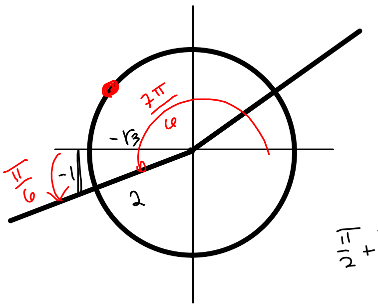
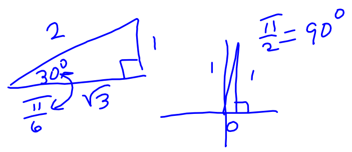
Basic Graphs of Sine and Cosine





x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{7\pi}{6}$
sin x	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$

$$\frac{\pi}{2}+\frac{\pi}{6}=\frac{3\pi+1\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}$$

Main focus
will be highs, lows,
midline, period, phase shift

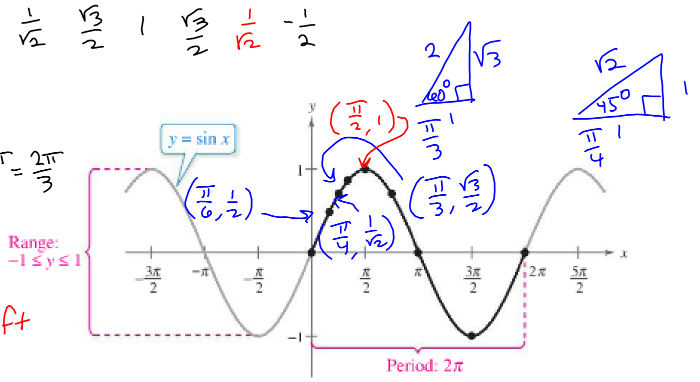
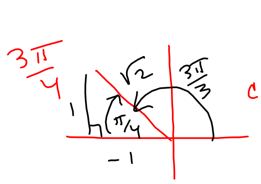
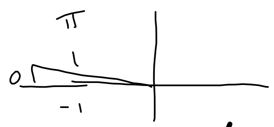


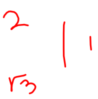
Figure 1.36



x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{7\pi}{6}$
cos x	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	-1	$-\frac{\sqrt{3}}{2}$

Sn





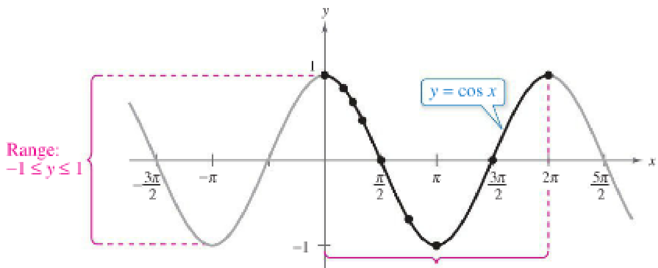
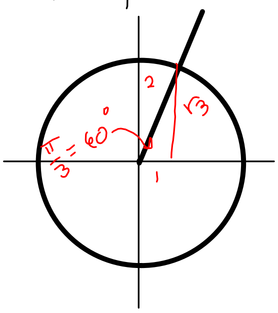
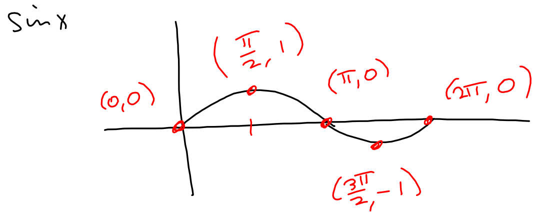
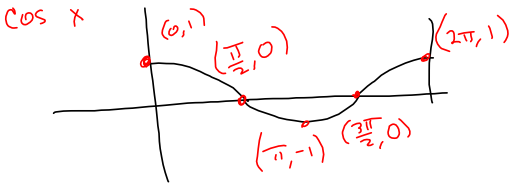


Figure 1.37



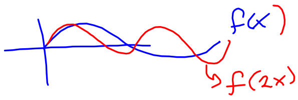


Label Key Points as ordered pairs.



