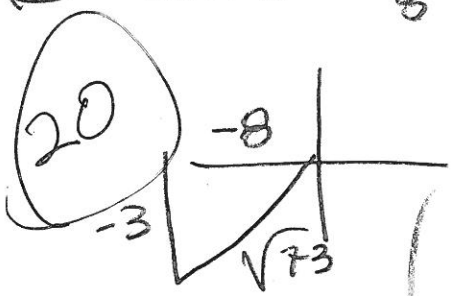
122 TEST 2 FALL, 2016

125 total

(1) $\tan u = \frac{3}{8}$ & $\sin u < 0$

20

$4 + 64 = 73$

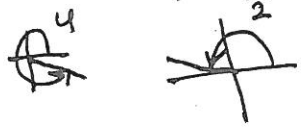


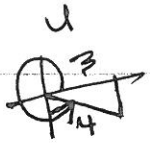
$\sin u = -\frac{3}{\sqrt{73}}$ $\quad \csc u = -\frac{\sqrt{73}}{3}$

$\cos u = \frac{-8}{\sqrt{73}}$ $\quad \sec u = -\frac{\sqrt{73}}{8}$

$\tan u = \frac{3}{8}$ $\quad \cot u = \frac{8}{3}$

$\frac{3\pi}{2} < u < 2\pi$ $\quad \frac{3\pi}{4} < \frac{u}{2} < \pi$





(2) $\cos u = \frac{3}{4}$ & $\frac{3\pi}{2} \le u < 2\pi$ QIV

so $\frac{3\pi}{4} \le \frac{u}{2} \le \pi$ QII

20

$\sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\cos u}{2}}$

QII: sine > 0, cosine < 0, tangent < 0

$= +\sqrt{\frac{1-\frac{3}{4}}{2}} = +\sqrt{\frac{\frac{1}{4}}{2}} = \boxed{+\sqrt{\frac{1}{8}} = +\frac{1}{2\sqrt{2}}}$ $\sin\left(\frac{u}{2}\right)$

$= \boxed{\frac{+\sqrt{2}}{4}}$

$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{3}{4}}{2}} = -\sqrt{\frac{\frac{7}{4}}{2}}$

$\boxed{-\sqrt{\frac{7}{8}} = -} \rightarrow \cos\left(\frac{u}{2}\right)$

$\tan\left(\frac{u}{2}\right) = \frac{+\frac{1}{2\sqrt{2}}}{-\sqrt{\frac{7}{8}}}$

$= -\frac{\sqrt{8}}{\sqrt{7}} \cdot \frac{1}{2\sqrt{2}} = \boxed{\frac{-1}{\sqrt{7}}}$ $\tan\left(\frac{u}{2}\right)$

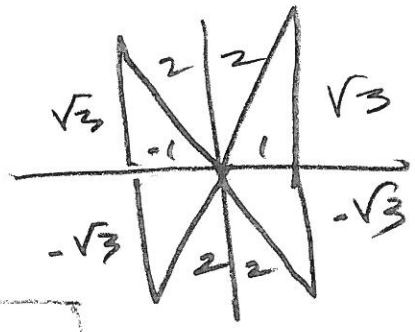
40

122 E2

(3) $4\sin^2 x - 3 = 0$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm \frac{\sqrt{3}}{2}$



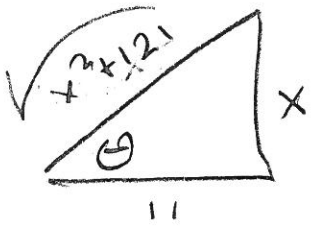
(2) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(2) $= 60^\circ, 120^\circ, 240^\circ, 300^\circ$

(b) $x = \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi, \quad n \in \mathbb{Z}$

(10) $= 60^\circ + 180^\circ n, 120^\circ + n\pi, \quad n \in \mathbb{Z}$

(4) $\sin\left(\tan^{-1}\left(\frac{x}{11}\right)\right)$



(10) $= \sin\theta$.

