MAT 122
100 Points

Test 4 – Fall, 2019

Name KEY
NO GRAPHING CALCULATORS!!!

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

1. Let $f(x)=3x^3-8x^2+19x-10$

a. (10 pts) Use synthetic division to find $f(3)$.

b. (10 pts) Use synthetic division to show that $x=1+2i$ is a solution of the equation $f(x)=0$.

c. (10 pts) Split f into linear factors, that is, factor f all the way.

2. Let $z=3\sqrt{6}-3\sqrt{6}i$

a. (10 pts) Find $z+\bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.

b. (10 pts) Express z in trigonometric form.

3. Let $z=16\left(\cos\left(\frac{5\pi}{4}\right)+i\sin\left(\frac{5\pi}{4}\right)\right)$

a. (10 pts) Express z in standard form.

b. (10 pts) Find the principal 4th root of z, i.e., find $\sqrt[4]{z}$. Leave z in trigonometric form for this.

c. (10 pts) Now, find the *other* 4th roots of z, in trigonometric form.

d. (10 pts) Find the trigonometric form of z^4.

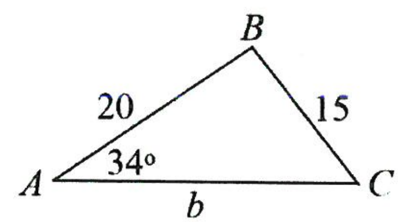
4. (10 pts) Find all solutions $\theta\in[0,2\pi)$ of the trig equation $4\sin^3(2\theta)+12\sin^2(2\theta)-3\sin(2\theta)-9=0$.
(Hint: If $f(x)=4x^3+12x^2-3x-9$, then $f(-3)=0$.)

Work up to 15 points' worth of bonus.

Bonus 1. Consider the triangle on the right.

a. (5 pts) Prove that there are two possible solutions to this triangle.

b. (5 pts) Use the Law of Sines to find the measure of angle C for the case where B is *acute*. (The case where B is obtuse is shown.) Give final answer accurate to 6 decimal places.



c. (5 pts) Use the Law of Cosines and your answer from part b to find the length of side b. Give final answer accurate to 6 decimal places.

Bonus 2. The vector $\bar{u}$ has a magnitude of $\|\bar{u}\|=60$ Newtons (N) and a direction angle $\theta=45^0$. The vector $\bar{v}$ has a magnitude of $\|\bar{v}\|=50$ and a direction angle of $\phi=120^0$.

a. (5 pts) Draw a diagram that describes this situation.

b. (5 pts) Express $\overline{u}$ and $\overline{v}$ in component form, in two ways: Give an exact answer, and an answer rounded to 3 decimal places.

c. (5 pts) Find the resultant force.

Bonus 3. (5 pts) Sketch the graph of $10\sin\left(\frac{\pi}{50}x-\frac{7\pi}{50}\right)-11$

Bonus 4. (5 pts) Find $\sin\left(\frac{u}{2}\right)$ and $\cos\left(\frac{u}{2}\right)$, given that $\cos(u)=\frac{3}{4}$ and $\sin(u)<0$. Give exact answers in simplified radical form for full credit.

Bonus 5. (5 pts) What quadrant does $2u$ lie in if $\cos(u)=\frac{3}{4}$ and $\sin(u)<0$?

Bonus 6. (5 pts) Find the cosine function that in one of its periods achieves a maximum at $(7,100)$ and a minumum at $(43,-200)$

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(1) $f(x) = 3x^3 - 8x^2 + 19x - 10$

(a) 10 pts

$$\begin{array}{r|rrrr} 3 & 3 & -8 & 19 & -10 \\ & & 9 & 3 & 66 \\ \hline & 3 & 1 & 22 & \boxed{56 = f(3)} \end{array}$$

(b) 10 pts

$$\begin{array}{r|rrrr} 1+2i & 3 & -8 & 19 & -10 \\ & & 3+6i & -17-4i & 10 \\ \hline 1-2i & 3 & -5+6i & 2-4i & 0 \\ & & 3-6i & -2+4i & \\ \hline & 3 & -2 & 0 & \end{array}$$