Basics of transformin graphs,
from $f(x)$ to $af(bx-c)+d$

1. $af(x)$ vertical stretch by factor of a: $(x,y) \longrightarrow (x, ay)$
2. $f(bx)$ horizontal stretch by " " $\frac{1}{b}$: $(x,y) \longrightarrow (\frac{1}{b}x, y)$
3. $f(x-c)$ Right phase shift by c units: $(x,y) \longrightarrow (x+c, y)$
4. $f(x)+d$ Up d units $(x,y) \longrightarrow (x, y+d)$



NOTES: 1. Stuff inside f SEEMS backwards, but $f(x-5)$ is a DELAY of 5 units,
$f(x-c)$, $f(bx)$
and $f(5x)$ makes f move 5 times "faster," which shrinks the period (SHRINKS pic towards y-axis)

2. Always do vertical shift LAST, in your analysis.

Two Main Methods:

(M1) Do $f(bx)$ before $f(x-c)$ BOOK METHOD/BOOK INTERPRETATION
(M2) " $f(x-c)$ " $f(bx)$

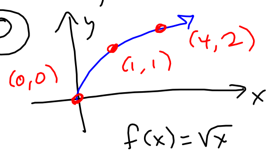
Graphing by Transforming

Basic Functions

An example from College Algebra

Graph $g(x) = 3\sqrt{2x-5} - 11$, step-by-step, one transformation per step.

Basic Graph: $f(x) = \sqrt{x}$ 0



$f(x) = \sqrt{x}$

METHOD 1 (Closer to Book Method

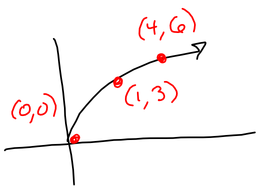
M1 $bx - c$ becomes $b(x - \frac{c}{b})$

& we do $f(bx)$, $f(b(x-\frac{c}{b}))$

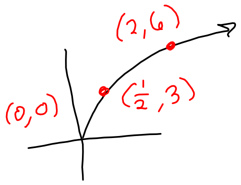
1 $af(x)$ 2 $f(bx)$ 3 $f(x-c)$ 4 $f(x)+d$

$3\sqrt{x}$ $3\sqrt{2x}$ $3\sqrt{2(x-\frac{5}{2})}$ $3\sqrt{2(x-5/2)} - 11$

1 $3f(x) = 3\sqrt{x}$

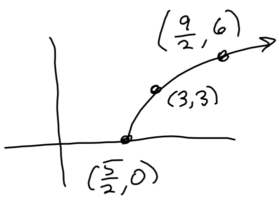


2 $3f(2x) = 3\sqrt{2x}$



3 $3f(2(x-\frac{5}{2}))$

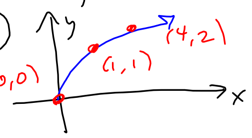
$2x-5 = 2(x-\frac{5}{2})$



METHOD 2! SAME example from College Algebra

Graph $g(x) = 3\sqrt{2x-5} - 11$, step-by-step, one transformation per step.

Basic Graph: $f(x) = \sqrt{x}$ (0)

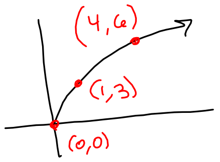


METHOD 2 we switch Steps 2 & 3

(1) $af(x)$ (3) $f(x-c)$ (2) $f(bx)$ (4) $f(x)+d$

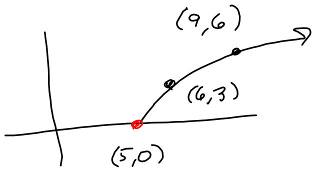
$3\sqrt{x}$ $3\sqrt{x-5}$ $3\sqrt{2x-5}$ $3\sqrt{2x-5} - 11$

