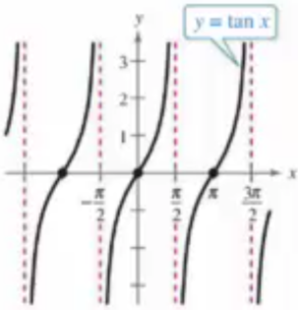
Graphs of tangent, cotangent, secant and cosecant.

We lay 'em out, for you, then show more organic ways of obtaining them, for yourself.

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan t$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\nexists$



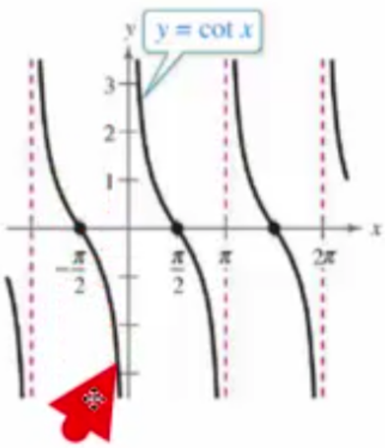
Period: π

Domain: all $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, \infty)$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

Symmetry: origin



Period: π

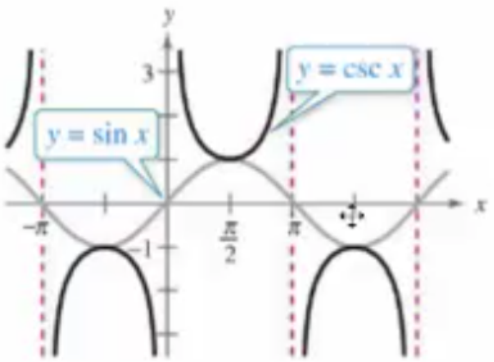
Domain: all $x \neq n\pi$

Range: $(-\infty, \infty)$

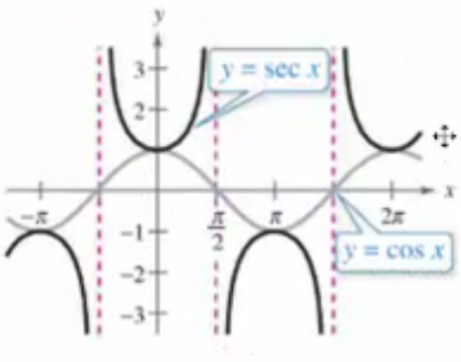
Vertical asymptotes: $x = n\pi$

Symmetry: origin

Key Interval $[0, \pi]$



Period: 2π
Domain: all $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Vertical asymptotes: $x = n\pi$
Symmetry: origin



Period: 2π
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$
Symmetry: y-axis

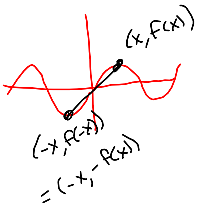
1. –/2 points LarTrig9 1.6.001. My Notes Ask Your

Fill in the blanks.

The tangent, cotangent, and cosecant functions are ---Select--- , so the graphs of these functions have symmetry with respect to the ---Select--- .

ODD

ORIGIN

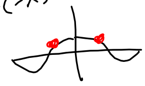


$\tan x = \frac{\sin x}{\cos x} = \frac{-}{+} = -$ ODD

means $f(-x) = -f(x)$

$\cos x$ is even

$f(-x) = f(x)$



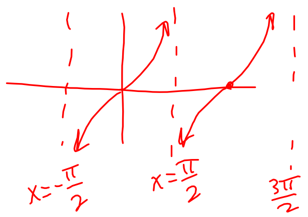
2. –/1 points LarTrig9 1.6.002. My No

Fill in the blank.

The graphs of the tangent, cotangent, secant, and cosecant functions all have ---Select--- asymptotes.

Vertical

e.g. $\tan x$

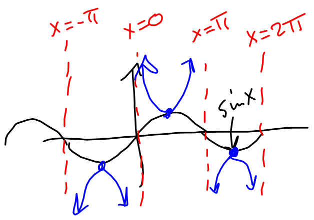


3. –/1 points LarTrig9 1.6.003. My

Fill in the blank.

To sketch the graph of a secant or cosecant function, first make a sketch of its ---Select--- function.

reciprocal



csc x by graphing sin x & then doing reciprocal thing

$\sin x \xrightarrow{x \to \pi^-} 0$ from + direction

$\therefore \csc x \xrightarrow{x \to \pi^-} +\infty$ from above

$\sin x \xrightarrow{x \to \pi^+} 0$ from − dir

$\csc x \xrightarrow{x \to \pi^+} -\infty$ from below

4. –/1 points LarTrig9 1.6.004. My Notes

Fill in the blank.

For the functions given by $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ---Select--- factor of the function $f(x)$.

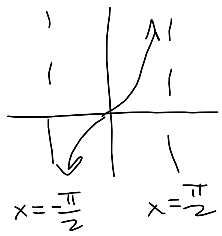
damping

$x \sin\left(\frac{1}{x}\right)$

5. –/1 points LarTrig9 1.6.005.

Fill in the blank.

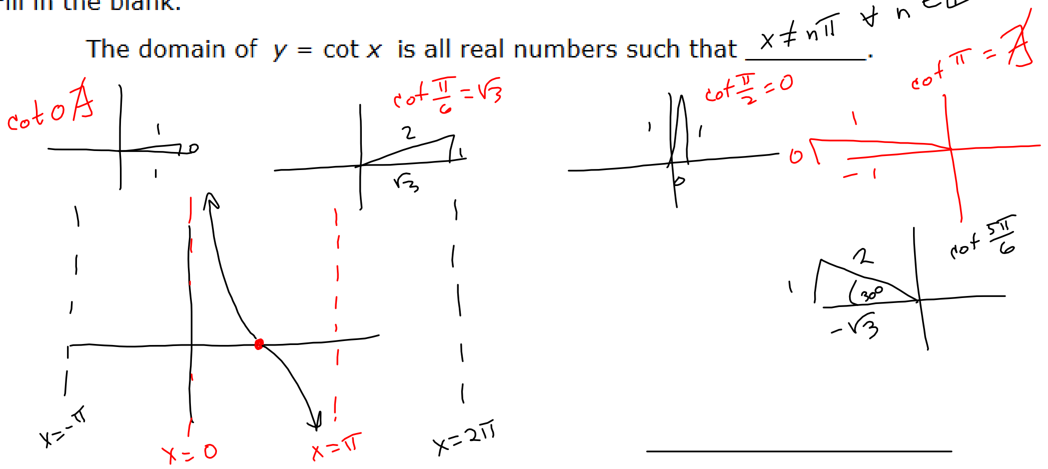
The period of $y = \tan x$ is π .



