

$$\sin \theta =$$

$$\csc \theta =$$

$$\cos \theta =$$

$$\sec \theta =$$

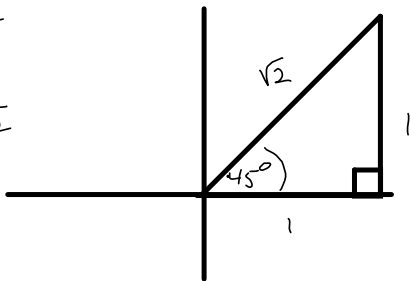
$$\tan \theta =$$

$$\cot \theta =$$

Section 1.4 - Trigonometric Functions of ANY Angle

Let's do the BASICS for Quadrant 1:

45-45 Right  
Triangle  
1-1- $\sqrt{2}$

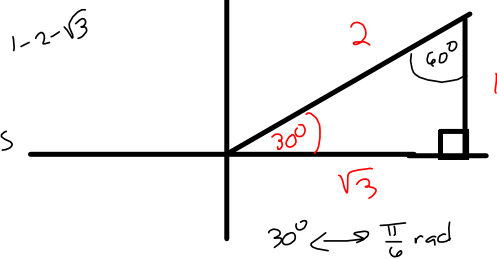


$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tan \frac{\pi}{4} = \frac{1}{1} = 1$$

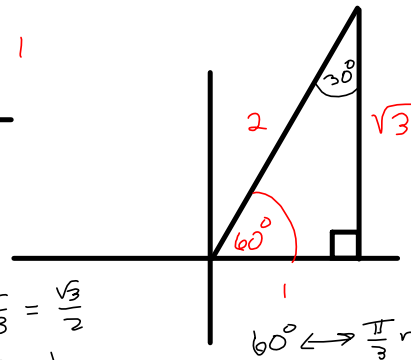
30-60 Right  
Triangle  
TWO ORIENTATIONS



$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

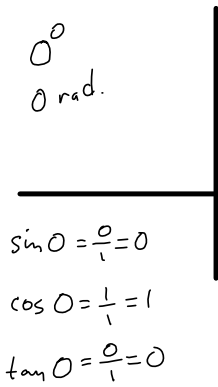


$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

Quadrant (al) Angles  
(Degenerate Triangles)



Hypotenuse is = 1 in all these. The "longer leg" is also "1", but it might be -1, depending on position

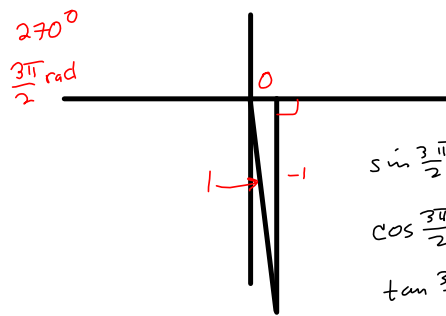


Learn to draw the triangles & memorize less.

$\sin \frac{\pi}{2} = 1$   
 $\cos \frac{\pi}{2} = 0$   
 $\tan \frac{\pi}{2} = \frac{1}{0}$

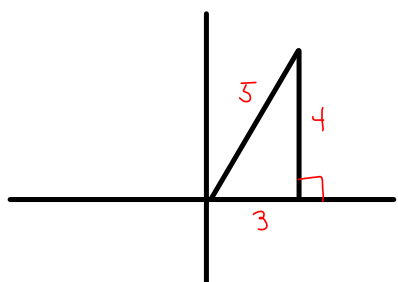


$\sin \pi = \frac{0}{1} = 0$   
 $\cos \pi = -1$   
 $\tan \pi = \frac{0}{-1} = 0$



$\sin \frac{3\pi}{2} = \frac{-1}{1} = -1$   
 $\cos \frac{3\pi}{2} = \frac{0}{1} = 0$   
 $\tan \frac{3\pi}{2} = \frac{-1}{0}$

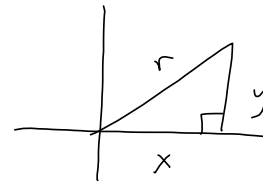
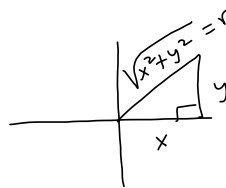
SPECIAL : 3-4-5 Right Triangle



1.  -1 points LarTrig9 1.4.001. My

Fill in the blank. Let  $\theta$  be an angle in standard position, with  $(x, y)$  a point on the terminal side of  $\theta$  and  $r = \sqrt{x^2 + y^2} \neq 0$ .

$$\sin \theta = \boxed{\phantom{000}} \quad r/y$$

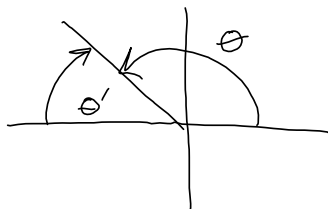


4. -1 points LarTrig9 1.4.008. My Notes Ask Your Tea

Fill in the blank.