(1) $3\sqrt{x} = 3f(x)$

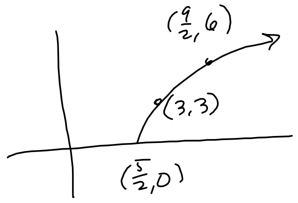


Right 5

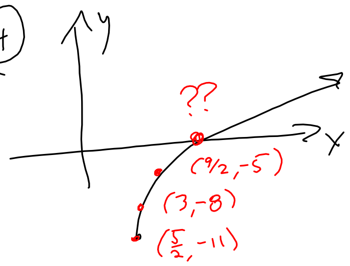
(3) $3\sqrt{x-5} = 3f(x-5)$



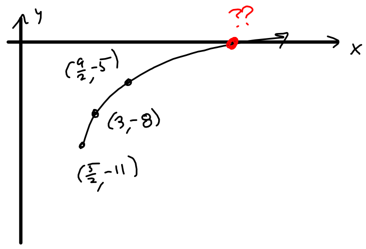
(3) $3\sqrt{2x-5}$



(4)



(4) $3f(2(x-\frac{5}{2})) - 11$

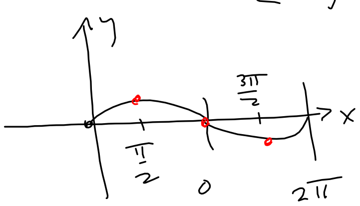


1. **Question Details**

Fill in the blank.

One period of a sine function is called ---Select--- of the sine curve.

one cycle

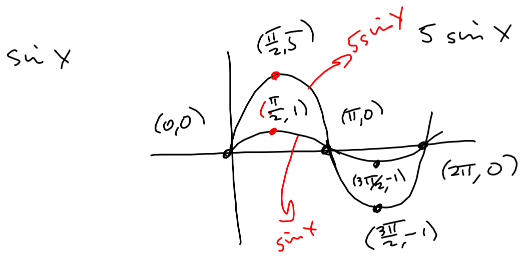


2. **Question Details**

Fill in the blank.

The ---Select--- of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.

amplitude



3. Question Details

Fill in the blank.

For the function $y = a\sin(bx - c)$, $\frac{c}{b}$ represents the ---Select--- of the graph of the function.

phase shift

$\sin(3x-6)$

$= \sin(3(x-2))$

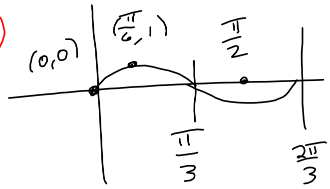
shrink by $\frac{1}{3}x$

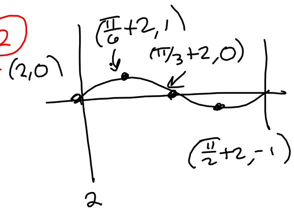
Right 2 units

$\boxed{M1}$

$\sin(3x)$

$\sin(3(x-2))$

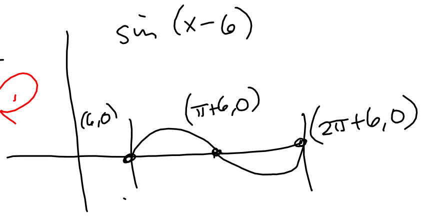




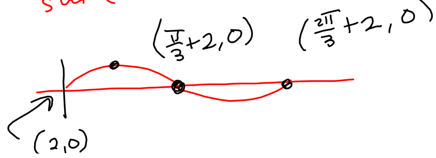
$\frac{\pi}{3} + \frac{\pi}{6} = \frac{3\pi}{6}$

M2

$\sin(x-6)$



$\sin(3x-6)$



4. **Question Details**

Fill in the blank.

For the function given by $y = d + a\cos(bx - c)$, d represents a ---Select--- of the graph of the function.

Vertical Shift

"Vertical Translation."

"up d units."

5. **Question Details**

Find the period and amplitude.

$y = 8\sin 7x$

$a = 8 = \text{amplitude}.$

$\boxed{\text{sine has period } 2\pi.}$

when is $7x = 2\pi$?

$\Rightarrow x = \frac{2\pi}{7} = \text{period}.$

6. **Question Details**

Find the period and amplitude.

