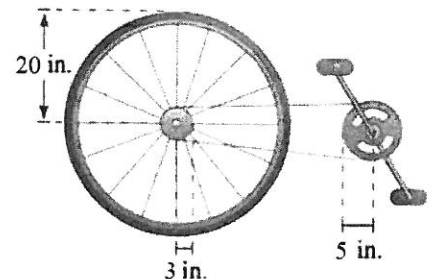


Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

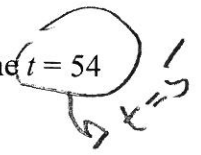
Leave space between problems. No prizes for saving paper, here. Figure this stuff out, and use your smarts to plant trees! Only use one column of work. Don't start a 2<sup>nd</sup> column to save paper. ALL I WANT ON THIS PAGE IS YOUR NAME.

1. (10 pts) Find two angles, between  $-2\pi$  and  $2\pi$  (i.e.,  $-360^\circ$  and  $360^\circ$ ) that are coterminal with  $\frac{71\pi}{6}$ . Give exact answers in degrees and radians.
2. Arc Length and Area of Sector. Suppose we have a circle of radius  $r = 7$  cm.
  - a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of  $892^\circ$ . Round to 3 decimal places.
  - b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of  $\theta = \frac{2\pi}{3}$ .
3. Basic concept: Draw the doggone pictures!
  - a. (5 points) Sketch two triangles that satisfy  $\sin(\theta) = \frac{-2}{\sqrt{13}}$ .
  - b. (5 pts) Assume the terminal side of the angle  $\theta$  lies in the 4<sup>th</sup> quadrant (Quadrant IV). Find the other five trigonometric functions of  $\theta$ .
  - c. (5 pts) Again, assuming  $\theta$ 's terminal side lies in Q IV, and  $0 \leq \theta < 2\pi$ , find  $\theta$ , in radians *and* degrees, rounded to 3 decimal places.
  - d. (5 pts) Give *all* solutions to the equation  $\sin(\theta) = \frac{-2}{\sqrt{13}}$ , in degrees *and* radians, rounded to three (3) decimal places. (Good one to skip and come back to, if time permits).
4. (10 pts) Sketch one period of the graphs of  $y = \cos(x)$  and  $y = \sec(x)$  on the same set of coordinate axes.
5. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 20 inches, respectively. A cyclist is pedaling at a rate of 4 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.

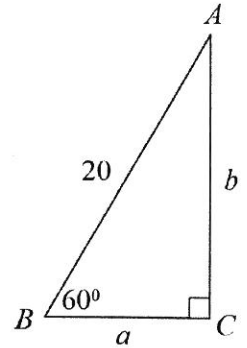


6. (10 pts) Sketch the graph of  $f(x) = -5 \sin\left(\frac{\pi}{11}x - \frac{7\pi}{11}\right) + 11$ .

7. (10 pts) Write the cosine function that achieves its maximum height of  $y = 70$  centimeters at time  $t = 54$  seconds and its minimum height of  $y = -10$  centimeters at  $t = 25$  seconds.



8. (5 pts) Solve the triangle on the right. That means, find all lengths and angles. Exact answers required.



9. Find the exact value of...

a. ... (5 pts)  $\cot\left(\arcsin\left(\frac{3}{7}\right)\right)$ .

b. ... (5 pts)  $\arccos\left(\sin\left(\frac{7\pi}{6}\right)\right)$

10. (5 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to

$$\csc\left(\arctan\left(\frac{\sqrt{9-4x^2}}{2x}\right)\right).$$

Assume that everything is taking place in the 1<sup>st</sup> quadrant.