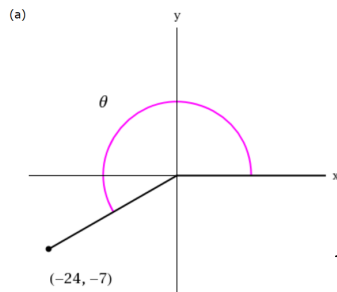
The acute positive angle that is formed by the terminal side of the angle  $\theta$  and the horizontal axis is called the  angle of  $\theta$  and is denoted by  $\theta'$ .

reference  
angle



5.  -12 points LarTrig9 1.4.010.

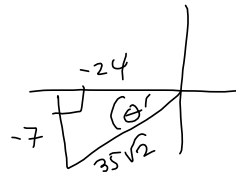
Determine the exact values of the six trigonometric functions of the angle  $\theta$ .



$$\sin \theta = \frac{-7}{35\sqrt{2}} = \frac{-1}{5\sqrt{2}}$$

$$\cos \theta = \frac{-24}{35\sqrt{2}}$$

$$\tan \theta = \frac{-7}{-24} = \frac{7}{24}$$



$$7^2 + 24^2 = 49 + 2401$$

$$= 2450$$

$$r = \sqrt{2450}$$

$$= 5.7\sqrt{2}$$

$$= 35\sqrt{2}$$

$\sin \theta$

$$\begin{array}{r} 3 \overline{) 49} \\ \underline{49} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{) 2450} \\ \underline{490} \\ 1960 \\ \underline{1960} \\ 0 \end{array}$$

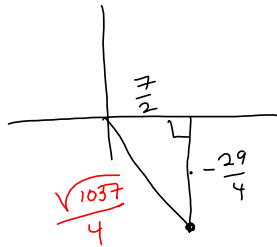
7. -6 points LarTrig9 1.4.018.

My Notes Ask

The point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

$$\left(3\frac{1}{2}, -7\frac{1}{4}\right)$$

$$3\frac{1}{2} = \frac{7}{2}, \quad 7\frac{1}{4} = \frac{29}{4}$$



$$\left(\frac{7}{2}\right)^2 + \left(-\frac{29}{4}\right)^2 = \frac{49}{4} + \frac{841}{16} = \frac{1037}{16}$$

$$\Rightarrow r = \frac{\sqrt{1037}}{4} \text{ in simplest terms}$$

$$\sin \theta = \frac{-\frac{29}{4}}{\frac{\sqrt{1037}}{4}} = -\frac{29}{4} \cdot \frac{4}{\sqrt{1037}} = \boxed{-\frac{29}{\sqrt{1037}} = \sin \theta}$$

$$\cos \theta = \frac{\frac{7}{2}}{\frac{\sqrt{1037}}{4}} = \left(\frac{7}{2}\right) \left(\frac{4}{\sqrt{1037}}\right) = \boxed{\frac{14}{\sqrt{1037}} = \cos \theta}$$

$$\tan \theta = \left(-\frac{29}{4}\right) \left(\frac{2}{7}\right) = \boxed{-\frac{29}{14} = \tan \theta}$$



9.  -1 points LarTrig9 1.4.020.

State the quadrant in which  $\theta$  lies.

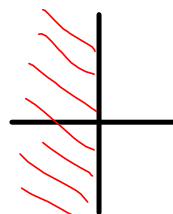
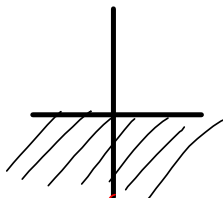
$$\sin \theta < 0 \text{ and } \cos \theta < 0$$

$$y < 0$$

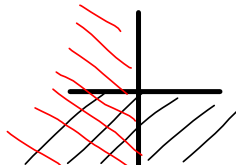
$$x < 0$$

$$\sin \theta < 0$$

$$\cos \theta < 0$$



*QIII  
satisfies  
both conditions.*



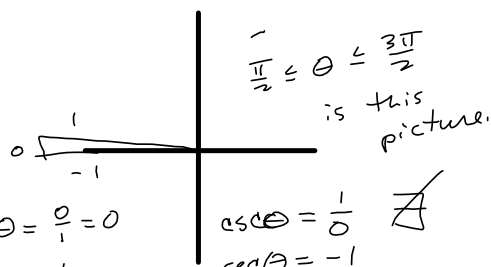
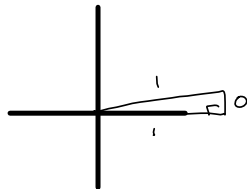
13. + -6 points LarTrig9 1.4.031. My No

Find the values of the six trigonometric functions of  $\theta$  with the given constraint. (If an answer is undefined, enter UNDEFINED.)