$= \frac{x}{\sqrt{x^2+121}}$

(5) $\sin x - 1 = \cos x$

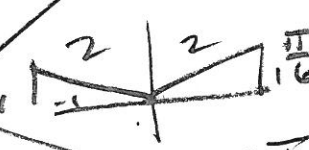
$(\sin x - 1)^2 = \sin^2 x - 2\sin x + 1 = \cos^2 x$

$= 1 - \sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2} \quad \sin x = -1$



122 E2

Check (5) (cont'd

$\sin\left(\frac{\pi}{6}\right) - 1 = \frac{1}{2} - 1 = -\frac{1}{2} \stackrel{?}{=} \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

No.

$\sin\left(\frac{5\pi}{6}\right) - 1 = \frac{1}{2} - 1 = -\frac{1}{2} \stackrel{?}{=} \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

NO

$\sin\frac{3\pi}{2} - 1 = -1 - 1 = -2$

oops!

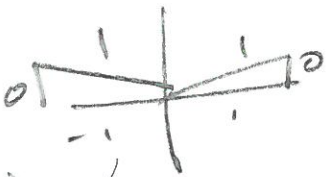
(5) $\sin^2 x - 2\sin x + 1 = \cos^2 x = 1 - \sin^2 x$

$2\sin^2 x - 2\sin x = 0$

$2\sin x(\sin x - 1) = 0$

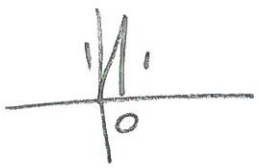
5pts

$\sin x = 0$



$0, \pi$

$\sin x = 1$



$\frac{\pi}{2}$

Check

$\sin(0) - 1 = -1 \stackrel{?}{=} \cos(0) = 1$

No

$\sin\frac{\pi}{2} - 1 = 0 \stackrel{?}{=} \cos\left(\frac{\pi}{2}\right) = 0$

Yes

$\sin\pi - 1 = -1 \stackrel{?}{=} \cos(\pi) = -1$

Yes

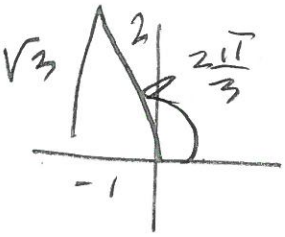
$x \in \left\{\frac{\pi}{2}, \pi\right\}$

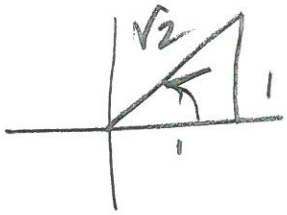
122 E2

(6) (a) $\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

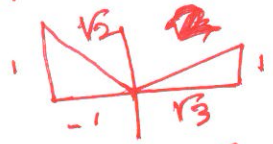
10

$= \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$





$\frac{9\pi}{12} - \frac{2\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$



$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{-1-\sqrt{3}}{2\sqrt{2}}}$ $= \frac{-\sqrt{2}-\sqrt{6}}{4}$

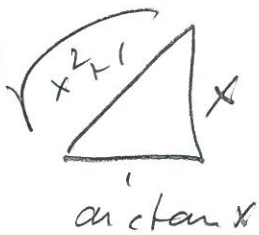
$\cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

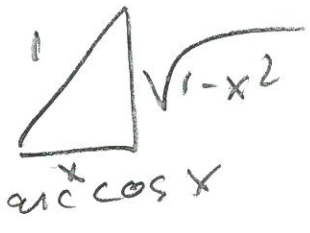
$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

(7) $\sin(\arctan(x) + \arccos(x))$

$= \sin(\arctan x)\cos(\arccos x) + \sin(\arccos x)\cos(\arctan x)$

(5)





$= \left(\frac{x}{\sqrt{x^2+1}}\right)x + \left(\sqrt{1-x^2}\right)\left(\frac{1}{\sqrt{x^2+1}}\right)$