6. –/1 points LarTrig9 1.6.006.

Fill in the blank.

The domain of $y = \cot x$ is all real numbers such that $x \neq n\pi \ \forall n \in \mathbb{Z}$.

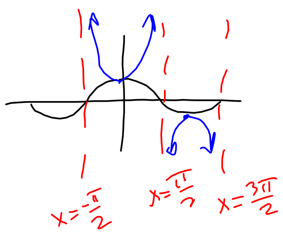


7. –/1 points LarTrig9 1.6.007.

Fill in the blank.

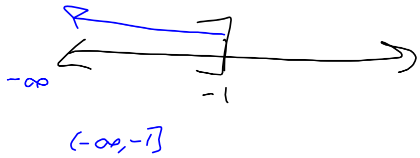
The range of $y = \sec x$ is ________.

How low can you go?



$[1,\infty) \cup (-\infty,-1]$

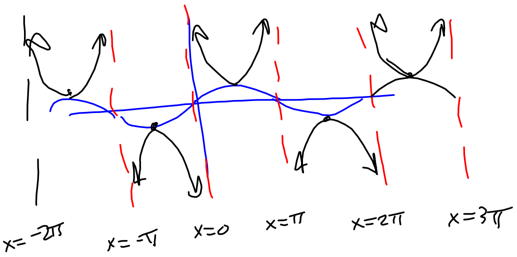
$(-\infty,-1] \cup [1,\infty)$



8. **–/1 points** LarTrig9 1.6.008.

Fill in the blank.

The period of $y = \csc x$ is Same as period of $\sin x$ $\boxed{2\pi}$



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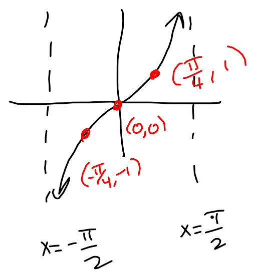
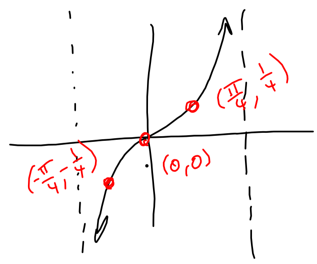
9. –/1 points LarTrig9 1.6.015.

Sketch the graph of the function. (Include two full periods.)

$$y = \frac{1}{4} \tan x$$





BASIC TANGENT GRAPH

$y = \tan x$

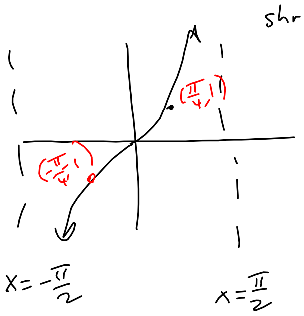
$y = \frac{1}{4} \tan x$

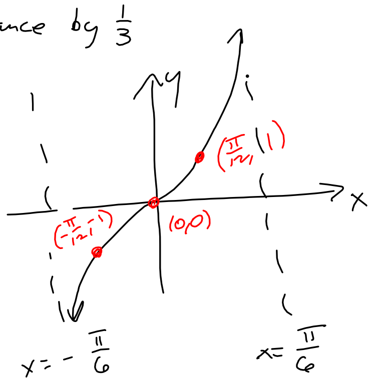
10. –/1 points LarTrig9 1.6.016.

Sketch the graph of the function. (Include two full periods.)

$y = \tan(3x)$ → 3 times as "fast"

shrink distance by $\frac{1}{3}$





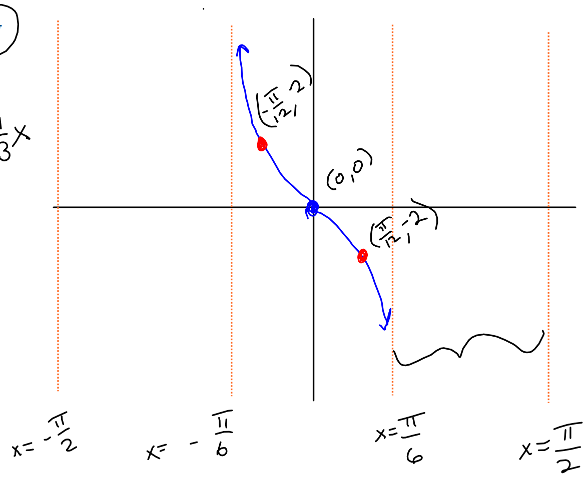
11. **–/1 points** LarTrig9 1.6.017.

Sketch the graph of the function. (Include two full periods.)

$y = -2 \tan 3x$

$-2y$

$\frac{1}{3}x$



$T = \frac{\pi}{3}$

$\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi + 2\pi}{6}$

$= \frac{3\pi}{6} = \frac{\pi}{2}$

12. –/1 points LarTrig9 1.6.018.

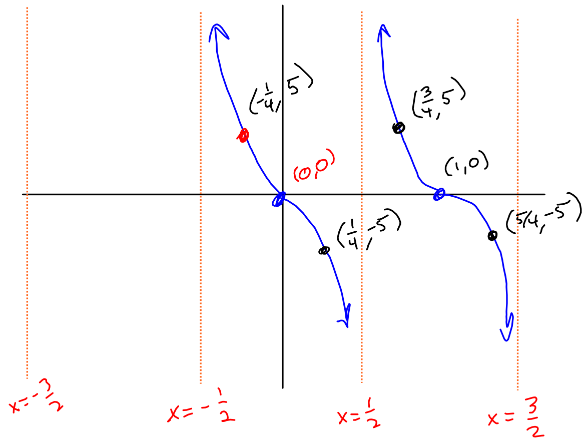
Sketch the graph of the function. (Include two full periods.)