<b>Function Value</b>	<b>Constraint</b>
$\cot \theta$ is undefined.	$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$



$\cot \theta = \frac{x}{y} = \frac{x}{0}$  to be undefined.



$$\sin \theta = \frac{0}{1} = 0$$

$$\cos \theta = \frac{-1}{1} = -1$$

$$\tan \theta = \frac{0}{-1} = 0$$

$$\csc \theta = \frac{1}{0} \quad \text{[crossed out]}$$

$$\sec \theta = -1$$

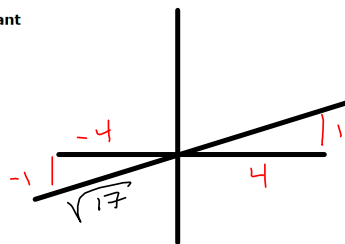
$$\cot \theta = \quad \text{[crossed out]}$$

14. -6 points LarTrig9 1.4.034.

My Notes Ask Your Tea

The terminal side of  $\theta$  lies on a given line in the specified quadrant. Find the values of the six trigonometric functions of  $\theta$  by finding a point on the line.

Line	Quadrant
$y = \frac{1}{4}x$	III



$$\sin \theta = -\frac{1}{\sqrt{17}}$$

$$\cos \theta = -\frac{4}{\sqrt{17}}$$

$$\tan \theta = \frac{1}{4}$$

$$m = \frac{1}{4} = \frac{\Delta y}{\Delta x} = \text{tangent!}$$

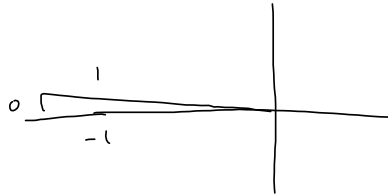
$$1^2 + 4^2 = 17$$

$$r = \sqrt{17}$$

15.  -1 points LarTrig9 1.4.037.

Evaluate the trigonometric function of the quadrant angle, if possible.

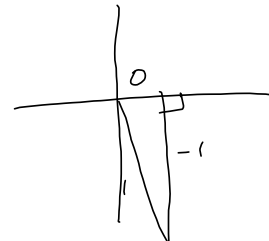
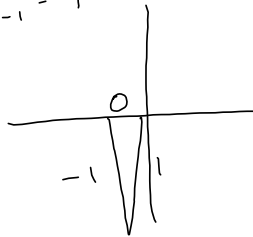
$$\cos \pi = -1$$



16.  -1 points LarTrig9 1.4.038.

Evaluate the trigonometric function of the quadrant angle.

$$\csc \frac{3\pi}{2} = \frac{1}{\sin(\frac{3\pi}{2})} = \frac{1}{-1} = -1$$

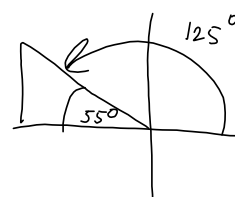
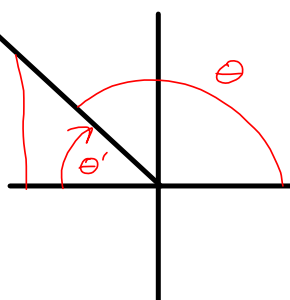


19.  -/2 points LarTrig9 1.4.045.

Find the reference angle  $\theta'$ .

$$\theta = 125^\circ$$

$$\begin{aligned}\theta' &= 180^\circ - 125^\circ \\ &= 55^\circ\end{aligned}$$



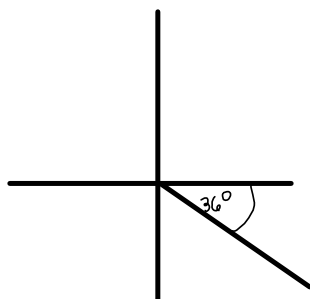
20.  -/2 points LarTrig9 1.4.046.