(c) 10 pts

So $\boxed{f(x) = (x-(1+2i))(x-(1-2i))(3x-2)}$

$$(1+2i)(-5+6i) = -5 + 6i - 10i + 12i^2$$
$$= -5 - 4i - 12 = -17 - 4i$$

$$(1+2i)(2-4i)$$
$$= 2(1+2i)(1-2i) = 2(1^2 + 2^2) = 10$$

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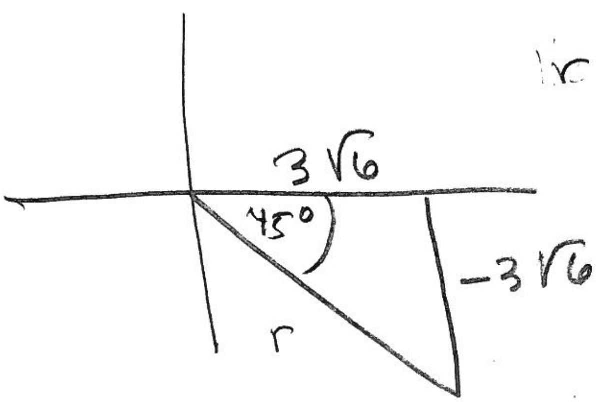
2. $z = 3\sqrt{6} - 3\sqrt{6}\, i = a+bi$

a) 10 pts $z + \bar{z} = \boxed{6\sqrt{6} = 2a = z+\bar{z}}$

$z\bar{z} = a^2+b^2 = (3\sqrt{6})^2 + (3\sqrt{6})^2$

$= 2(9\cdot 6) = 2(54) = \boxed{108 = z\bar{z}}$

b) $3\sqrt{6} - 3\sqrt{6}\, i$



$r = \sqrt{z\bar{z}} = \sqrt{108} = 6\sqrt{3}$

2 | 108
2 | 54
3 | 27
3 | 9
3

$r(\cos\theta + i\sin\theta)$

$= 6\sqrt{3}\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$ OR

$\boxed{6\sqrt{3}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = z}$

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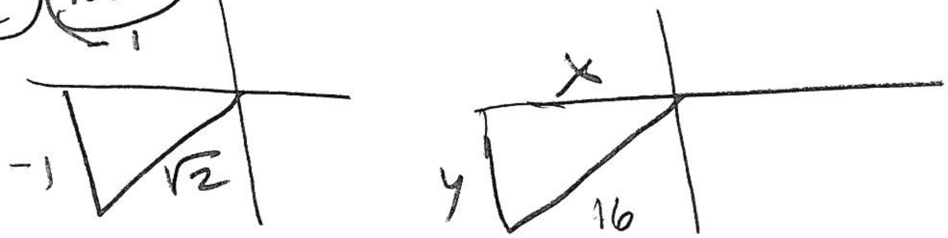
3 $z = 16\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$

a 10pts



$16 = \frac{16}{\sqrt{2}} \cdot \sqrt{2}$ so $x = -1\left(\frac{16}{\sqrt{2}}\right) = \frac{-16}{\sqrt{2}} = \frac{-16\sqrt{2}}{2}$

$= -8\sqrt{2} = x$

so $y = -8\sqrt{2}$, also

$$\boxed{z = -8\sqrt{2} - 8\sqrt{2}\, i}$$

b 10pts $\sqrt[4]{z}$; $16^{\frac{1}{4}} = 2 = r^{\frac{1}{4}}$

$\frac{\theta}{4} = \frac{\frac{5\pi}{4}}{4} = \frac{5\pi}{4} \cdot \frac{1}{4} = \frac{5\pi}{16}$

$$\boxed{\sqrt[4]{z} = 2\left(\cos\left(\frac{5\pi}{16}\right) + i \sin\left(\frac{5\pi}{16}\right)\right)}$$

C 10 pts The other 3 fourth roots:

Increment $= \frac{2\pi}{4} = \frac{8\pi}{16}$

$\frac{5\pi}{16} + \frac{8\pi}{16} = \frac{13\pi}{16}$

$\frac{13\pi}{16} + \frac{8\pi}{16} = \frac{21\pi}{16}$

$\frac{21\pi}{16} + \frac{8\pi}{16} = \frac{29\pi}{16}$

Check: $\frac{29\pi}{16} + \frac{8\pi}{16} = \frac{37\pi}{16} = \frac{32\pi + 5\pi}{16}$

$= 2\pi + \frac{5\pi}{16}$ Yes Want it coterminal with $\frac{5\pi}{16}$ & it is!