$y = -5 \tan \pi x$

$-5y$ $\quad \frac{1}{\pi}x$

$T: \pi x = \pi$

$x = 1$



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13. -/1 points LarTrig9 1.6.019.

Sketch the graph of the function. (Include two full periods.)

$y = -\frac{1}{2}\sec x$

$= -\frac{1}{2}\cdot\frac{1}{\cos x}$ → $-\frac{1}{2}y$

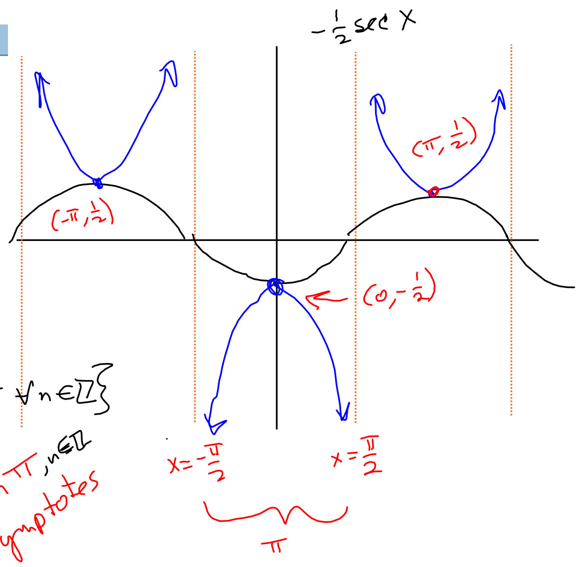
Tangent has cosine downstairs so does secant

$\mathcal{D}$ = Domain

$= \{x \mid x \neq \frac{\pi}{2} + n\pi \ \forall n \in \mathbb{Z}\}$

$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$

= asymptotes



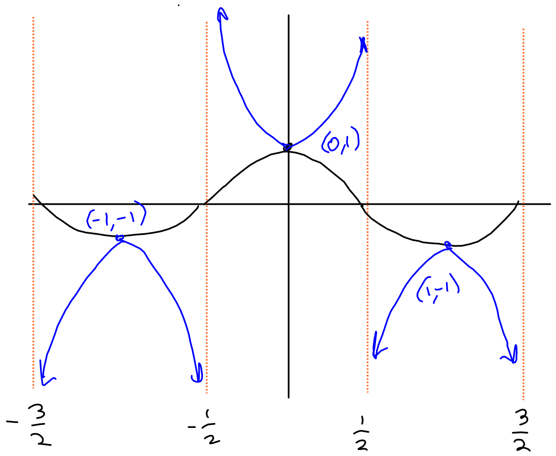
14. –/1 points LarTrig9 1.6.021.

Sketch the graph of the function. (Include two full periods.)

$y = \sec \pi x$

$T:\ \pi x = 2\pi$

$x = 2$



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15. –/1 points LarTrig9 1.6.022.

Sketch the graph of the function. (Include two full periods.)

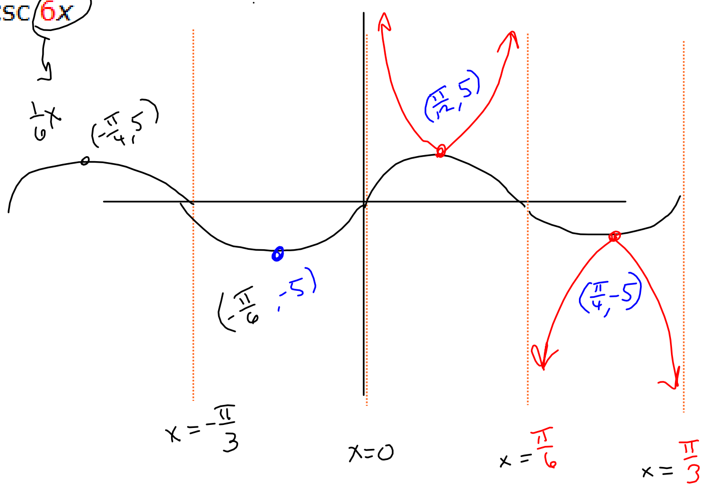
$y = 5 \csc 6x$

$5y$ $\quad$ $\frac{1}{6}x$

$\frac{1}{6}\pi$

$\frac{1}{6}(-\pi)$

$\frac{1}{6}(2\pi)$



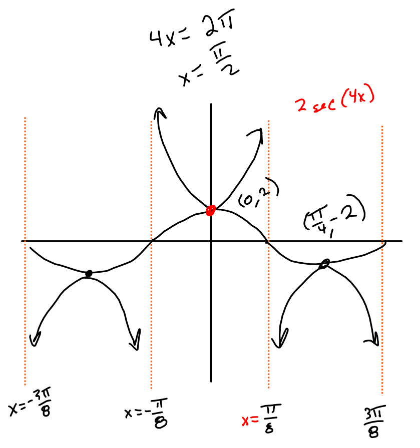
$$\frac{\frac{\pi}{6} + \frac{2\pi}{6}}{2} = \frac{\frac{3\pi}{6}}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$$

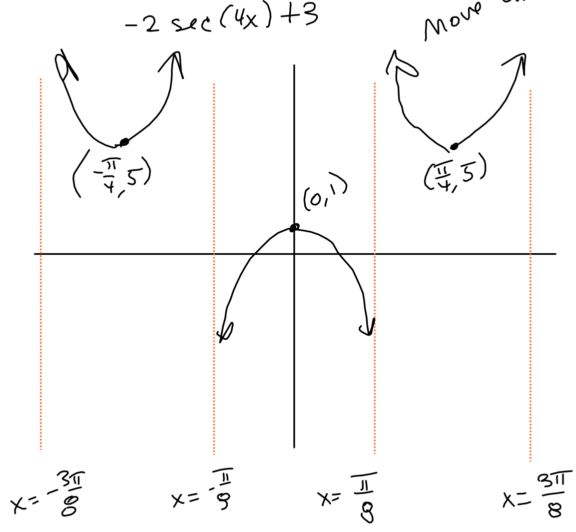
16. –/1 points LarTrig9 1.6.024.

