

Work any three (3) problems for up to 30 bonus points. You will be given 10 more minutes than it takes me to do all five (5) of them.

Another option for you is to just solve #6 for all birds with one stone.

1. Find the *exact* value of  $\cos\left(\frac{23\pi}{12}\right)$  in two ways: (Hint: If degrees are easier for you, use *degrees*.)

a. (5 pts) Use a Sum identity.

b. (5 pts) Use a Half-Angle identity.

2. (10 pts) Find  $\sin(2u)$ ,  $\cos(2u)$  and  $\tan(2u)$ , given that  $\cos(u) = -\frac{3}{7}$  and  $\sin(u) > 0$ .

Are you smarter than the average bear?



3. (10 pts) Build a cosine function that achieves its maximum height of  $y = 71$  meters at time  $x = 5$  seconds and its minimum height of  $y = -9$  meters at  $x = 41$  seconds.

4. (10 pts) Find all solutions of the equation  $4\sin^2(2x) - 3 = 0$  in the interval  $[0, 2\pi)$ .

5. (10 pts) Sketch the graph of  $-5\sin\left(-\frac{11}{2\pi}x - \frac{11}{26\pi}\right) + 5$ .

6. (30 pts) Find all solutions of the equation  $\cos^3(\theta) - \cos^2(\theta) - \frac{1}{2}\cos(\theta) + \frac{1}{2} = 0$  in the interval  $[0, 2\pi)$ .  
Hint:  $\theta = 0$  is one solution.

①  $\cos\left(\frac{23\pi}{12}\right) = \cos\left(\frac{21\pi}{12} + \frac{2\pi}{12}\right)$

②  $\cos\left(\frac{7\pi}{4} + \frac{\pi}{6}\right) =$

$= \cos\left(\frac{7\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{7\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

2.9659258263

$\frac{4}{\sqrt{6} + \sqrt{2}}$

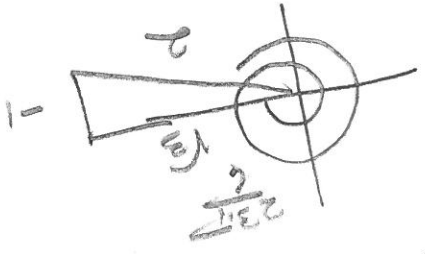
③  $\cos\left(\frac{12}{23\pi}\right) = \cos\left(\frac{1}{2}\left(\frac{2}{23\pi}\right)\right)$

$= \sqrt{\frac{1 + \cos\left(\frac{2}{23\pi}\right)}{2}}$

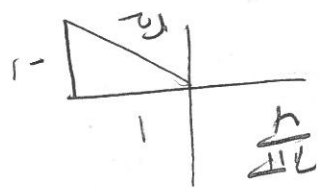
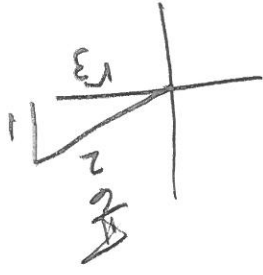
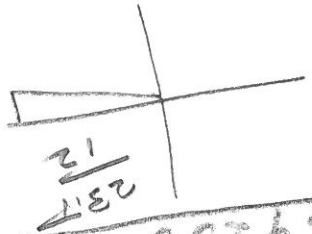
$= \sqrt{\frac{1 + \frac{2\pi}{23\pi}}{2}}$

$= \sqrt{\frac{2 + \sqrt{3}}{2}}$

$= \sqrt{\frac{2 + \sqrt{3}}{2}}$



$\frac{2}{23\pi} < \frac{1}{12} = \theta < \frac{2}{23\pi}$   
 $3\pi < \frac{6}{23\pi} < 4\pi$



E2 B

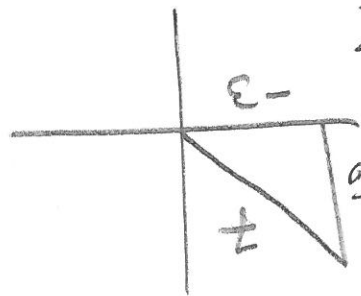
(2)

$\sin u > 0, \cos(u) = -\frac{7}{25}$

So  $\frac{\pi}{2} < u < \frac{3\pi}{2}$

$\pi < 2u < \frac{3\pi}{2}$

III



$\sqrt{49} = 25$

$\sin(2u) = 2 \sin u \cos u$

$= 2 \left( \frac{7}{25} \right) \left( -\frac{3}{25} \right) = -\frac{42}{625}$

ANSWER

$\cos(2u) = 2 \cos^2 u - 1 = 2 \left( \frac{3}{25} \right)^2 - 1 = \frac{18}{625} - \frac{625}{625} = -\frac{607}{625}$