$= \boxed{\frac{x^2 + \sqrt{1-x^2}}{\sqrt{x^2+1}}}$

15

6b

E2

6b

$\cos\frac{11\pi}{12} = \cos\left(\frac{1}{2}\cdot\frac{11\pi}{6}\right)$

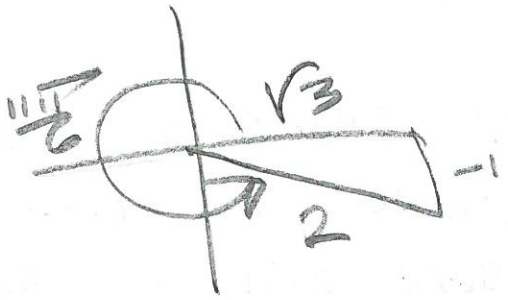
10

$= -\sqrt{\frac{1+\cos\frac{11\pi}{6}}{2}}$

$\frac{11\pi}{12}$ in QII

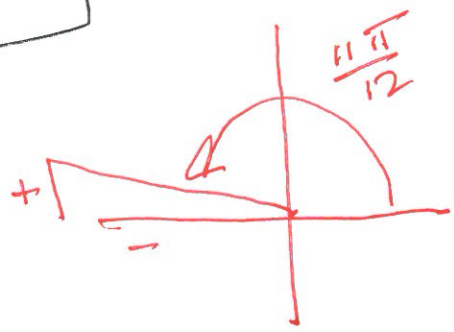
cosine's negative.

$= -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$



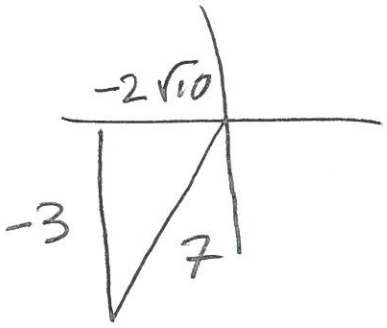
$= -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}}$



122 E2

(8) $\sin u = -\frac{3}{7}$ & $\cos u < 0$



$49 - 9 = 40$

$\sqrt{40} = 2\sqrt{10}$

$\tan(2u) = \frac{\frac{12\sqrt{10}}{49}}{\frac{31}{49}}$

$\tan(2u) = \boxed{\frac{12\sqrt{10}}{31}}$

(10)

$\sin(2u) = 2\sin u \cos u$

$= 2\left(-\frac{3}{7}\right)\left(-\frac{2\sqrt{10}}{7}\right) = \boxed{\frac{12\sqrt{10}}{49}}$ $\sin(2u)$

$\cos(2u) = 1 - 2\sin^2(u)$

$= 1 - 2\left(-\frac{3}{7}\right)^2 = 1 - 2\left(\frac{9}{49}\right)$

$= 1 - \frac{18}{49} = \frac{49-18}{49} = \boxed{\frac{31}{49}}$ $\cos(2u)$

122 E2

(9) $s = r\theta = (8)(210)\left(\frac{\pi}{180}\right)$

$= \frac{(8)(210\pi)}{18} = \frac{4(210\pi)}{9} = \frac{4(70\pi)}{3} = \boxed{\frac{280\pi}{3}}$

(5)

293,215 ≈ 143

B1 $\frac{91\pi}{12} = \frac{84\pi + 7\pi}{12} = 7\pi + \frac{7\pi}{12}$

$= 6\pi + \pi + \frac{7\pi}{12} = \boxed{\frac{19\pi}{12}} + 6\pi$

B2

$77 - 10 = 67$

$(67)(2) = 134 = T$

$T = 134 \rightarrow$

$6x = 2\pi$ when $x = 134$

$134b = 2\pi$

$b = \frac{2\pi}{134} = \frac{\pi}{67}$

$\cos\left(\frac{\pi}{67}x\right)$

Starts @ $x = 10$

$\cos\left(\frac{\pi}{67}(x-10)\right)$

$\frac{27-(-8)}{2} = \frac{35}{2} = Amp$

$\frac{27+(-8)}{2} = \frac{19}{2} = mid$

$\frac{35}{2}\cos\left(\frac{\pi}{67}(x-10)\right)$

$\boxed{y = \frac{35}{2}\cos\left(\frac{\pi}{67}(x-10)\right) + \frac{19}{2}}$

$\boxed{y = \frac{35}{2}\cos\left(\frac{\pi}{67}(x-10)\right) + \frac{19}{2}}$

5

122 E2

B3 $S = r\theta$ $\quad A = \frac{1}{2}r^2\theta = \frac{1}{2}(20)^2(71^{10})\left(\frac{\pi}{180}\right)$