Sketch the graph of the function. (Include two full periods.)

$y = -2 \sec 4x + 3$

-2y $\frac{1}{4}x$ up 3





17. **–/1 points** LarTrig9 1.6.027.

Sketch the graph of the function. (Include two full periods.)

$y = 5 \cot 4x$

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18. –/1 points LarTrig9 1.6.028.

Sketch the graph of the function. (Include two full periods.)

$$y = 5 \cot \frac{\pi x}{2}$$

Sep 9-9:27 AM

19. –/1 points LarTrig9 1.6.033.

Sketch the graph of the function. (Include two full periods.)

$$y = 5\csc(x - \pi)$$

20. **–/1 points** LarTrig9 1.6.034.

Sketch the graph of the function. (Include two full periods.)

$$y = \csc(3x - \pi)$$

Sep 9-9:29 AM

21. –/1 points LarTrig9 1.6.048.

Use a graphing utility to graph the function. (Include two full periods.)

$$y = \frac{1}{3}\sec\left(\frac{\pi x}{8} + \frac{\pi}{2}\right)$$

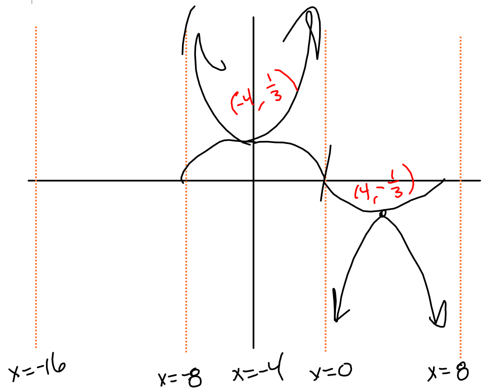
$\sec x$

$\sec \frac{\pi}{8}x$

$\sec \frac{\pi}{8}(x+4)$

$\frac{\pi x}{8} + \frac{\pi}{2} =$

$\frac{\pi}{8}(x+4)$



$\frac{\pi}{8}(x+4) = 0$

@ $x=-4$

is where it

all starts

$\dfrac{\frac{\pi}{2}}{\frac{\pi}{8}} = \frac{\pi}{2}\cdot\frac{8}{\pi} = 4$

$\frac{\pi}{8}x = 2\pi$

$x = (2\pi)\left(\frac{8}{\pi}\right) = 16$

$\left(\frac{-3\pi}{2}\right)\left(\frac{8}{\pi}\right)$ $= -12$

$\left(\frac{8}{\pi}\right)\left(\frac{-\pi}{2}\right)$ $= -4$

$\frac{\pi}{2}\cdot\frac{8}{\pi}$ $= 4$

$\frac{3\pi}{2}\cdot\frac{8}{\pi}$ $= 12$

would get me to $\sec\left(\frac{\pi x}{8}\right)$

we want $\sec\left(\frac{\pi}{8}(x+4)\right)$ so, move

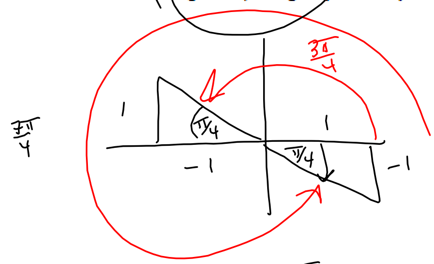
it left 4 units

$-12 \to -16$ $\quad -4 \to -8$ $\quad 4 \to 0$ $\quad 12 \to 8$

22. **–/1 points** LarTrig9 1.6.049.

Use a graph to solve the equation on the interval $[-2\pi, 2\pi]$. (Enter your answers as a comma-separated list.)

$\tan x = -1$



$$\frac{3\pi - 8\pi}{4} = -\frac{5\pi}{4}$$

$$\frac{7\pi}{4} - 2\pi = \frac{7\pi - 8\pi}{4} = -\frac{\pi}{4}$$

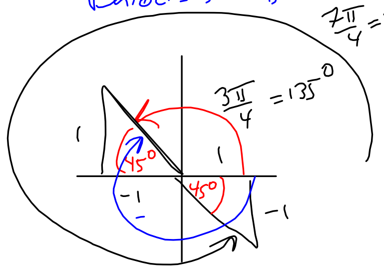
$$\left\{-\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$$

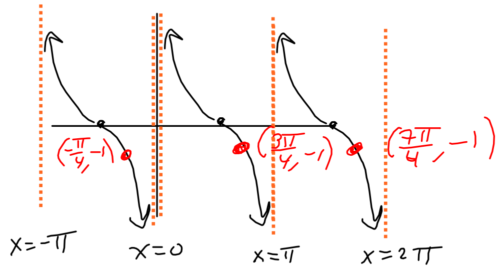
23. –/1 points LarTrig9 1.6.051.

Use a graph to solve the equation on the interval $[-2\pi, 2\pi]$. (Enter your answers as a comma-separated list.)

$\cot(x) = -1 \Rightarrow \tan(x) = -1$

Period $= T = \pi$





Period of cotangent is Pi.

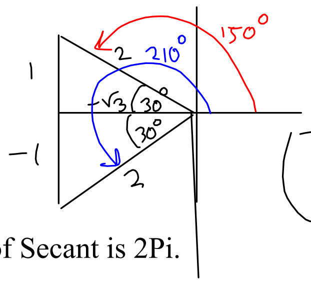
$x \in \{\frac{3\pi}{4}, \frac{7\pi}{4}, -\frac{\pi}{4}, -\frac{5\pi}{4}\}$

After 1st 2, I'm just adding & subtracting $180^\circ = \pi$.

24. –/1 points LarTrig9 1.6.053.

Use a graph to solve the equation on the interval $[-2\pi, 2\pi]$. (Enter your answers as a comma-separated list.)