Find the reference angle  $\theta'$ .

$$\theta = 324^\circ$$

$$\theta' = 36^\circ$$

$$360^\circ - 324^\circ = 36^\circ$$



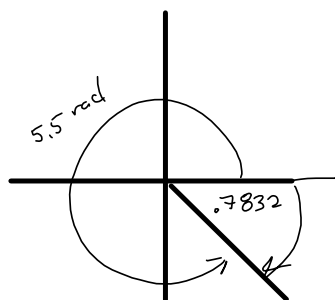
23.  -/2 points LarTrig9 1.4.051.

Find the reference angle  $\theta'$ . (Round your answer to four decimal places.)

$$\theta = 5.5$$

$$2\pi - 5.5 \approx 0.783185\dots$$

$$\approx .7832$$



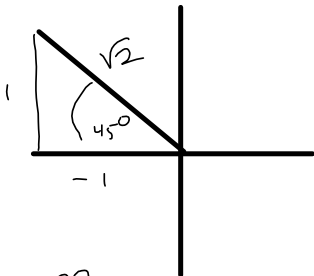
25. + -/3 points LarTrig9 1.4.058.

Evaluate the sine, cosine, and tangent of the angle without using a calculator.

1215°

$$\begin{array}{r} 1215 \\ -1080 \\ \hline 1350 \end{array}$$
  
 360  $\sqrt{1215}$  3r something  

$$\begin{array}{r} 360 \\ 3 \\ \hline 1080 \end{array}$$
  
 $1215 - 1080 = \text{ref. angle.}$



360	1
720	2
1080	3
1440	4

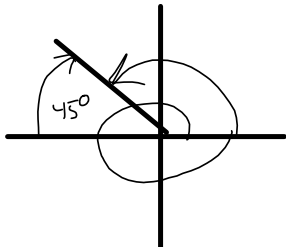
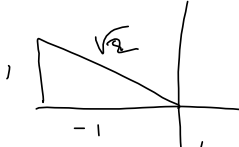
$\sin \theta = \frac{1}{\sqrt{2}}$   
 $\cos \theta = -\frac{1}{\sqrt{2}}$   
 $\tan \theta = \frac{1}{-1} = -1$

30. + -/3 points LarTrig9 1.4.068.

Evaluate the sine, cosine, and tangent of the angle without using a calculator.

$\theta = -\frac{11\pi}{4}$

$$\frac{\frac{11\pi}{4}}{2\pi}$$
  
 $= \frac{11}{4} \cdot \frac{1}{2} = \frac{11}{8} = 1\frac{3}{8}$ 
  
 once around plus

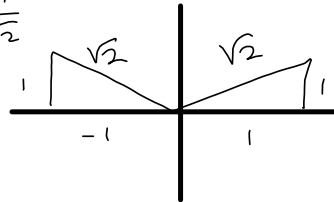
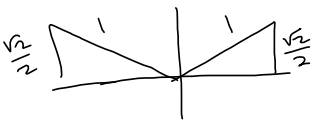
The picture.  
etc.

$$\frac{11\pi}{4} - 1(2\pi) = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$$

33. + -4 points LarTrig9 1.4.092. My Notes + Ask Your Teac

Find two solutions of the equation. Give your answers in degrees ( $0^\circ \leq \theta < 360^\circ$ ) and in radians ( $0 \leq \theta < 2\pi$ ). Do not use a calculator. (Do not enter your answers with degree symbols. Enter your answers as comma-separated lists.)

(a)  $\sin \theta = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$



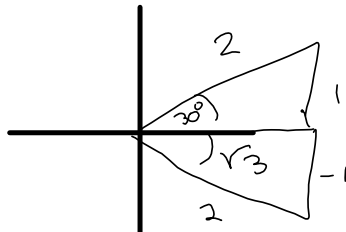
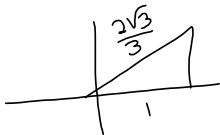
$\frac{\pi}{4}, \frac{3\pi}{4}$   
 $45^\circ, 135^\circ$

34. + -4 points LarTrig9 1.4.093. My Notes + Ask Your Teac

Find two solutions of each equation. Give your answers in degrees ( $0^\circ \leq \theta < 360^\circ$ ) and in radians ( $0 \leq \theta < 2\pi$ ). Do not use a calculator. (Do not enter your answers with degree symbols. Enter your answers as comma-separated lists.)

(a)  $\sec \theta = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

$\cos \theta = \frac{\sqrt{3}}{2}$



$30^\circ, 330^\circ$   
 $\frac{\pi}{6}, \frac{11\pi}{6}$

