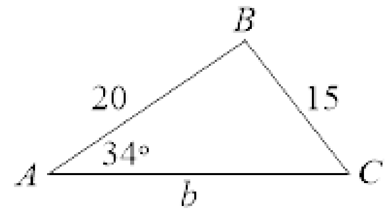
MAT 122 Test 3 – Fall, 2019 Name_________________

100 Points NO GRAPHING CALCULATORS!!!

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.



1. Consider the triangle in the figure on the right. Do not use rounded results in your calculations for new results. Only round the final answers.

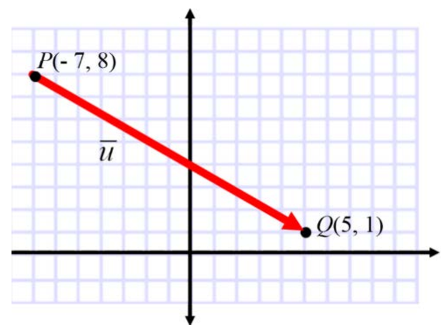
   a. (10 pts) This triangle is oriented a bit differently than others you've seen for this SSA situation. But you can still show there are 2 solutions to this triangle. Do so.

   b. (10 pts) Use the Law of Sines to solve for the acute angle C for the picture shown, to as many digits as your calculator can display. Then round the measure of angle C to 3 decimal places. Circle both.

   c. (10 pts) Use your work, above, to find the measure of angle B and the length of side b.

   d. (5 pts) Use your (un-rounded) result for the *obtuse* angle B and the Law of Cosines to find the length of side b. You can check your answer using the Law of Sines, but I insist on seeing the Law of Cosines, here. Give the length b rounded to 3 decimal places.

   e. (10 pts) Draw the picture for the case where angle B is acute. What is the measure of angle C in *this* case? Your work in part b, above, will be helpful. Your angle C better be obtuse. Circle this result and then round to 3 places and circle *that*.



2. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:

   a. (10 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.

   b. (10 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical $\bar{v}$ form.

   c. (5 pts) Find the direction angle of $\bar{u}$ (the positive angle measured from the positive x-axis). Give an exact answer, which may involve an arctangent, and an approximate answer, in degrees, to 3 decimal places.

3. Let $\bar{u} = \langle -6, 2 \rangle$.

   a. (5 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.

   b. (5 pts) What's another word for the sum of 2 vectors?

4. The wind is blowing at 30 knots ($\left(\|\bar{u}\| = 30 \text{ mph} \right)$ from the West and a pilot in an extreme bush plane is cruising due North at 130 knots $\left(\|\bar{v}\| = 130 \text{ knots} \right)$.

   a. (5 pts) Draw a diagram and Tell me her heading (in degrees East of due North, e.g., "North, 40 degrees East.").

   b. (5 pts) What is her ground speed?

BONUS Answer up to four (4) 5-pointers for up to 20 bonus points.

Bonus 1. Let $\bar{u} = \langle 5,3 \rangle$ and $\bar{v} = \langle 2,7 \rangle$.

   a. (5 pts) What is the angle between $\bar{u}$ and $\bar{v}$, to the nearest 1/100th of a degree?

   b. (5 pts) Find the projection of $\bar{u}$ onto $\bar{v}$, that is, find $\text{proj}_{\bar{v}}\bar{u}$. Draw a rough sketch showing $\bar{u}$, $\bar{v}$, and $\text{proj}_{\bar{v}}\bar{u}$.

Bonus 2. (5 pts) Find the *exact* value of $\sin\left(\frac{u}{2}\right)$ and $\cos\left(\frac{u}{2}\right)$, if $u = \frac{5\pi}{6}$.

Bonus 3. (5 pts) In what quadrant does 2*u* lie, if $\tan(u) = -\frac{2}{3}$ and $\sin(u) > 0$? Full credit for reasoning your way on general considerations. 4 out of 5 for just using a calculator to find *u*, multiplying by 2, and observing the quadrant in which 2*u* lies.

Bonus 4. (5 pts) Find all solutions in $[0, 2\pi)$ to the equation $\sin(2x) = \frac{1}{2}$

Bonus 5. Let $f(x) = 6x^4 - 25x^3 + 32x^2 + 3x - 10$.

   a. (5 pts) Use synthetic division to show that $x = 2 + i$ is a solution of the equation $f(x) = 0$.

   b. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

Bonus 6. Let $z = -1 - i$.

   a. (5 pts) Find $z + \bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.

   b. (5 pts) Express z in trigonometric form.