$= \frac{400(71)\pi}{\cancel{2}\cancel{(180)}} = \frac{10(71\pi)}{9} = \frac{710\pi}{9} \approx$

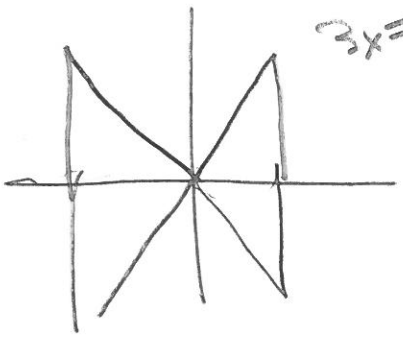
247.8368

B4 $4\sin^2(3x) - 3 = 0 \qquad x \in [0, 2\pi)$

So, $3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

But $x \in [0, 2\pi)$ says we need

$3x \in [0, 6\pi)$ to get 'em all.



$3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3},$

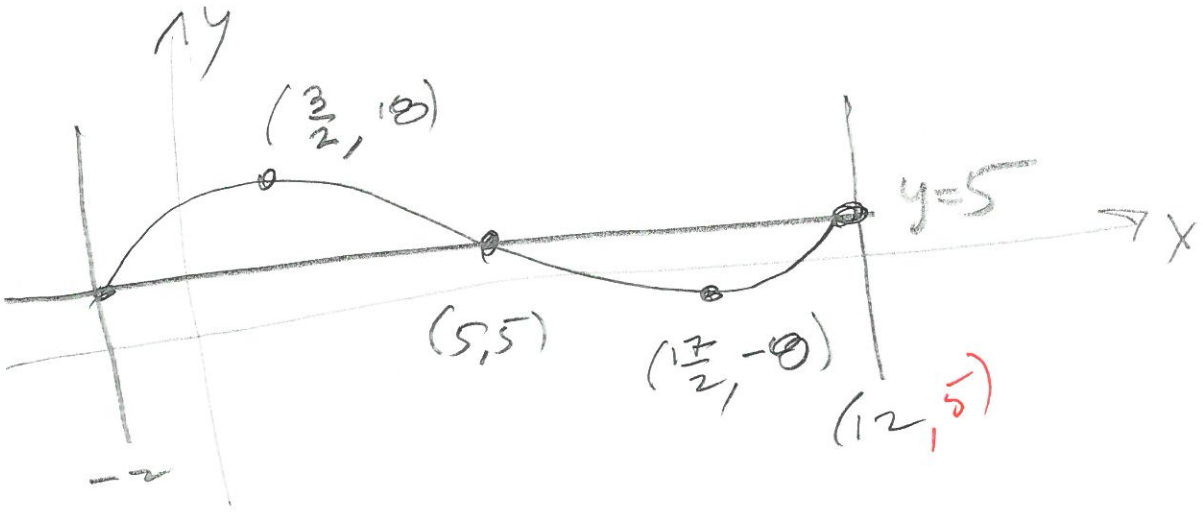
$\frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}$

So, $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9},$

$\frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

122 E2

(5) $y = 13\sin\left(\frac{\pi}{7}x + \frac{2\pi}{7}\right) + 5$



$y = 13\sin\left(\frac{\pi}{7}(x+2)\right) + 5$

$\frac{\pi}{7}x = 2\pi$ when $x = 14 = T$

START: $x = -2$

$\frac{12-2}{2} = \frac{10}{2} = 5$

$\frac{5-2}{2} = \frac{3}{2}$

$\frac{12+5}{2} = \frac{17}{2}$

$5+13 = 18$

$5-13 = -8$