MAT 122 Final Test, Part 1 – Fall, 2019 Name_____
100 Points Comprehensive thru Chapter 4 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Leave space between problems. No prizes for saving paper, here. Figure this stuff out, and use your smarts to plant trees! Only use one column of work. Don't start a 2nd column to save paper. ALL I WANT ON THIS PAGE IS YOUR NAME.

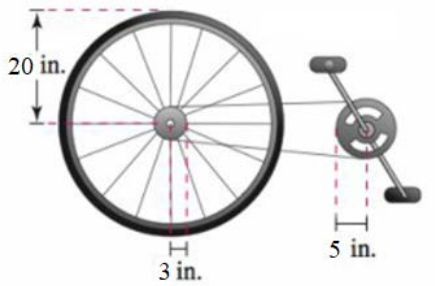
1. Basic concept: Draw the doggone pictures!
   a. (5 points) Sketch two triangles that satisfy $\sin(\theta)=\frac{-2}{\sqrt{11}}$.

   b. (5 pts) Assume the terminal side of the angle θ lies in the 4th quadrant (Quadrant IV). Find the other five trigonometric functions of θ.

   c. (5 pts) Again, assuming θ's terminal side lies in Q IV, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

   d. (5 pts) Give *all* solutions to the equation $\sin(\theta)=\frac{-2}{\sqrt{11}}$, in degrees *and* radians, rounded to three (3) decimal places.

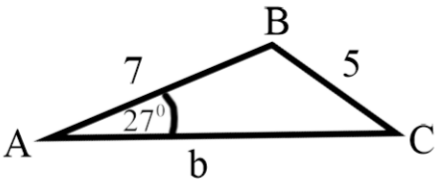
2. (20 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 20 inches, respectively. A cyclist is pedaling at a rate of 2 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.



3. (20 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u)=-\frac{\sqrt{2}}{3}$ and $\cos(u)>0$. Assume $0<u<2\pi$. Give final answers in simplified radical form.

4. Consider the triangle in the figure. Assume lengths are in centimeters.
   a. (10 pts) Prove that there are two possible solutions to this triangle.

   b. (10 pts) Use the Law of Sines to find angle C to 4 decimal places.



5. Let $z = 16\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)$

   a. (10 pts) Find all 4th roots of *z*.

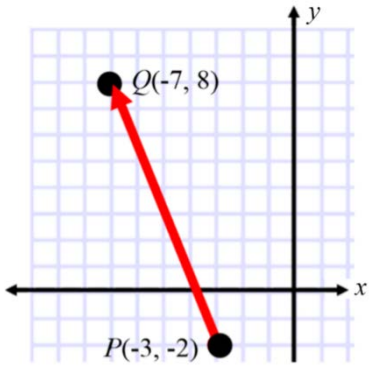
   b. (10 pts) Let $w = 3\left(\cos\left(\frac{\pi}{7}\right) + i\sin\left(\frac{\pi}{7}\right)\right)$ and find the product *zw* in trigonometric form.

Bonus: Answer *two* of the following problems, for *up to* 20 points:

1. (10 pts) Find all solutions θ of the trigonometric polynomial $4\sin^3(\theta) + 2\sin^2(\theta) - 2\sin(\theta) - 1 = 0$ in the interval $[0, 2\pi)$.

2. (10 pts) If you worked the first bonus problem, above, you're half done with this one. Find all solutions θ of the trigonometric polynomial $4\sin^3(2\theta) + 2\sin^2(2\theta) - 2\sin(2\theta) - 1 = 0$ in the interval $[0, 2\pi)$.

3. (10 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to $\sin\left(\text{arcsec}\left(\frac{\sqrt{9+4x^2}}{2x}\right)\right)$. Assume that everything is taking place in the 1st quadrant.



4. (10 pts) Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. Find the direction angle of $\vec{u}$. Use degrees, rounded to 4 places.

5. (10 pts) Build a cosine function that achieves its maximum height of $y = 200$ meters at time $x = 7$ seconds and its minimum height of $y = -130$ meters at $x = 19$ seconds.