$y = \frac{5}{6}\cos\frac{x}{2}$

$\frac{x}{2} = 2\pi$

$x = 4\pi = T$

$$\boxed{a = \frac{5}{6} \qquad T = 4\pi}$$

7. **Question Details**

Find the period and amplitude.

$$y = \frac{4}{7} \sin \frac{\pi x}{6}$$

$\boxed{a = \frac{4}{7} \quad T = 12}$

Repetitive...

$$\frac{\pi x}{6} = 2\pi$$

$$x = \frac{12\pi}{\pi} = 12 = T$$

8. **Question Details**

Find the period and amplitude.

$$y = \frac{1}{9} \sin 2\pi x$$

$\boxed{a = \frac{1}{9} \quad T = 1}$

9. **Question Details**

Describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

$$f(x) = \sin x$$
$$g(x) = \sin(x - 5\pi)$$

Book does multiple choice.

* gag *

g is right shift by 5π units of f.

DELAY

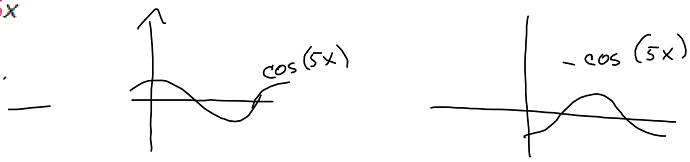
10. Question Details

Describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

$f(x) = \cos 5x$
$g(x) = -\cos 5x$

g is Vertical flip of f.

g is f reflected about the x-axis.

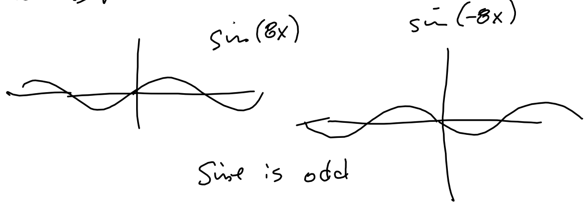


11. Question Details

Describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

$f(x) = \sin 8x$
$g(x) = \sin(-8x)$

g is reflection about y-axis of f.
"horizontal flip"



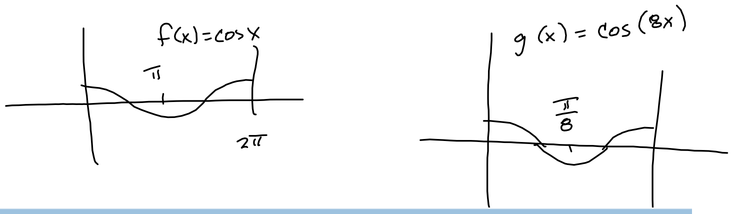
Sine is odd

12. **Question Details**

Describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

$f(x) = \cos x$
$g(x) = \cos 8x$

g is horizontal "shrink" of f by $\frac{1}{8}$ - factor



13. **Question Details**

Describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

$f(x) = \sin 8x$
$g(x) = 4 + \sin 8x$

g is vertical shift by 4 units up of f.