Bonus: Answer *two* of the following 4 problems, for *up to* 10 points:

1. (5 pts) If possible, sketch the picture(s) corresponding to the following equations. If not possible, explain why.

a.  $\sin(x) = \frac{5}{3}$

d.  $\tan(x) = -\sqrt{3}$

b.  $\cos(x) = 1$

e.  $\sec(x) = \frac{1}{2}$

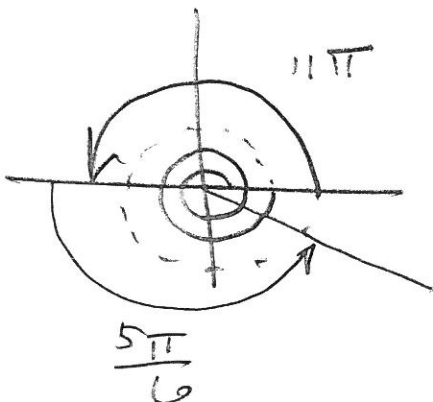
c.  $\cos(x) = \frac{5}{11}$



2. (5 pts) Sketch the graph of one period of  $y = \cos(x)$  (restricted to make it 1-to-1) *and*  $y = \arccos(x)$  on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted cosine function and its inverse.
3. (5 pts) Find all solutions  $\theta$  of the trigonometric polynomial  $\cos^2(\theta) + \cos(\theta) = 0$  in the interval  $[0, 2\pi)$ .
4. (5 pts) Super-duper bonus Find all solutions  $\theta$  of the trigonometric polynomial  $4\sin^3(\theta) + 2\sin^2(\theta) - 2\sin(\theta) - 1 = 0$  in the interval  $[0, 2\pi)$ .

① 10 pts

$$\Theta = \frac{71\pi}{6} = \frac{166\pi + 5\pi}{6} = 11\pi + \frac{5\pi}{6}$$



11 is odd, so  $11\pi$  ends @ -x-axis.

Add  $\frac{5\pi}{6}$

So,  $\pi + \frac{5\pi}{6}$  is where

it lies

$$\frac{11\pi}{6}, -\frac{\pi}{6}$$

$$330^\circ, -30^\circ$$

② a) 5 pts

Arc length,  $r=7$ ,  $\Theta=892^\circ$

$$S = r\Theta = (7)(892^\circ)\left(\frac{\pi}{180}\right) = \frac{1561\pi}{45}$$

$$\approx 108.9783585 \text{ cm}$$

$$\approx 108.978 \text{ cm}$$

Common mistake =  
6244 cm, b/c NO  $\frac{\pi}{180}$   
conversion.

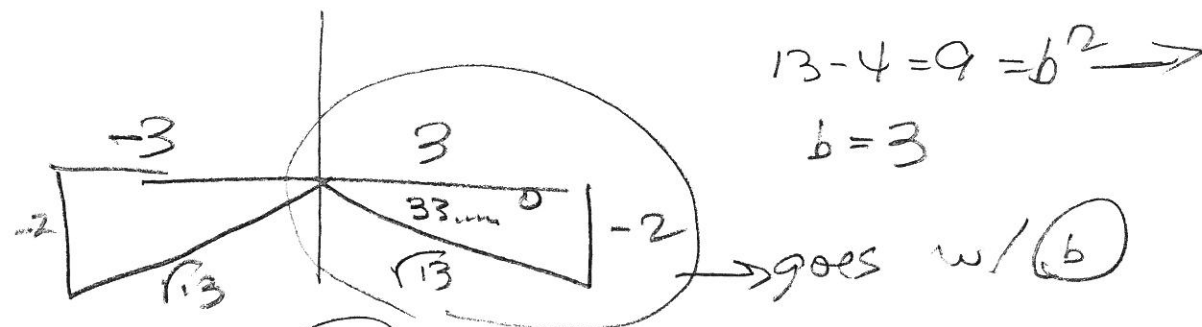
b) 5 pts

$$A = \frac{1}{2}r^2\Theta = \frac{1}{2}(7)^2\left(\frac{2\pi}{3}\right)$$

$$= \frac{49\pi}{3} \text{ cm}^2$$

(3) (a) (5 pts)

$$\sin \theta = -\frac{2}{\sqrt{13}}$$



(b) (5 pts) Says use Q IV, so

$$\sin \theta = -\frac{2}{\sqrt{13}}$$

$$\csc \theta = -\frac{\sqrt{13}}{2}$$

$$\cos \theta = \frac{3}{\sqrt{13}}$$

$$\sec \theta = \frac{\sqrt{13}}{3}$$

$$\tan \theta = -\frac{2}{3}$$

$$\cot \theta = -\frac{3}{2}$$

(c) (5 pts)  $\arcsin\left(-\frac{2}{\sqrt{13}}\right) \approx -33.69006753^\circ$

