122 TEST 4 Spring, 2019

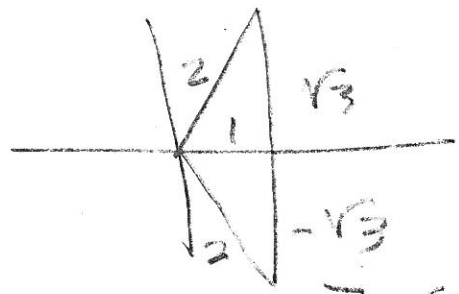
1 5pts

$2\cos(x) - 1 = 0$

$2\cos(x) = 1$

$\cos(x) = \frac{1}{2}$

$\boxed{x \in \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}}$



$2\pi - \frac{\pi}{3} = \frac{6-1}{3}\pi$

2 5pts $2\cos(2x) - 1 =$

$x \in [0, 2\pi) \longrightarrow 2x \in [0, 4\pi)$

$2x = \frac{\pi}{3} + 2\pi = \frac{\pi + 6\pi}{3} = \frac{7\pi}{3}$

$2x = \frac{5\pi}{3} + 2\pi = \frac{5+6}{3}\pi = \frac{11\pi}{3}$

$\Longrightarrow \boxed{x \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}}$

3 5pts

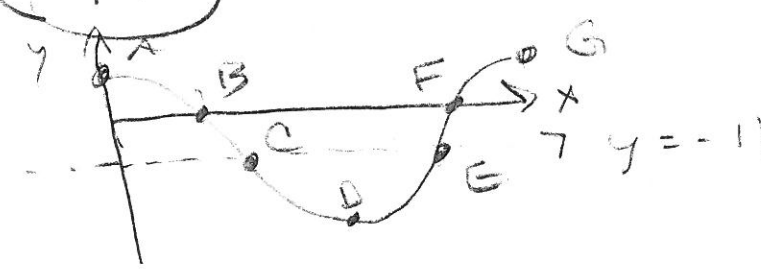
$\boxed{x \in \left\{ \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi \;\middle|\; n \in \mathbb{Z} \right\}}$

4 10pts $f(x) = 2\cos(2x) - 1$

$y = -1$ mid

Amp $= 2$

Period $= \frac{2\pi}{2} = \pi$



$A = (0, 1)$ $D = \left(\frac{\pi}{2}, -3\right)$

$B = \left(\frac{\pi}{6}, 0\right)$ $E = \left(\frac{3\pi}{6}, -1\right)$

$C = \left(\frac{\pi}{4}, -1\right)$ $F = \left(\frac{5\pi}{6}, 0\right)$

$G = (\pi, 1)$

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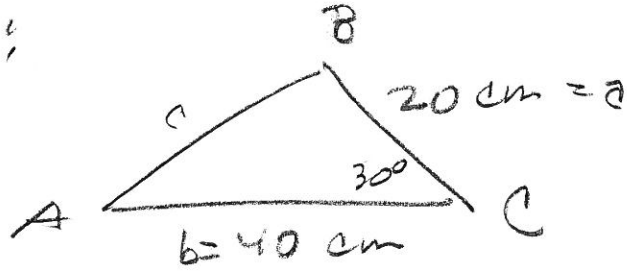
(5) The intended:



(a) (10 pts) $c^2 = a^2 + b^2 - 2ab\cos C$

$= 20^2 + 40^2 - 2(20)(40)\left(\frac{\sqrt{3}}{2}\right)$

$= 400 + 1600 - 1600\frac{\sqrt{3}}{2}$

$= 2000 - 800\sqrt{3}$

$\Rightarrow c = \sqrt{2000 - 800\sqrt{3}}$

$\approx 24.78627349854 5$ cm

$\boxed{c \approx 24.7863 \text{ cm}}$

(b) (5 pts) $c = \sqrt{2000 - 800\sqrt{3}} = \sqrt{400(5 - 2\sqrt{3})}$

$\boxed{= 20\sqrt{5 - 2\sqrt{3}} \text{ cm}}$

(c) (10 pts) $\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin A = \frac{20 \cdot \frac{1}{2}}{20\sqrt{5 - 2\sqrt{3}}} = \frac{1}{2\sqrt{5 - 2\sqrt{3}}}$