$g(x) = f(x) + 4$

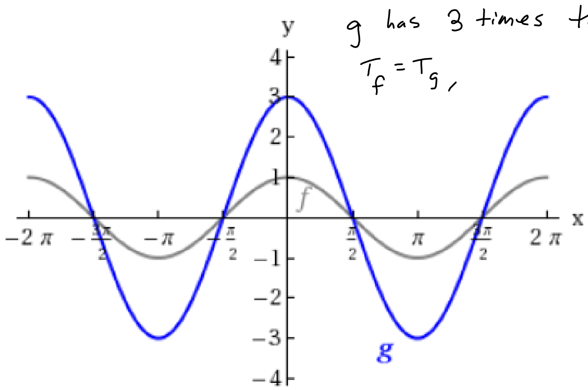
14. **Question Details**

Describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

g has 3 times the amplitude

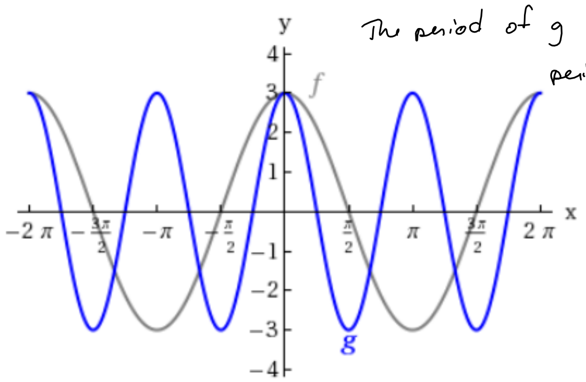
$T_f = T_g$,

$g(x) = 3f(x)$



15. Question Details

Describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

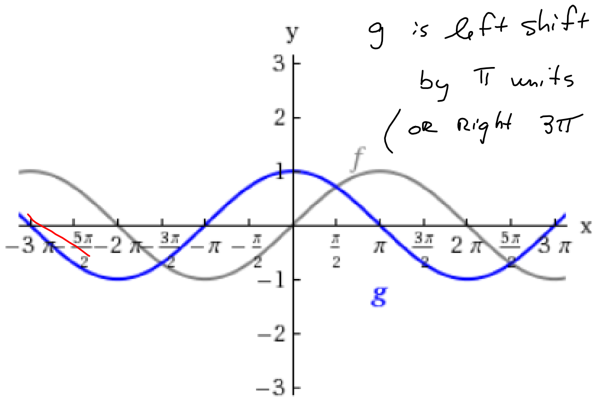


The period of g is $\frac{1}{2}$ the period of f.

$g(x) = f(2x)$

16. Question Details

Describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

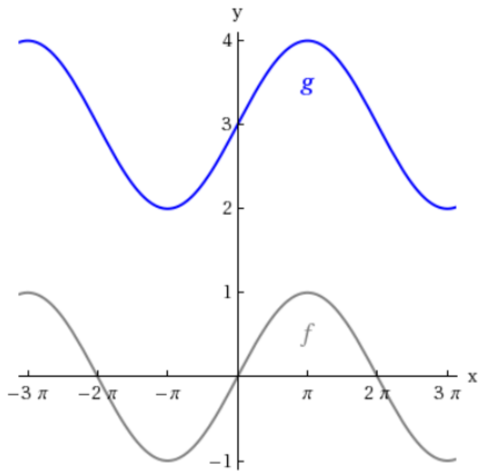


g is left shift of f by π units (or Right 3π)

$g(x) = f(x+\pi)$

17. Question Details

Describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.



$g(x) = f(x) + 3$

g is f(x) translated upward 3 units

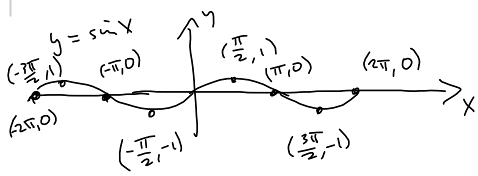
18. Question Details

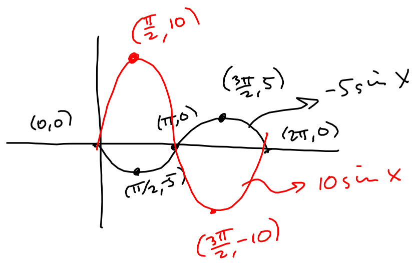
Sketch the graphs of f and g in the same coordinate plane. (Include two full periods.)

$f(x) = -5 \sin x$

$g(x) = 10 \sin x$

ONE PERIOD





19. **Question Details**

Sketch the graphs of f and g in the same coordinate plane. (Include two full periods.)

Sucks

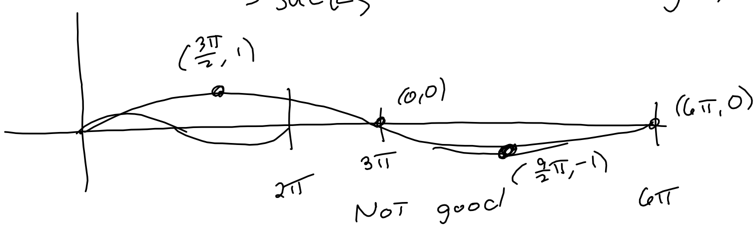
ugh!