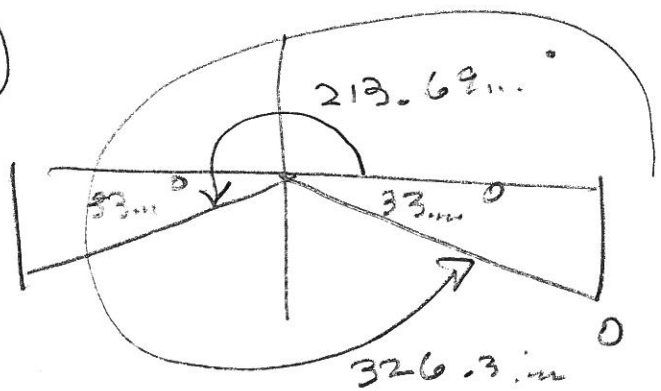
We want  $0 \leq \theta < 2\pi$ , so,

$$360 - 33.69006753^\circ \approx 326.3099325^\circ$$

$$\approx 326.310^\circ \approx \theta$$

$$\approx 5.695 \text{ radians}$$

3d



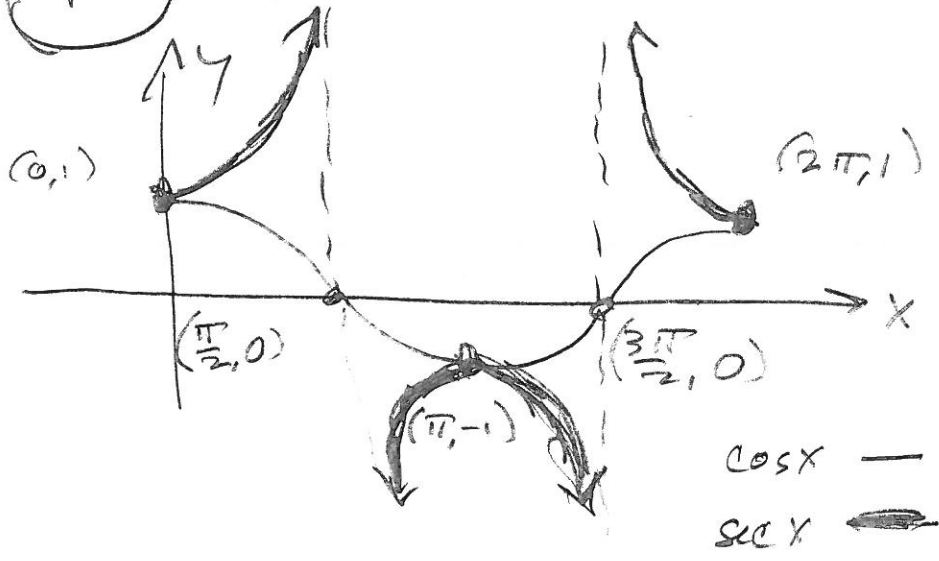
So  $\theta \in \{ 213.69^\circ + 360^\circ n \text{ or } 326.31^\circ + 360^\circ n \mid n \in \mathbb{Z} \}$

OR in radians :

$\{ 5.695 + 2n\pi \text{ or } 3.730 + 2n\pi \mid n \in \mathbb{Z} \}$

4 10pts

cos x & sec x



cos x —  
 sec x —

(5) (10 pts)

$\frac{4 \text{ revs front}}{\text{sec}}$

$$\left( \frac{4 \text{ revs front}}{1 \text{ sec}} \right) \left( \frac{5 \text{ revs back}}{3 \text{ rev front}} \right) \left( \frac{2\pi}{1 \text{ rev}} \right) (20 \text{ in}) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right)$$

$$= \frac{4(5)(2\pi)(20)}{3(12)} = \frac{200\pi}{9} \frac{\text{ft}}{\text{sec}} \approx 69.81317008$$

$$\approx 69.8 \frac{\text{ft}}{\text{sec}}$$

$$\left( 69.81317008 \frac{\text{ft}}{\text{sec}} \right) \left( \frac{60 \frac{\text{mi}}{\text{hr}}}{33 \frac{\text{ft}}{\text{sec}}} \right) \approx 47.59988869 \text{ mi/hr}$$

$$\approx 47.6 \frac{\text{mi}}{\text{hr}}$$

(6) (10 pts)