$\cos(2u) = \frac{-31}{49}$

$\sin(2u) = \frac{\sin(2u)}{\cos(2u)}$

$= \frac{-\frac{42}{625}}{-\frac{607}{625}} = \frac{42}{607}$

$\frac{12\sqrt{10}}{31} = \frac{12\sqrt{10}}{31} = \sin(2u)$

$$40 \cos\left(\frac{36}{11}(x-5)\right) + 31$$

$$71 - 31 = 40 = \text{amp}$$

$$b = \frac{36}{11} = 9$$

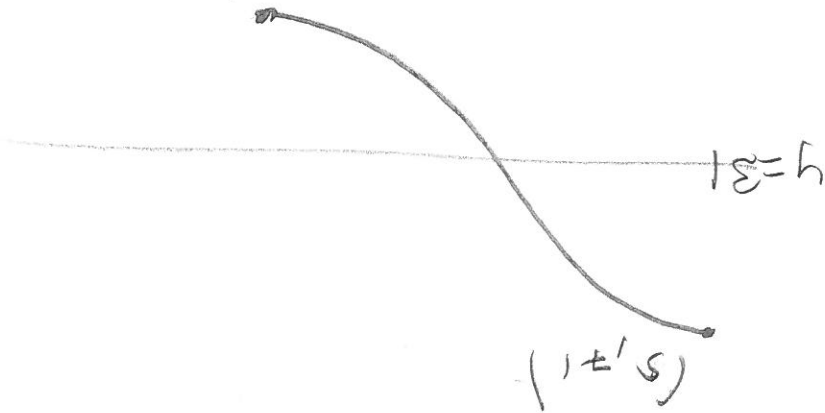
$$72b = 2\pi$$

$$bx = 2\pi \text{ when } x = 72$$

$$T = \frac{2(41-5)}{2(36)} = 72$$

$$71 - 9 = \frac{2}{b} = 31 = \text{mid}$$

(41, 9)



E2B

(3)

(4)

$$4 \sin^2(2x) - 3 = 0 \quad x \in [0, 2\pi)$$

$$x \in [0, 2\pi)$$

$$2x \in [0, 4\pi)$$

$$\text{SO, } 2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

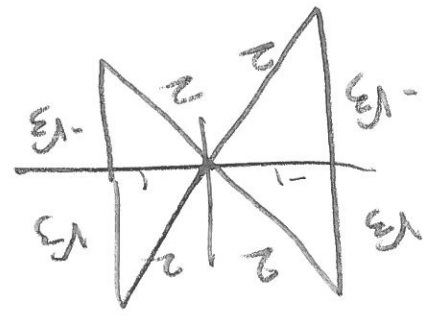
$$2x + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$2x + \frac{2\pi}{3} = \frac{8\pi}{3}$$