$f(x) = \sin x$

$g(x) = \sin \frac{x}{3}$

$\frac{x}{3} = 2\pi$

$x = 6\pi = T$

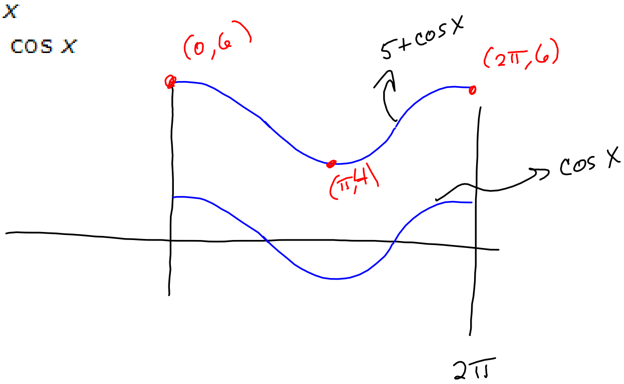


20. **Question Details**

Sketch the graphs of f and g in the same coordinate plane. (Include two full periods.)

$f(x) = \cos x$

$g(x) = 5 + \cos x$

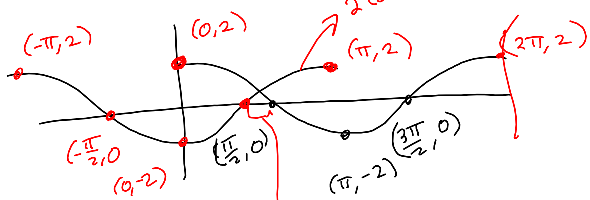


21. **Question Details**

Sketch the graphs of f and g in the same coordinate plane. (Include two full periods.)

$$f(x) = 2\cos x$$
$$g(x) = 2\cos(x + \pi)$$

$2\cos(x+\pi)$ LEFT π!



Should be same point. I graph badly.

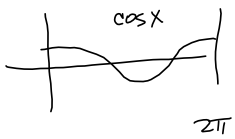
22. Question Details

Sketch the graph of the function. (Include two full periods.)

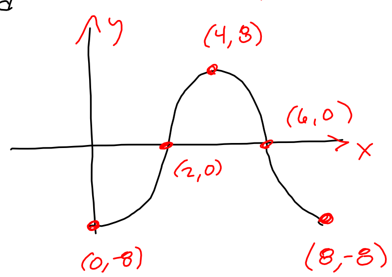
$$y = -8\cos\frac{\pi x}{4} \Rightarrow \frac{\pi x}{4} = 2\pi$$

$$x = \frac{8\pi}{\pi} = \boxed{8 = T} = \text{period}$$

vertical flip
vertical stretch
factor of 8



$-8\cos\left(\frac{\pi x}{4}\right)$



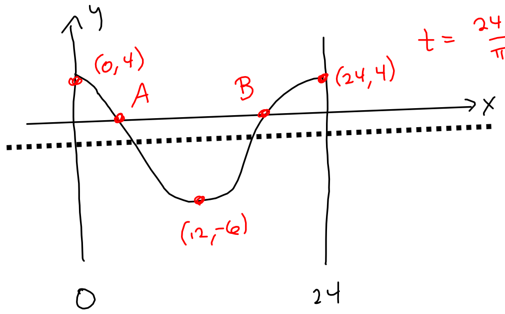
23. Question Details

Sketch the graph of the function. (Include two full periods.)

$$y = -1 + 5\cos\frac{\pi t}{12}$$

$\frac{\pi t}{12} = 2\pi$

$t = \frac{24\pi}{\pi} = 24 = T$



A & B:
Solve $5\cos\left(\frac{\pi t}{12}\right) - 1 = 0$ for x-intercepts.

LATER. Not Now!

24. Question Details

Sketch the graph of the function. (Include two full periods.)

$y = -2\cos(10x + \pi)$

Phase Shift!