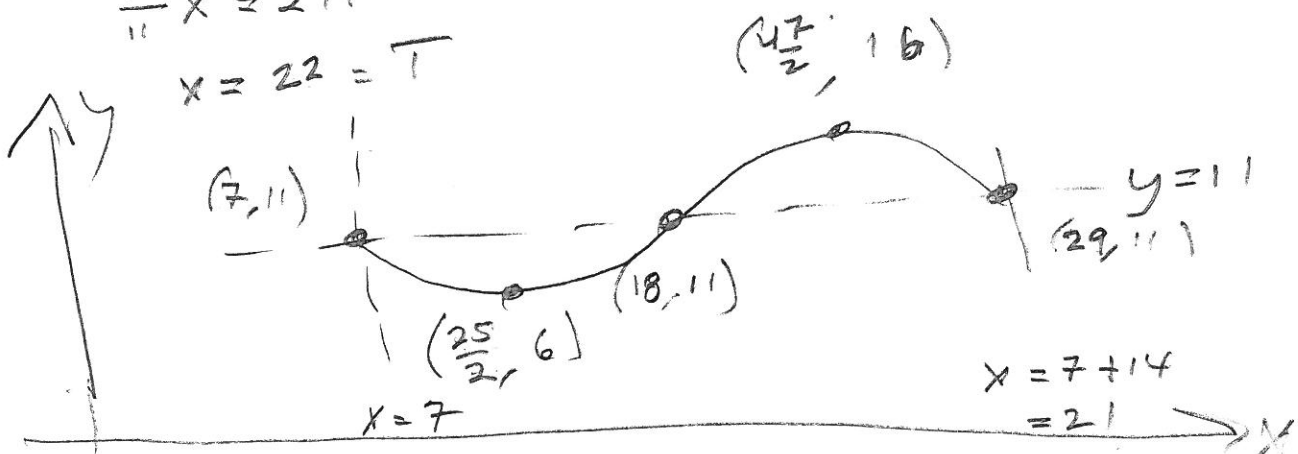
$$f(x) = -5 \sin\left(\frac{\pi}{11}(x-7)\right) + 11$$

↑ Amp
 ↑ start
 ↑ mid

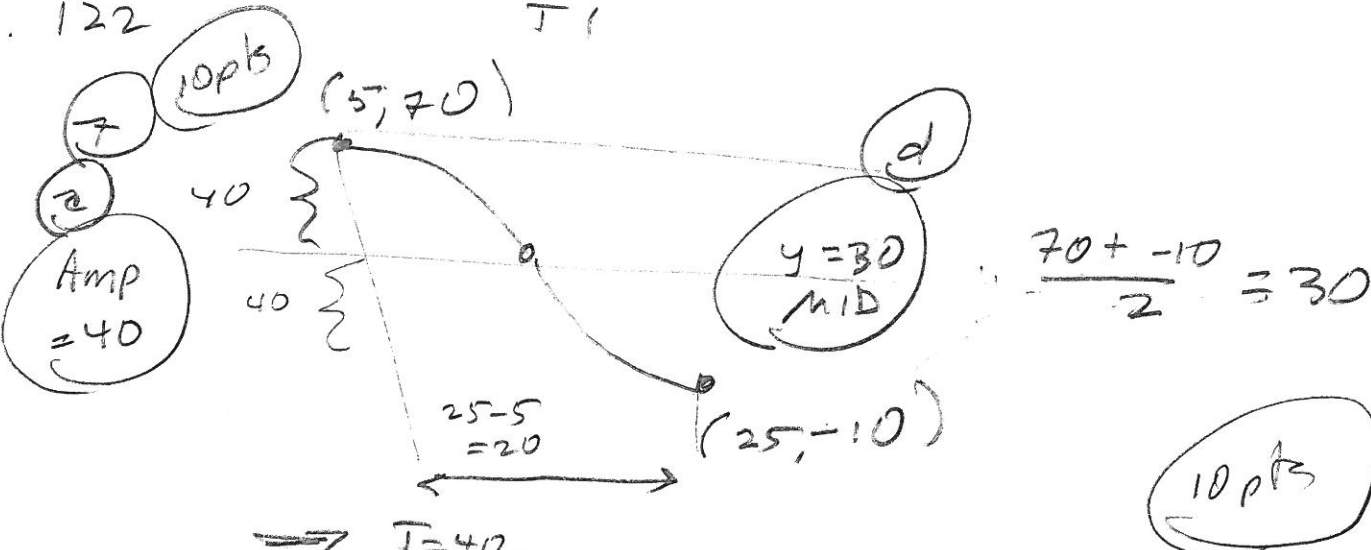
Period:

$$\frac{\pi}{11}x = 2\pi$$

$$x = 22 = T$$



$$\frac{18+7}{2} = \frac{25}{2} = 12.5 \quad \frac{18+29}{2} = \frac{47}{2} = 23.5$$



$\Rightarrow T = 40$

$b \cdot x = 2\pi$  when  $x = 40$

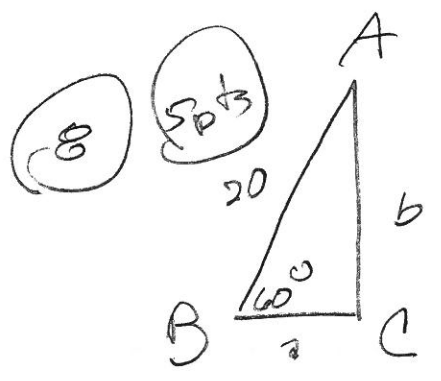
$40b = 2\pi$

$b = \frac{\pi}{20}$

start @  $x = 5 = C$

$2 \cos(b(x - C)) + d$

$= 40 \cos\left(\frac{\pi}{20}(x - 5)\right) + 30$



$A = 30^\circ$

$B = 60^\circ$  Given

$C = 90^\circ$



$\frac{a}{20} = \cos(60^\circ)$

$a = 20 \cos 60^\circ = 20 \cdot \frac{1}{2} = 10 = a$

$\frac{b}{20} = \sin \theta \rightarrow$

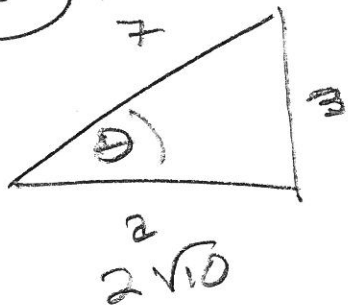
$b = 20 \sin(60^\circ) = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3} = b$

17,32050608

$20 = c$  given

(9)  $\cot(\arcsin(\frac{3}{7})) = \cot(\theta)$

5pts



$$7^2 - 3^2 = 49 - 9 = 40 = a^2$$

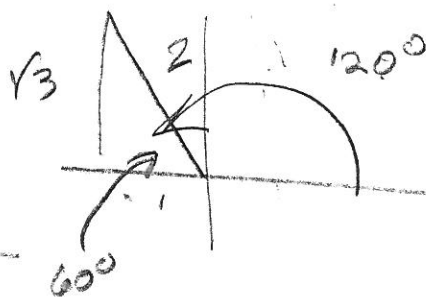
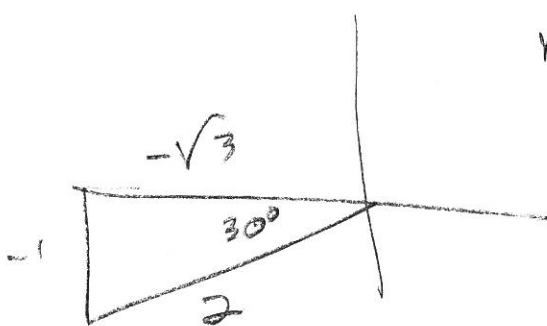
$$\Rightarrow a = \sqrt{40} = 2\sqrt{10}$$