So:

$$\boxed{\begin{aligned} &2\left(\cos\left(\frac{13\pi}{16}\right) + i\sin\left(\frac{13\pi}{16}\right)\right), \\ &2\left(\cos\left(\frac{21\pi}{16}\right) + i\sin\left(\frac{21\pi}{16}\right)\right), \quad \& \\ &2\left(\cos\left(\frac{29\pi}{16}\right) + i\sin\left(\frac{21\pi}{16}\right)\right) \end{aligned}}$$

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3d 10 pts $z^4 = 16^4\left(\cos\left(4\cdot\frac{5\pi}{4}\right) + i\sin\left(\frac{4\cdot 5\pi}{4}\right)\right)$

$= \boxed{65536\left(\cos(5\pi) + i\sin(5\pi)\right)}$

$= \boxed{-65536}$!

4 10 pts All sol'ns $\theta \in [0, 2\pi)$ of

$4\sin^3(2\theta) + 12\sin^2(2\theta) - 3\sin(2\theta) - 9 = 0$

$4x^3 + 12x^2 - 3x - 9 = 0$ when $x = -3$, so

$$\begin{array}{r|rrrr} -3 & 4 & 12 & -3 & -9 \\ & & -12 & 0 & 9 \\ \hline & 4 & 0 & -3 & 0 \\ & x^2 & x & c & r \end{array}$$

$4x^2 - 3 = 0$

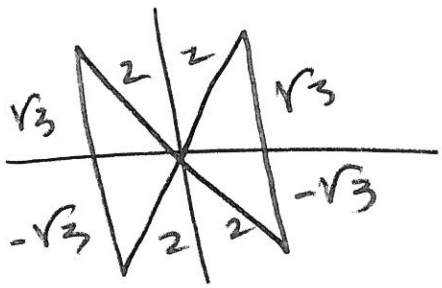
$4x^2 = 3$

$x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2} = \sin(2\theta)$

4 cnt'd. Now, $\theta \in [0, 2\pi) \Longrightarrow 2\theta \in [0, 4\pi)$

$$\sin(2\theta) = \pm \frac{\sqrt{3}}{2}$$



$$2\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

& Keep going

$$2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$$

$$2\pi + \frac{2\pi}{3} = \frac{8\pi}{3}$$

$$2\pi + \frac{4\pi}{3} = \frac{10\pi}{3}$$

$$2\pi + \frac{5\pi}{3} = \frac{11\pi}{3}$$

So, $2x$ = the above (8) sol'ns, giving us

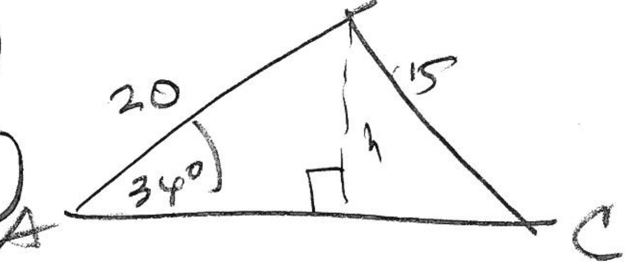
$$\boxed{x \in} \left\{ \frac{\pi}{6}, \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6} \right\}$$

$$= \boxed{\left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6} \right\}}$$

122 T4 (7)

B1

(a)



$\frac{h}{20} = \sin 34^\circ \Longrightarrow$

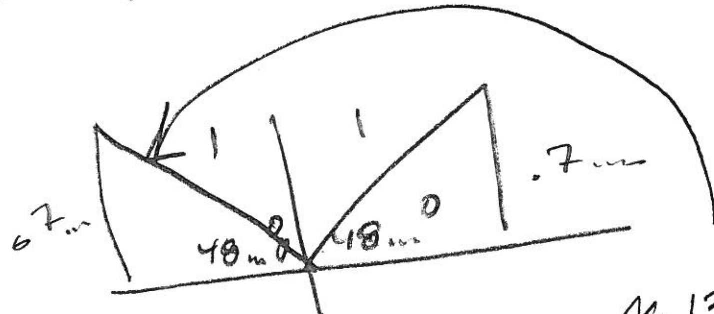
$h = 20 \sin 34^\circ \approx 11.18385807 < 15 < 20$