Flip & stretch vertically by factor of 2. (or just $-2y$)

$10x + \pi$

$= 10\left(x + \frac{\pi}{10}\right)$ → Left $\frac{\pi}{10}$

Horizontal Shrink: $\frac{1}{10}x$

T: $10x = 2\pi$

$x = \frac{\pi}{5} = T$

Start point:

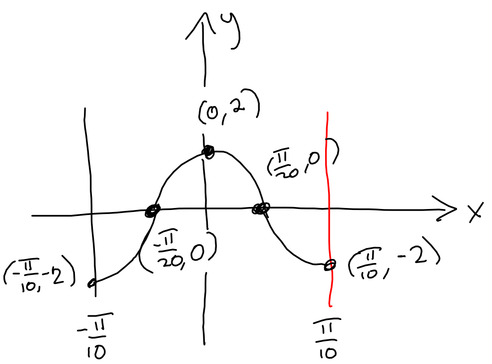
$x = -\frac{\pi}{10}$

Scratch:

$10x + \pi = 0$

$10x = -\pi$

$x = -\frac{\pi}{10}$



$-\frac{\pi}{10} + \frac{\pi}{5} = \frac{\pi}{10}$

25. Question Details

Sketch the graph of the function. (Include two full periods.)

$$y = \frac{1}{2}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{2}\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$$

$a = \frac{1}{2}$

Right $\frac{\pi}{2}$

(start @ $\frac{\pi}{2}$)

Scratch:

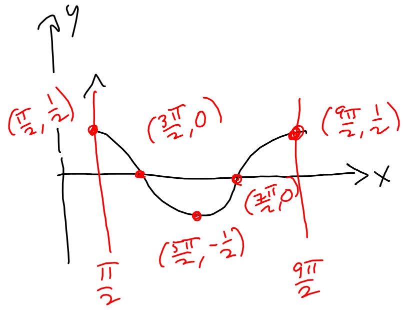
$\frac{x}{2} - \frac{\pi}{4} = \frac{1}{2}\left(x - \frac{\frac{\pi}{4}}{\frac{1}{2}}\right) = \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

$\frac{1}{2}x = 2\pi$

$x = 4\pi = T$

$\frac{\pi}{2} + 4\pi = \frac{\pi}{2} + \frac{8\pi}{2} = \frac{9\pi}{2}$

End



26. Question Details

The function g is related to a parent function $f(x) = \sin(x)$.

$g(x) = \sin(2x + \pi) = \sin\left(2\left(x+\frac{\pi}{2}\right)\right)$

(a) Describe the sequence of transformations from f to g.

The function $g(x)$ is obtained by a ---Select--- of 2 and a ---Select--- of $\frac{\pi}{2}$ to the left.

horiz. shrink phase shift

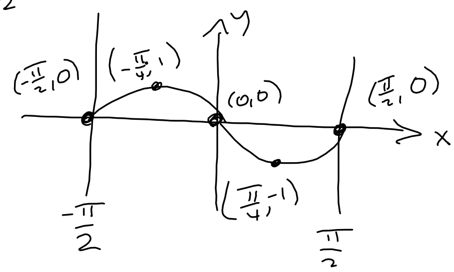
(b) Sketch the graph of g.

$2x = 2\pi$

$x = \pi = T$

left $\frac{\pi}{2}$

$-\frac{\pi}{2} + \pi = \frac{-\pi + 2\pi}{2} = \frac{\pi}{2}$



27. Question Details

Use a graphing utility to graph the function. Include two full periods. Be sure to choose an appropriate viewing window.

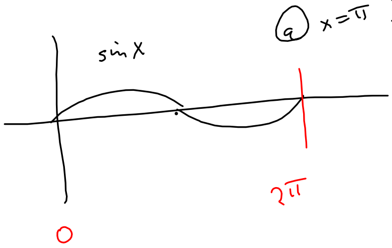
$y = -4 \sin\left(\frac{1}{2}x - \frac{\pi}{2}\right) = -4 \sin\left(\frac{1}{2}(x - \pi)\right)$

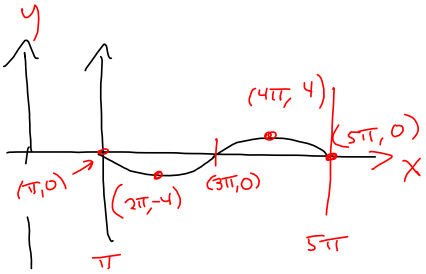
$-4y$ $\quad 2x \quad$ RIGHT π

(start "shape" @ $x = \pi$)

$\frac{1}{2}x = 2\pi$

$x = 4\pi = T$





$\pi + 4\pi = 5\pi$

28. Question Details LarTrig9 1.5.082. [2550698]

Use a graphing utility to graph y_1 and y_2 in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers x such that $y_1 = y_2$. (Enter your answers as a comma-separated list.)

