

$$g(x) = \arcsin\frac{x}{2}$$

2x

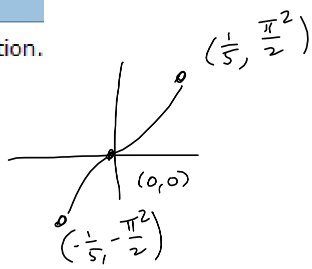




36. –/1 points LarTrig9 1.7.090.

Use a graphing utility to graph the function.

$$f(x) = \pi \arcsin(5x)$$



37. –/1 points LarTrig9 1.7.091.

Use a graphing utility to graph the function.

$f(x) = \arctan(3x - 2) = \arctan\left(3\left(x - \frac{2}{3}\right)\right)$

① shrink $\frac{1}{3}x$ $x + \frac{2}{3}$ ② Right $\frac{2}{3}$

38. –/1 points LarTrig9 1.7.094.

Use a graphing utility to graph the function.

$f(x) = \frac{\pi}{2} + \cos^{-1}\left(\frac{1}{\pi}\right)$

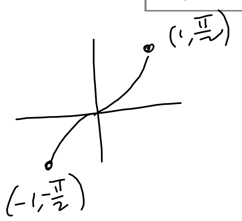
Horizontal line

$\cos^{-1}\left(\frac{1}{\pi}\right)$ is just a ~~number~~ constant.

39. –/1 points LarTrig9 1.7.097.

Fill in the blank. (Enter your answer in radians.)

As $x \to 1^-$, the value of arcsin $x \to$ $\boxed{\frac{\pi}{2}}$.



$\arcsin(x) \xrightarrow{x \to 1^-} \frac{\pi}{2}$

$\lim_{x \to 1^-} \arcsin(x) = \frac{\pi}{2}$

40. –/1 points LarTrig9 1.7.102.

Fill in the blank. (Enter your answer in radians.)

As $x \to -\infty$, the value of arctan $x \to$ $\boxed{}$.

$\arctan(x) \xrightarrow{x \to -\infty} -\frac{\pi}{2}$

$\lim_{x \to -\infty} \arctan(x) = -\frac{\pi}{2}$