$-\frac{2}{\sqrt{3}} = \boxed{\sec x = -\frac{2\sqrt{3}}{3}} \Rightarrow \cos(x) = -\frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{3\sqrt{3}}{2 \cdot 3} = \boxed{-\frac{\sqrt{3}}{2} = \cos(x)}$



Decimal
radians
non-intuitive (suck)

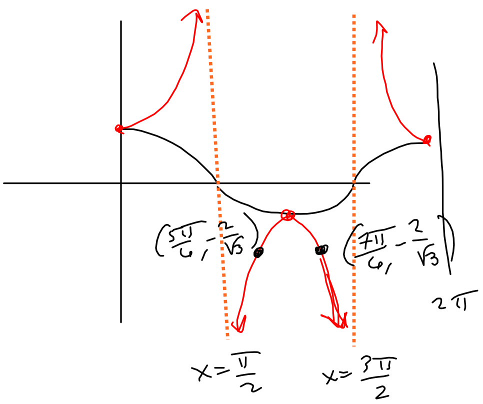
$\arccos(x) = \cos^{-1}(x)$

$$\boxed{x \in \{\frac{5\pi}{6}, \frac{7\pi}{6}, -\frac{7\pi}{6}, -\frac{5\pi}{6}\}}$$

Period of Secant is 2Pi.

$\frac{5\pi}{6} - 2\pi = \frac{5\pi - 12}{6} = -\frac{7\pi}{6}$

$\frac{7\pi}{6} - 2\pi = \ldots = -\frac{5\pi}{6}$



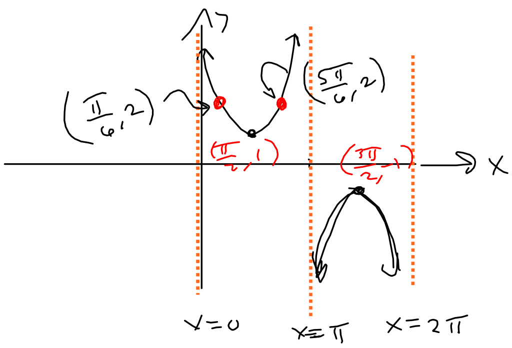

```
tan⁻¹(-1)
                -45
cos⁻¹(-√(3)/2)
                150
```

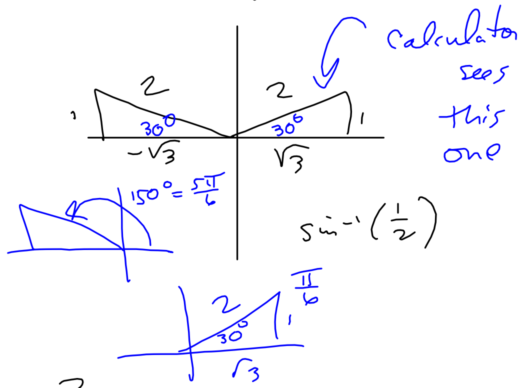
25. −/1 points LarTrig9 1.6.055.

Use a graph to solve the equation on the interval $[-2\pi, 2\pi]$. (Enter your answers as a comma-separated list.)

$\csc(x) = 2 \Longrightarrow \sin(x) = \frac{1}{2}$

Graph of csc(x) by reciprocal graph of sin(x) $\sqrt{2^2 - 1^2} = \sqrt{4-1} = \sqrt{3}$





$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}\right\}$

$T = 2\pi$, so $\frac{\pi}{6} - \frac{2\pi}{1} = \frac{-11\pi}{6}$, $\frac{5\pi}{6} - \frac{12\pi}{6} = -\frac{7\pi}{6}$

26. –/1 points LarTrig9 1.6.062.MI.

Use the graph of the function to determine whether the function is even, odd, or neither. Verify your answer algebraically.

$f(x) = x^2 - 3\sec x$ = Even

Evens: x^{2n}, $n \in \mathbb{Z}$

$\cos(x)$, $\sec(x)$

$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{-}{+} = -$ ODD

+ Even: $f(-x) = f(x)$

− odd: $f(-x) = -f(x)$

Even ± Even = Even

ODD ± ODD = ODD

ODDS: x^{2n+1}, $n \in \mathbb{Z}$

$\sin(x)$, $\csc(x)$, $\tan(x)$, $\cot(x)$

$f(x) = \sin x + \tan x$ = ODD

$f(-x) = \sin(-x) + \tan(-x) = -\sin(x) - \tan(x) = -(\sin x + \tan x) = -f(x)$

27. –/1 points LarTrig9 1.6.063.

Use the graph of the function to determine whether the function is even, odd, or neither. Verify your answer algebraically.

$g(x) = x\csc x = (-)(-) = +$ = Even

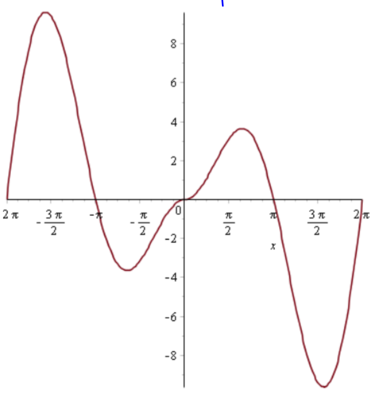
28. **–/2 points** LarTrig9 1.6.067.

Match the function with its graph.

$g(x) = |2x| \sin x$

Bleah
Not testing this skill by hand.

$|2x|$



29. **–/2 points** LarTrig9 1.6.072.

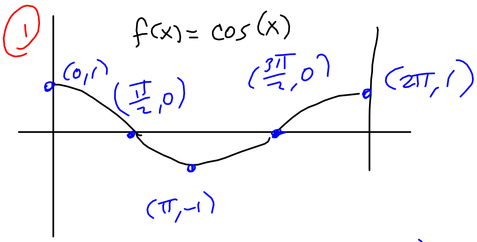
Graph the functions f and g.

$$f(x) = \cos^2 \frac{\pi x}{2}, \quad g(x) = \frac{1}{2}(1 + \cos \pi x)$$

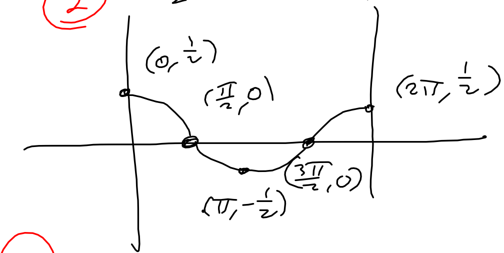
$\cos^2 x = \dfrac{1 + \cos(2x)}{2}$ are the same!