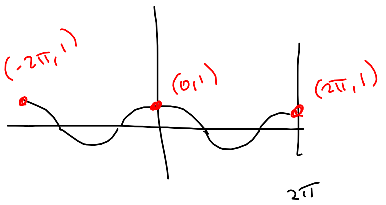
$y_1 = \cos x$
$y_2 = 1$

http://dlippman.imathas.com/graphcalc/graphcalc.html

DON'T click for free online grapher

Can't solve $\cos(x) = 1$

Lippman blew it!



$\{-2\pi, 0, 2\pi\}$

wolfram-alpha will graph. Solve separately

https://www.wolframalpha.com/input/?i=solve(cos(x)%3D1)

$\cos(x) = 1$ OR $\cos(x) - 1 = 0$

x = 2 π n and n element Z

$x = 2\pi n \;\; \forall\, n \in \mathbb{Z}$

29. Question Details

Write an equation for the function that is described by the given characteristics.

A sine curve with a period of 4π, an amplitude of 5, a left phase shift of $\pi/3$, and a vertical translation down 4 units.

Recall $\sin(bx)$'s period is found by solving

$bx = 2\pi$ for x (b was given)

If we know period, but not 'b', it's still the same equation,

Solve $bx = 2\pi$ for b, where $x = T = 4\pi$

$b \cdot 4\pi = 2\pi$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$: $\sin\left(\frac{1}{2}x\right)$ sitch from $T = 4\pi$.

$a = 5$: $5\sin\left(\frac{1}{2}x\right)$

left $\frac{\pi}{3}$: $5\sin\left(\frac{1}{2}\left(x + \frac{\pi}{3}\right)\right)$

Down 4 : $\boxed{5\sin\left(\frac{1}{2}\left(x + \frac{\pi}{3}\right)\right) - 4}$

30. **Question Details**

The table shows the maximum daily high temperatures in Las Vegas L and International Falls I (in degrees Fah

Month, t	Las Vegas, L	International Falls, I
1	57.1	13.8
2	63.0	22.4
3	69.5	34.9
4	78.1	51.5
5	87.8	66.6
6	98.9	74.2
7	104.1	78.6
8	101.8	76.3
9	93.8	64.7
10	80.8	51.7
11	66.0	32.5
12	57.3	18.1

HIGH: 104.1, $t = 7$

Low: 57.1, $t = 1$

Gives amplitude:

$104.1 - 57.1 = 47$

$\frac{47}{2} = 23.5$

$23.5 \cos(*)$

Midline:

$\frac{104.1 + 57.1}{2} = \frac{161.2}{2} = 80.6$

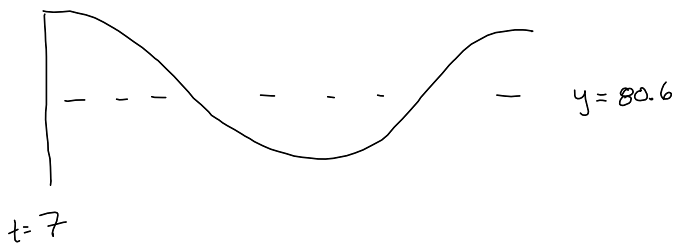
$y = 80.6$

$23.5 \cos(*) + 80.6$

(a) A model for the temperature in Las Vegas is given by

$$L(t) = 80.60 + 23.50 \cos\left(\frac{\pi t}{6} - 3.67\right).$$

Find a trigonometric model for International Falls. (Round all numerical values to one decimal place.)



Period: T=12 months.

$b(12) = 2\pi$

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

$23.6 \cos\left(\frac{\pi}{6}(t-7)\right)$

$= 23.6 \cos\left(\frac{\pi}{6} t - 3.67\right)$

oops! Now move it up!

$$\boxed{23.6 \cos\left(\frac{\pi}{6}(t-7)\right) + 80.6}$$

$\left(\frac{\pi}{6}\right)(7) \approx$ 3.665191429188092111539750613826086698230030965937623457804...