$\Longrightarrow$ 2 solutions

(b) $\frac{\sin C}{20} = \frac{\sin 34^\circ}{15} \Longrightarrow$

$\sin C = \frac{20 \sin 34^\circ}{15} \approx 0.745590538$

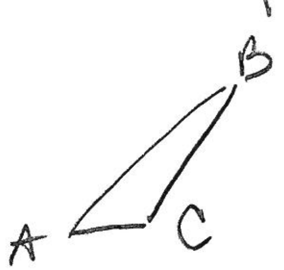
$\Longrightarrow C \approx \arcsin(0.745590538)$ $\boxed{\approx 48.20984677^\circ \approx C \text{ for obtuse } B}$



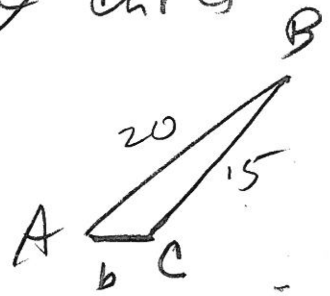
$180^\circ - 48.\ldots^\circ$

$\approx 131.7901532^\circ$

$\approx \boxed{131.7901532^\circ \approx C}$



B/C cnt'd



$A = 34^\circ$

$B = 180^\circ - A - C$

$\approx 180^\circ - 34^\circ - 131.7901532^\circ$

$\approx 14.20984677^\circ$

$b^2 = a^2 + b^2 - 2ab\cos B$

$= 15^2 + 20^2 - 2(15)(20)\cos B$

$\approx 625 - 600\cos(14.20984677^\circ)$

$\approx 625 - 581.6419064$

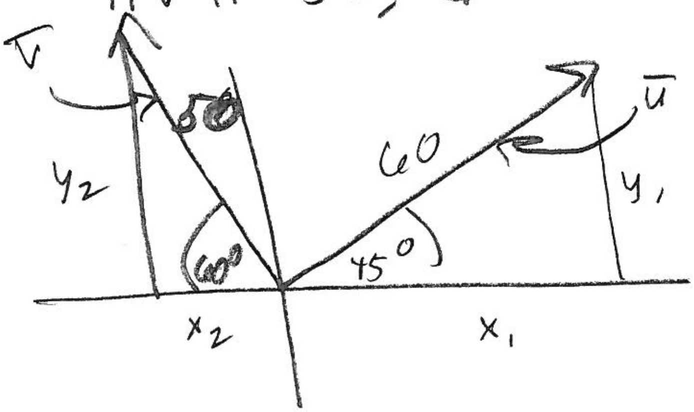
$\approx 43.35809355 \longrightarrow$

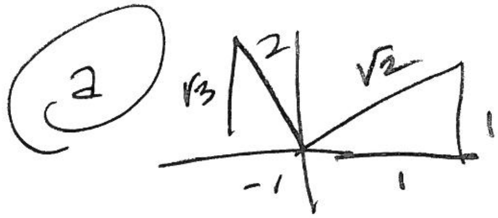
$b \approx \sqrt{\text{above}} \approx 6.584686291$

$\approx \boxed{6.584686 \approx b}$

B2 $\|\bar{u}\| = 60, \ \theta = 45^\circ$

$\|\bar{v}\| = 50, \ \phi = 120^\circ$





$$\bar{u} = \langle x_1, y_1 \rangle = 60 \langle \cos 45^\circ, \sin 45^\circ \rangle$$

$$= \boxed{60 \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \bar{u}}$$

$$= 60 \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle = 30 \langle \sqrt{2}, \sqrt{2} \rangle = \bar{u}$$

$$= \langle 30\sqrt{2}, 30\sqrt{2} \rangle$$

$$\bar{v} = 50 \langle \cos 120^\circ, \sin 120^\circ \rangle$$

$$= 50 \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle = \boxed{25 \langle -1, \sqrt{3} \rangle = \bar{v}}$$

$$= \langle -25, 25\sqrt{3} \rangle = \bar{v}$$

$$\boxed{\begin{aligned} \bar{u} &\approx \langle 42.42640687, 42.42640687 \rangle \\ &\approx \langle 42.426, 42.426 \rangle \end{aligned}}$$

$$\boxed{\begin{aligned} \bar{v} &= \langle -25, 43.30127019 \rangle \\ &\approx \langle -25, 43.301 \rangle \end{aligned}}$$