$= \frac{1}{2} + \frac{1}{2}\cos(\pi x) = g(x)$

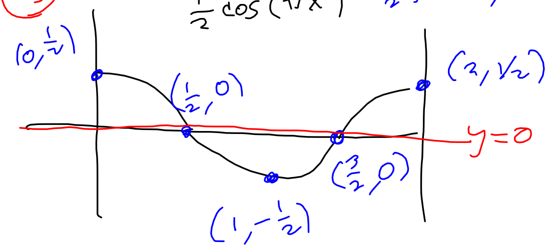
1 $f(x) = \cos(x)$



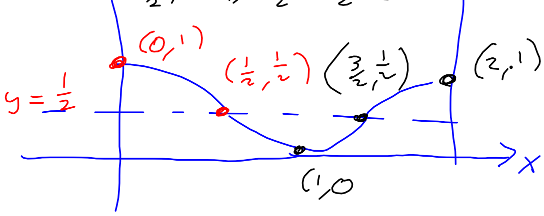
2 $\frac{1}{2}f(x) = \frac{1}{2}\cos(x)$



3 $\frac{1}{2}\cos(\pi x) = \frac{1}{2}f(\pi x)$



4 $\frac{1}{2}f(\pi x)+\frac{1}{2} = \frac{1}{2}\cos(\pi x)+\frac{1}{2}$



is same as $\cos^2(\frac{\pi}{2}x)$, by an identity you don't know yet.

30. –/2 points LarTrig9 1.6.073.

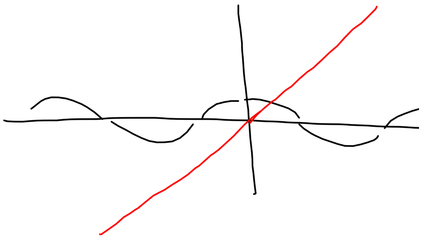
Use a graphing utility to graph the function and the damping factor of the function in the same viewing window.

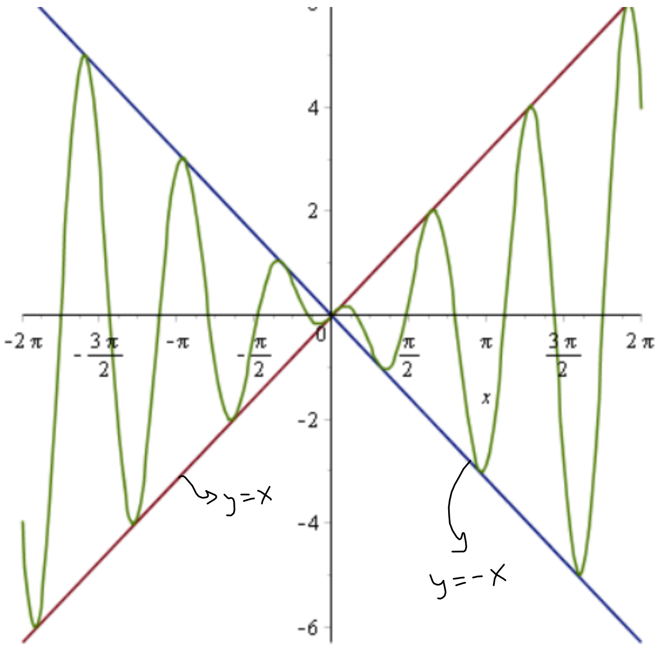
$g(x) = x \cos \pi x$

$\to y = x$ (I did $y = x$ & $y = -x$)

Damping Function

When $|x| < 1$, then $x \cos(\pi x)$ is a dampened cosine.





31. –/2 points LarTrig9 1.6.074.

Use a graphing utility to graph the function and the damping factor of the function in the same viewing window.

$f(x) = x^2 \cos x$

Same deal, only we bracket $f(x)$ between

$y = +x^2$ & $y = -x^2$

As previous : $x\cos(\pi x)$

NOTE:

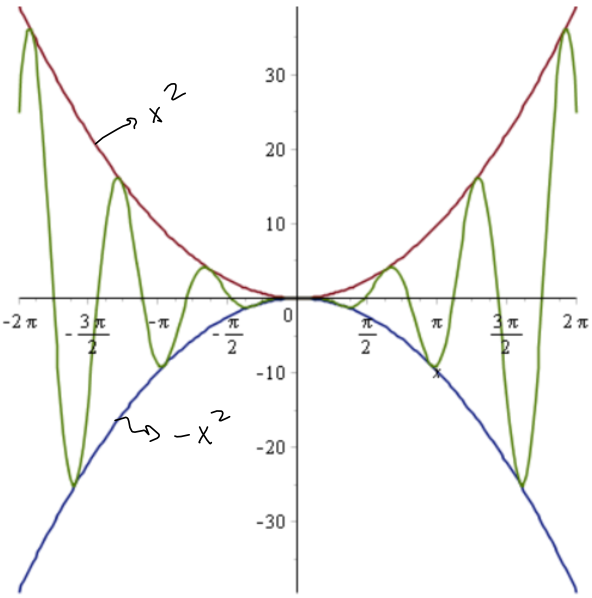
$x \cos(\pi x)$ is odd

$(-)(+) = -$ is odd

$(-x)\cos(\pi(-x)) = -x\cos(\pi x) = -(x\cos(\pi x))$

$x^2\cos(x)$ is EVEN:

$(+)(+) = +$ is even.



32. **–/2 points** LarTrig9 1.6.078.

Use a graphing utility to graph the function.