$$\Rightarrow \cot \theta =$$

$$\frac{2\sqrt{10}}{3}$$

(b)  $\arccos(\sin(\frac{\pi}{6})) = \arccos(-\frac{1}{2})$

5pts

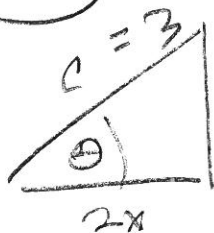


$$= 120^\circ$$

$$\text{OR } \frac{2\pi}{3}$$

(10)  $\csc(\arctan(\frac{\sqrt{9-4x^2}}{2x})) = \csc(\theta)$

5pts



$$\sqrt{9-4x^2}$$

$$c^2 = \sqrt{9-4x^2}^2 + (2x)^2$$

$$= 9 - 4x^2 + 4x^2 = 9$$

$$\Rightarrow c = 3$$

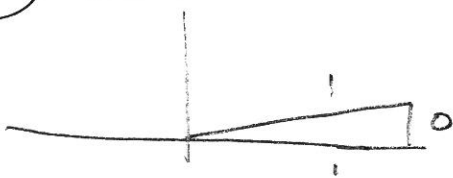
$$\Rightarrow \csc \theta =$$

$$\frac{3}{\sqrt{9-4x^2}}$$



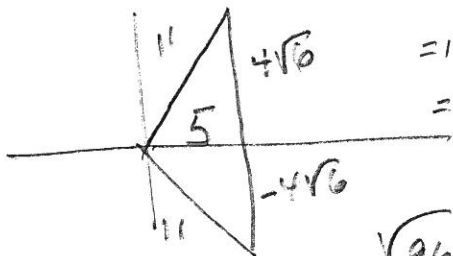
(B1) a)  $\sin(x) = \frac{5}{3}$  ~~---~~

b)  $\cos(x) = 1$



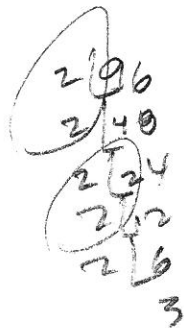
So B

c)  $\cos(x) = \frac{5}{11}$

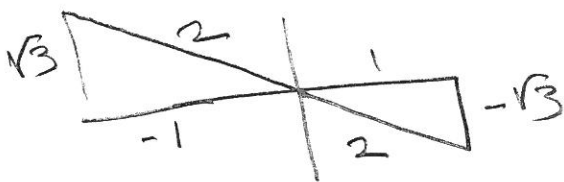


$11^2 - 5^2$   
 $= 121 - 25$   
 $= 96$

$\sqrt{96} = 4\sqrt{6}$

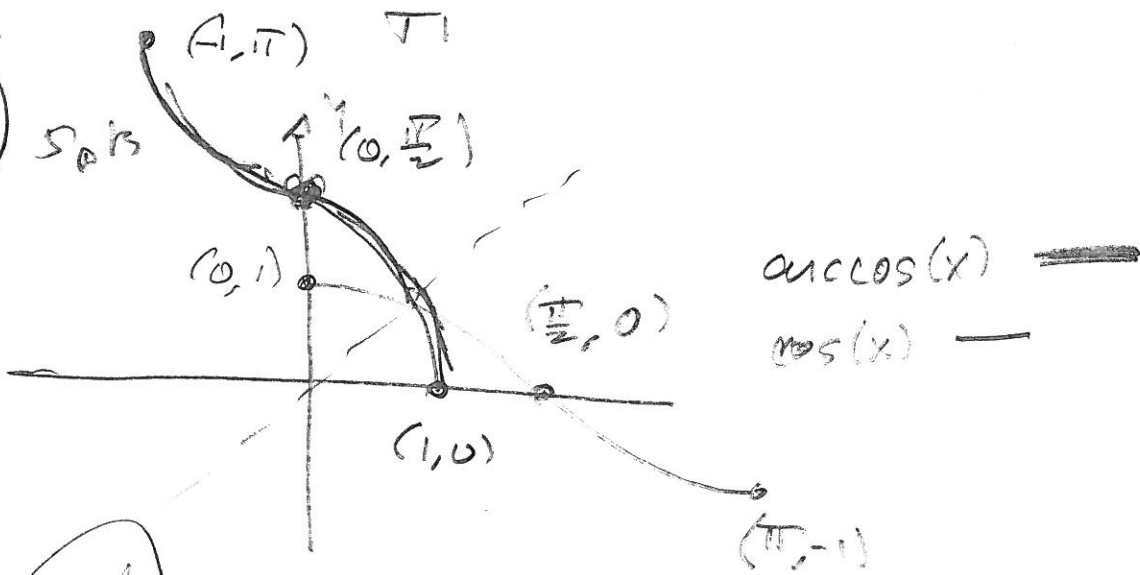


d)  $\tan(x) = -\sqrt{3}$



e)  $\sec(x) = \frac{1}{2}$  ~~---~~

(B2)