B2 cnt'd

C $\vec{u} + \vec{v} = \boxed{\langle 30\sqrt{2} - 25,\ 30\sqrt{2} + 25\sqrt{3} \rangle}$

$$\boxed{\approx \langle 17.42640687,\ 85.72767706 \rangle \approx \vec{u} + \vec{v}}$$

B3 $10 \sin\left(\frac{\pi}{50}x - \frac{7\pi}{50}\right) - 11$

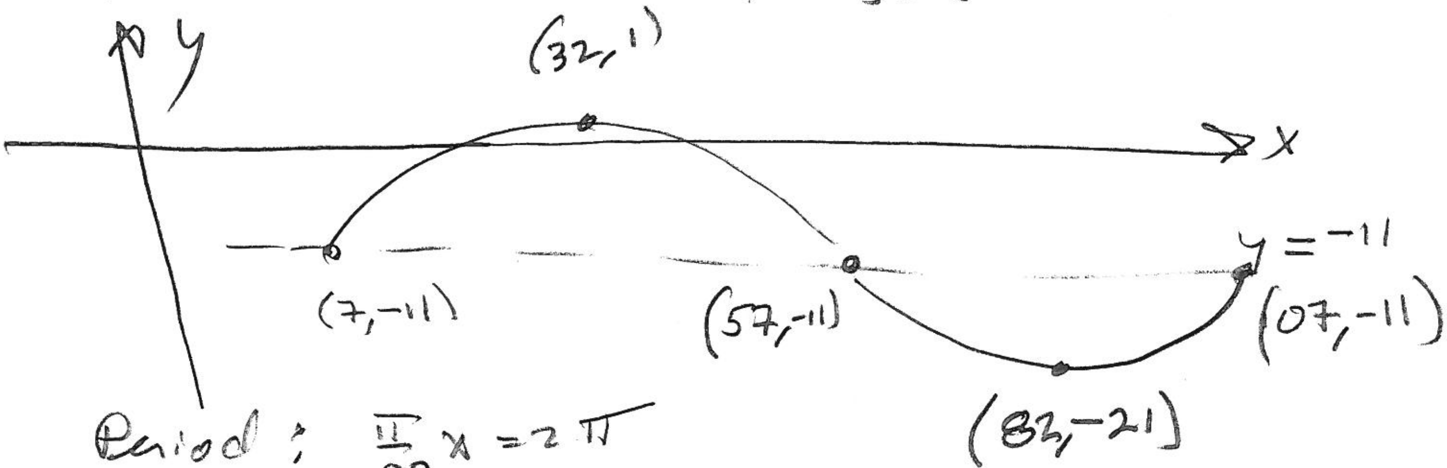
Amp $= 10$, mid: $y = -11$

$-11 + 10 = -1$

$-11 - 10 = -21$

$\frac{\pi}{50}x - \frac{7\pi}{50}$

$= \frac{\pi}{50}(x - 7)$



Period: $\frac{\pi}{50}x = 2\pi$

$x = 100 = T$

$\frac{T}{4} = 25$

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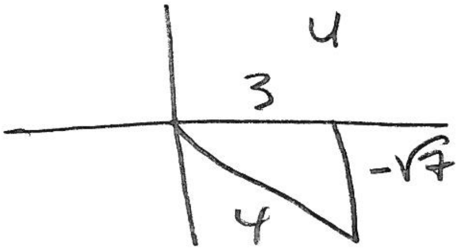
B4 $\cos u = \frac{3}{4}$ & $\sin u < 0$