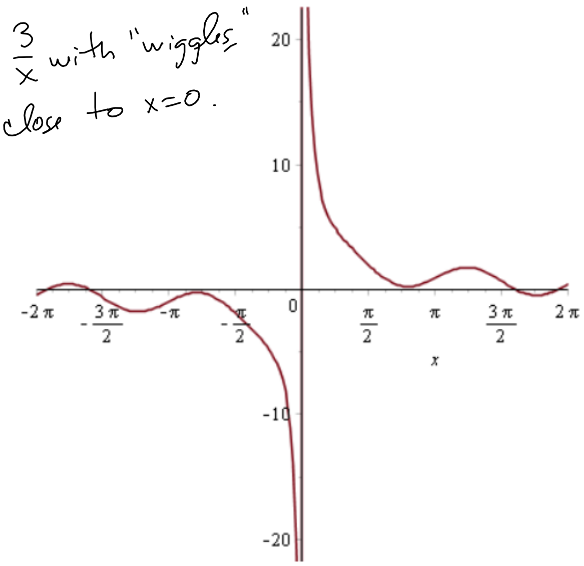
$$y = \frac{3}{x} + \sin 2x, \quad x > 0$$

Dominates as $|x| \to$ BIG

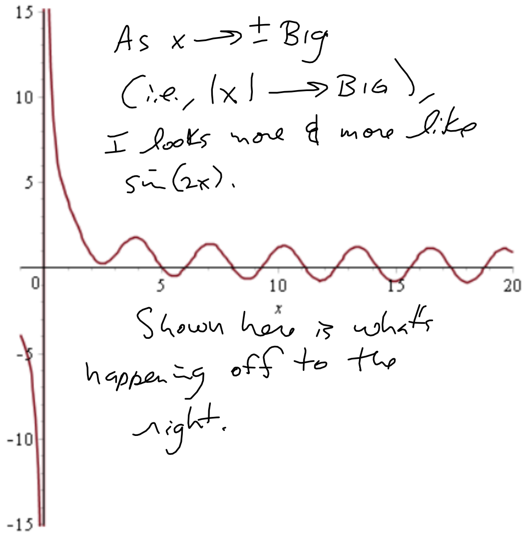
Blows up at $x = 0$ and dwindles to nothing as $|x| \to$ BIG

$$\frac{3}{x} \xrightarrow{x \to BIG} \text{SMALL}$$

$\frac{3}{x}$ with "wiggles" close to $x=0$.



As $x \to \pm$ Big (i.e., $|x| \to$ BIG), I looks more & more like $\sin(2x)$.



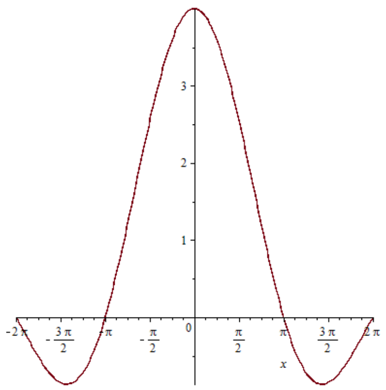
Shown here is what's happening off to the right.

33. –/2 points LarTrig9 1.6.079.

Use a graphing utility to graph the function.

$$g(x) = \frac{4 \sin x}{x}$$

This is an important example. It is not DEFINED at $x = 0$, but it approaches $y = 4$, as x approaches 0!



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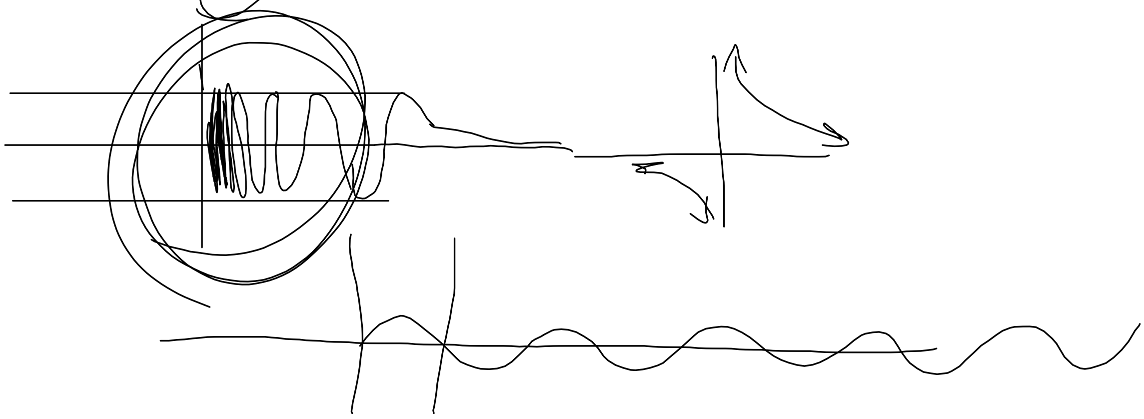
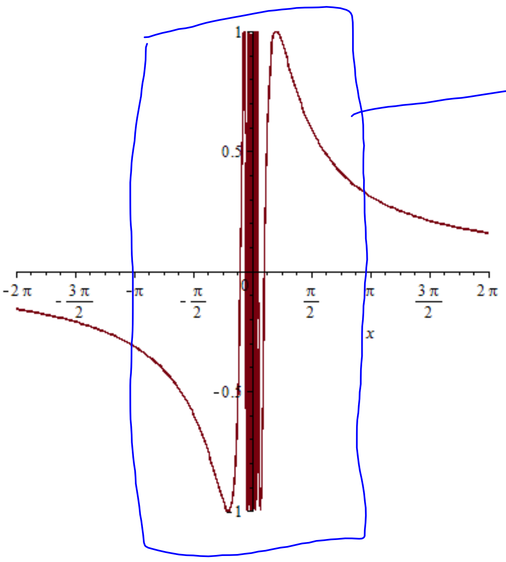
34. **–/2 points** LarTrig9 1.6.081.

Use a graphing utility to graph the function.

$f(x) = \sin \dfrac{1}{x}$

as $x \to 0^+$, $\dfrac{1}{x} \longrightarrow \infty$!





Infinitely many "waves" packed in here, between 0 & $\frac{\pi}{2}$.

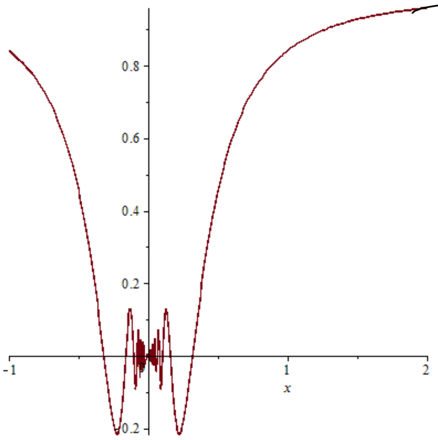
35. **–/2 points** LarTrig9 1.6.082.