(B3)

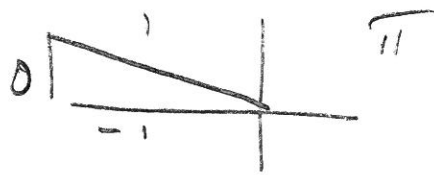
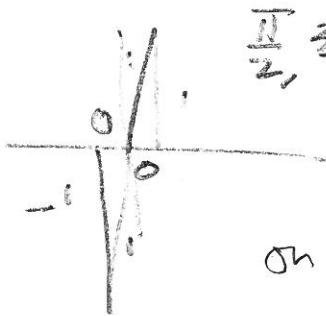
s.p.s

$$\cos^2 \theta + \cos \theta = 0$$

$$\cos \theta (\cos \theta + 1) = 0$$

$$\cos \theta = 0$$

$$\cos \theta = -1$$



on  $[0, 2\pi)$  :  $\theta \in \left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$

$$\left\{ \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z} \right\} \cup \left\{ \pi + 2n\pi \mid n \in \mathbb{Z} \right\}$$

$$= \left\{ 90^\circ + 180^\circ n \mid n \in \mathbb{Z} \right\} \cup \left\{ 180^\circ + 360^\circ n \mid n \in \mathbb{Z} \right\}$$

(B4) (4pb)

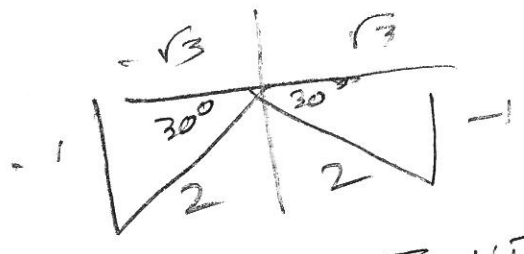
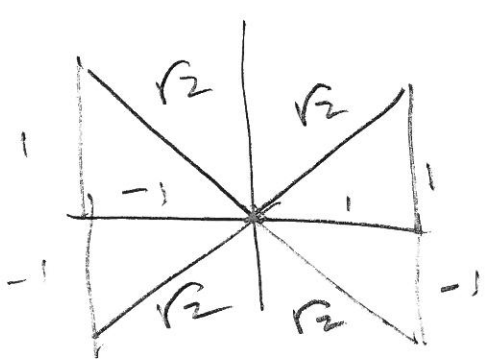
$$4\sin^3 \theta + 2\sin^2 \theta - 2\sin \theta - 1 = 0$$

$$2\sin^2 \theta (2\sin \theta + 1) - 1(2\sin \theta + 1) = 0$$

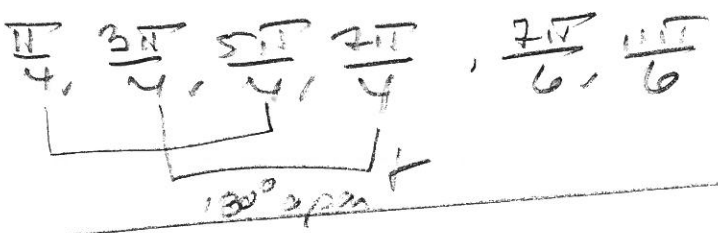
$$(2\sin^2 \theta - 1)(2\sin \theta + 1) = 0$$

$$(\sqrt{2}\sin \theta - 1)(\sqrt{2}\sin \theta + 1)(2\sin \theta + 1) = 0$$

$$\sin \theta = \frac{1}{\sqrt{2}}, \sin \theta = -\frac{1}{\sqrt{2}}, \sin \theta = -\frac{1}{2}$$



$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$



$$\theta \in \left\{ x \mid x = \frac{\pi}{4} + n\pi \text{ OR } x = \frac{3\pi}{4} + n\pi \text{ OR } x = \frac{7\pi}{6} + 2n\pi \right.$$

$$\left. \text{OR } x = \frac{11\pi}{6} + 2n\pi \text{ and } n \in \mathbb{Z} \right\}$$