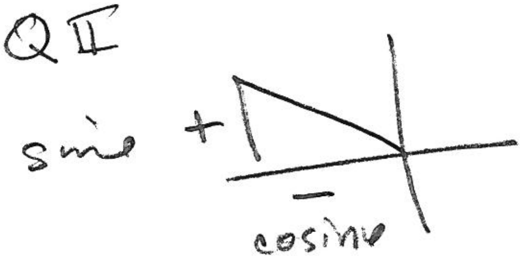


$4^2 - 3^2 = 16 - 9 = 7$

$270^\circ < u < 360^\circ$ $\qquad \frac{3\pi}{2} < u < 2\pi$

$135^\circ < \frac{u}{2} < 180^\circ$ $\qquad \frac{3\pi}{4} < \frac{u}{2} < \pi$

Q II $\qquad$ Q II



$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{3}{4}}{2}} = \sqrt{\frac{\frac{1}{4}}{2}}$$

$$= \sqrt{\frac{1}{8}} = \sqrt{\frac{1}{8}\cdot\frac{2}{2}} = \frac{\sqrt{2}}{\sqrt{16}} = \boxed{\frac{\sqrt{2}}{4} = \sin\frac{u}{2}}$$

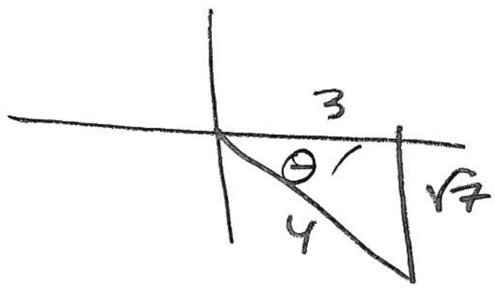
$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{3}{4}}{2}} = -\sqrt{\frac{\frac{7}{4}}{2}}$$

$$= -\frac{\sqrt{7}}{\sqrt{8}} = \frac{-\sqrt{14}}{\sqrt{16}} = \boxed{-\frac{\sqrt{14}}{4} = \cos\frac{u}{2}}$$

122 TY

(2)

(35) $\cos u = \frac{3}{4}$ & $\sin u < 0$. What quadrant is $2u$ in?



$\sqrt{7} < 3$, so

$\theta' < 45°$, i.e.

$315° < u < 360°$

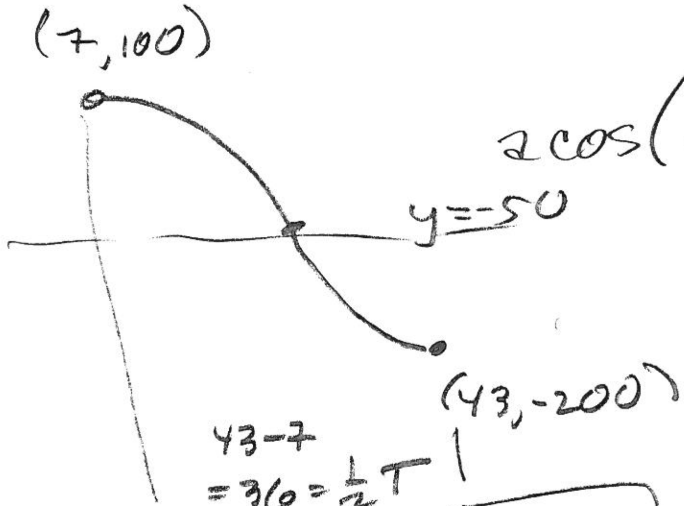
$630° < 2u < 720°$

$630° = 360° + 270° \longrightarrow 270°$

$720° = 360° + 360° \longrightarrow 360°$

So $270° < 2u < 360°$

$2u \in QIV$.



$100 - (-50)$
$\boxed{= 150 = a}$

$a\cos(b(x-c)) + d$

$\frac{100-200}{2} = -\frac{100}{2}$

$= -50 =$ midline

$\boxed{d = -50}$

$43 - 7$
$= 36 = \frac{1}{2}T$

$\Rightarrow T = 72$ & $\boxed{b = \frac{\pi}{36}}$

Start @
$\boxed{c = 7}$

So $\boxed{f(x) = 150\cos\left(\frac{\pi}{36}(x-7)\right) - 50}$