Use a graphing utility to graph the function.

$$h(x) = x \sin \frac{3}{x}$$

When we dampen this "Topologist's Sine Curve," it still wiggles infinitely often in the vicinity of $x = 0$; however, the x times the sin(1/x) dampens it down to ZERO near $x = 0$.

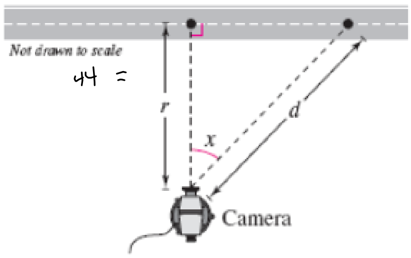
In Calculus, we show that this actually has a limit as x approaches zero, and the limit is zero.

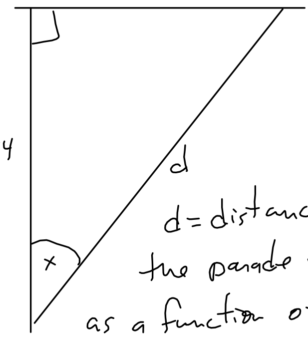


Approaches $y = 1$ as $x \to \infty$.

36. –/2 points LarTrig9 1.6.084.MI.

A television camera is on a reviewing platform r = 44 meters from the street on which a parade will be passing from left to right (see figure).





d = distance to the parade in meters, as a function of the angle x, given in radians.

$d = d(x) =$

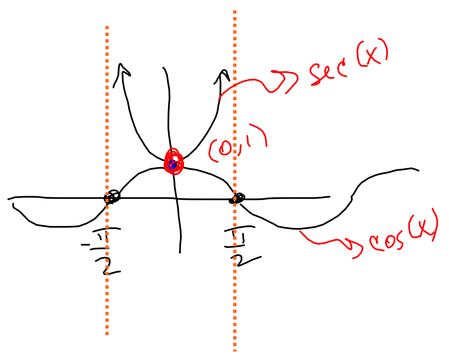
$\cos(x) = \frac{44}{d} \implies d = \frac{44}{\cos(x)}$

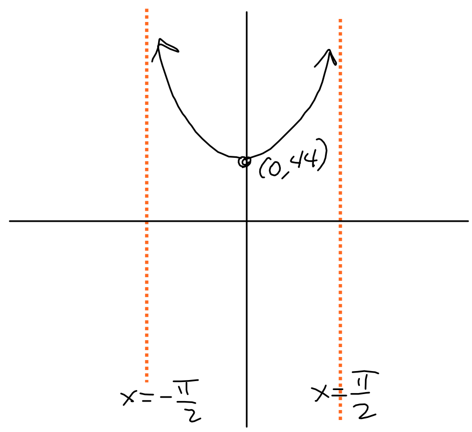
$= d = 44\sec(x)$

Write the distance d from the camera to a particular unit in the parade as a function of the angle x. (Consider x as negative when a unit in the parade approaches from the left.)

Graph the function over the interval $-\pi/2 < x < \pi/2$.

$44\sec(x)$

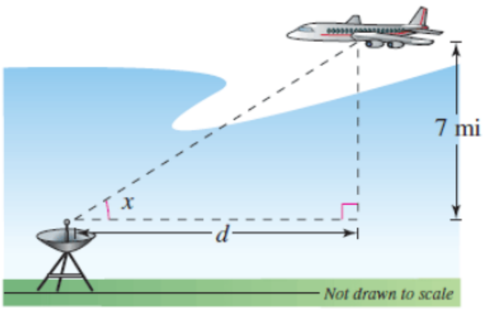




37. –/2 points LarTrig9 1.6.085.

A plane flying at an altitude of 7 miles above a radar antenna will pass directly over the radar antenna (see figure). Let d be the ground distance from the antenna to the point directly under the plane and

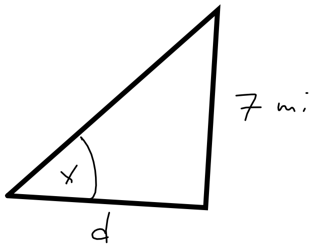
let x be the angle of the elevation to the plane from the antenna. (d is positive as the plane approaches the antenna.)



d = ground distance from plane to antenna, in miles

x = angle of elevation to the plane. in radians

Want $d = d(x)$

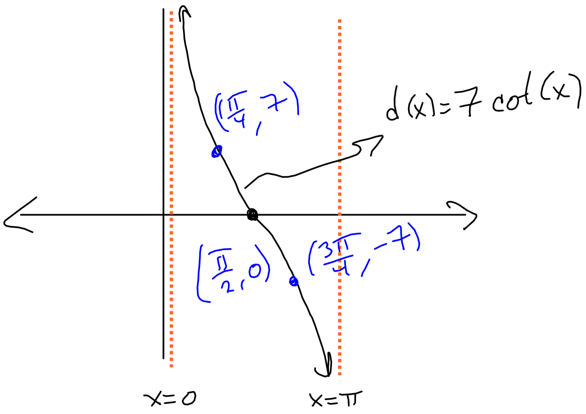


$\frac{7}{d} = \tan(x)$

$\frac{7}{\tan(x)} = d = \boxed{7\cot(x)}$

Write d as a function of x.

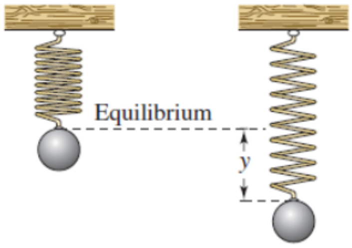
Graph the function over the interval $0 < x < \pi$.



38. –/2 points LarTrig9 1.6.506.XP.

An object weighing W pounds is suspended from the ceiling by a steel spring (see figure).

The weight is pulled downward (positive direction) from its equilibrium position and released. The resulting motion of the weight is described by the function $y = \frac{1}{2}e^{-t/7}\cos 7t$, $t > 0$, where y is the distance (in feet) and t is the time (in seconds).



(a) Use a graphing utility to graph the function.

(b) Describe the behavior of the displacement function for increasing values of time t.

As t increases, $y \to$ ______ .

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