$$2x + \frac{4\pi}{3} = \frac{10\pi}{3}$$

$$2x + \frac{5\pi}{3} = \frac{11\pi}{3}$$

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$$



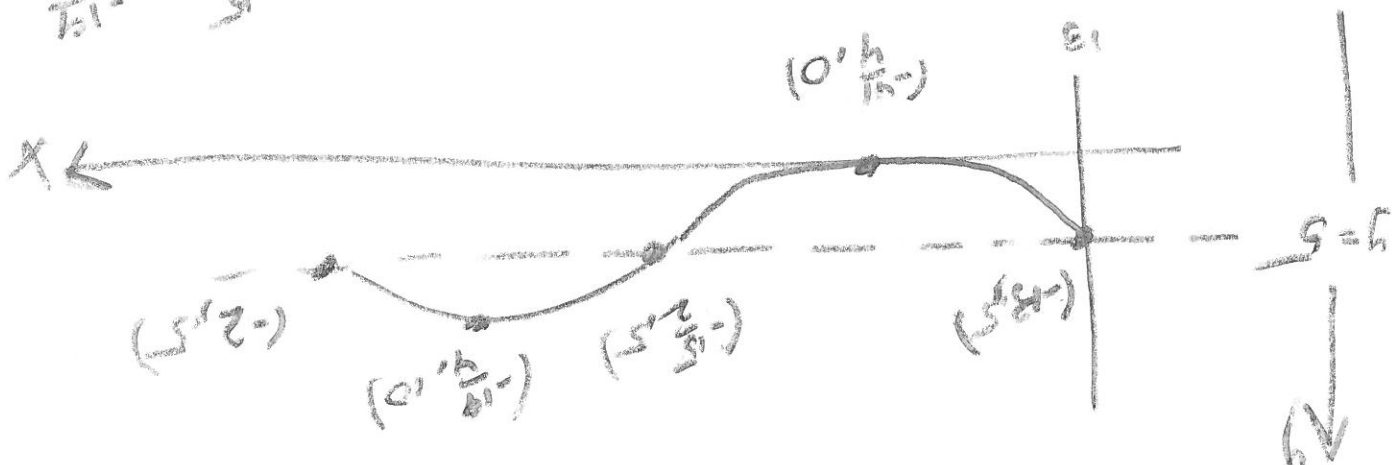
$$\sin(2x) = \frac{1}{2}$$

$$\sin^2(2x) = \frac{1}{4}$$

$$\text{SO } x = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6}$$

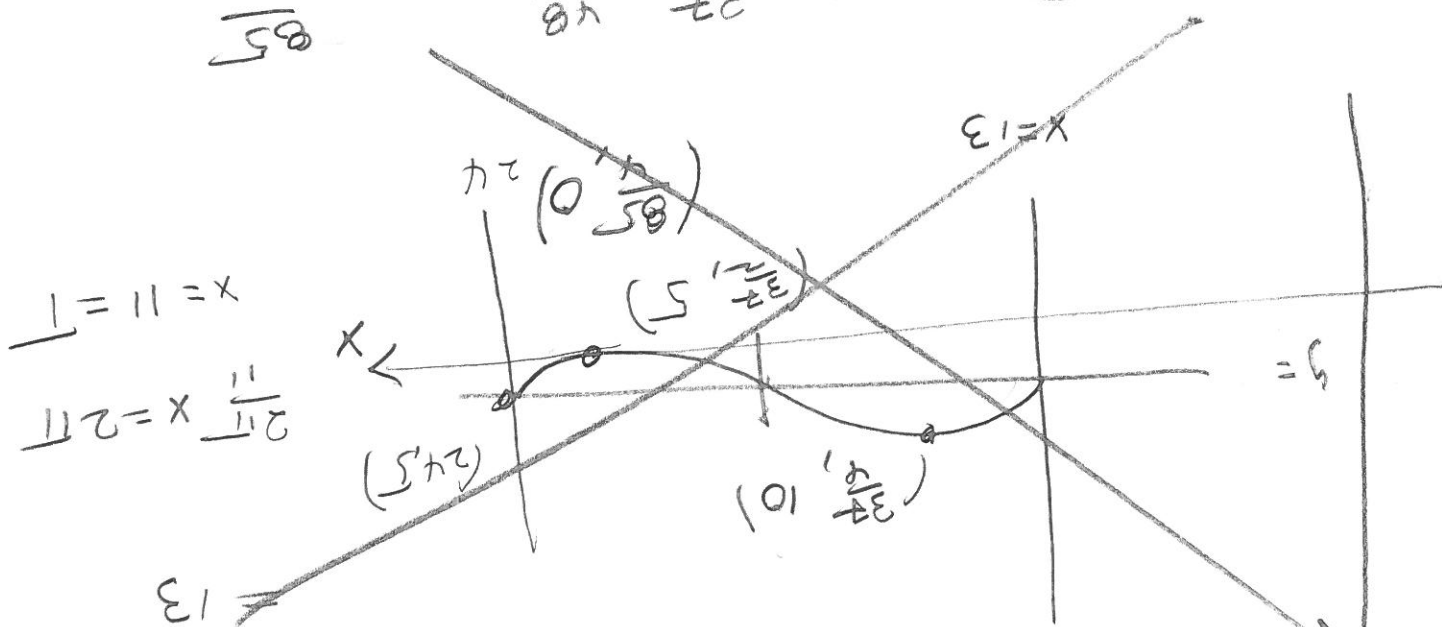
$$\frac{h}{11} = \frac{2}{2-5} = \frac{2}{-3} = -\frac{2}{3}$$

$$\frac{h}{11} = \frac{2}{2+13} = \frac{2}{15}$$



$$\frac{2}{4} = \frac{2}{\frac{37}{48} + \frac{2}{2}}$$

No. 5



$$x = 11 = 1$$

$$2\frac{11}{11} x = 2\frac{11}{11}$$

$$26\frac{11}{11} \cdot \frac{11}{2\frac{11}{11}}$$

$$= 5 \sqrt{2} \left( \frac{11}{2\frac{11}{11}} (x-13) \right) + 5$$

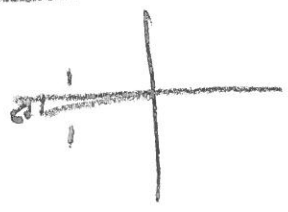
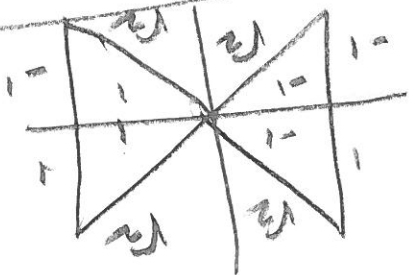
$$= -5 \sqrt{2} \left( -2\frac{11}{11} (x-13) \right) + 5$$

$$f(x) = -5 \sqrt{2} \left( -2\frac{11}{11} x - 26\frac{11}{11} \right) + 5$$

5

$$\frac{h}{\sin \theta} = \frac{h'}{\sin \theta} = \frac{h''}{\sin \theta} = \frac{h'''}{\sin \theta} = \frac{h''''}{\sin \theta} = \theta$$

$$Q = \theta$$



$$\frac{2}{T} = \theta \cos \theta$$

$$1 = \theta \sin \theta$$

$$0 = \left(\frac{2}{T} - \theta \cos \theta\right) (1 - \theta \sin \theta)$$

$$0 = (1 - \theta \sin \theta) \frac{2}{T} - (1 - \theta \sin \theta) \theta \cos \theta$$

$$0 = \frac{2}{T} + \theta \cos \theta - \theta \cos^2 \theta - \theta \sin^2 \theta$$

(9)

E2 B