$\approx .40344911067753 7$

$\rightarrow \sin^{-1}(\sin(A)) \approx 23.7939768869968 4^\circ$

$\approx \boxed{A \approx 23.7940^\circ}$

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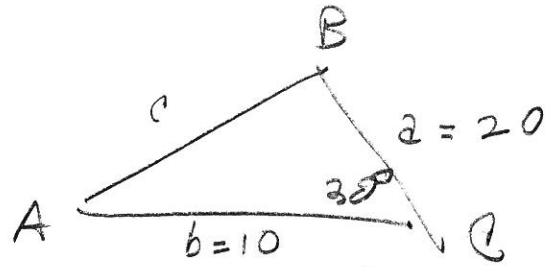
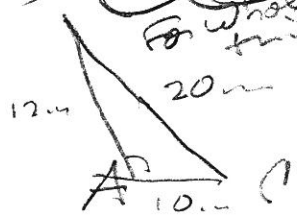
5 a 10 pts

70% if they miss the A is obtuse is a bad picture



A is OBTUSE!

B 5 pts for whole thing



$c^2 = a^2 + b^2 - 2ab\cos C$

$= 20^2 + 10^2 - 2(20)(10)\cos(30^\circ)$

$= 500 - 400\left(\frac{\sqrt{3}}{2}\right)$

$= 500 - 200\sqrt{3} \approx 500 - 200(.8660254037 8)$

$\approx 153.5898386 \text{ cm}^2$

$\Rightarrow c \approx 12.39313674927 \text{ cm}$

$\boxed{\approx 12.3931 \text{ cm}}$

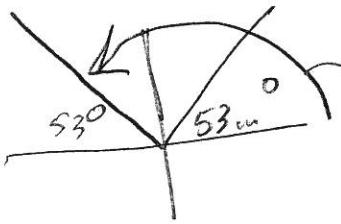
By this, we see that A is the largest angle, C is opposite the largest side

b 5 pts $\boxed{c = \sqrt{500 - 200\sqrt{3}} \text{ cm}}$

c 10 pts $\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin A = \frac{20 \sin 30^\circ}{c}$

$\approx \frac{10}{12.39313674927} \approx .8068922135 5$

$\Rightarrow \arcsin(\sin A) \approx 53.7939768869 8 5 8 1^\circ$



$\Rightarrow A = 180^\circ - \text{previous} \approx 126.206023113^\circ$

$\boxed{A \approx 126.2060^\circ}$

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6a 10pts $3+2i$ is a zero of $f(x)$

$= 2x^3 - 9x^2 + 8x + 39$

$3+2i$	2	-9	8	39
		$6+4i$	$-17+6i$	-39
	2	$-3+4i$	$-9+6i$	0

$3+2i$ is a zero

b 5pts $3-2i$

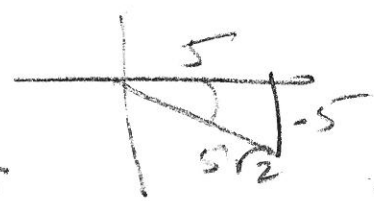
	2	$-3+4i$	$-9+6i$
		$6-4i$	$9-6i$
	2	3	0

$$\boxed{f(x) = (2x+3)(x-3+2i)(x-3-2i)}$$

① $(-3+4i)(3+2i) = -9-6i+12i+8i^2 = -9-8+6i$
$= -17+6i$

② $(-9+6i)(3+2i) = -27-18i+18i-12$
$= -39$

7 5pts $z = 5-5i$



$\arctan\left(\frac{-5}{5}\right) = \frac{\pi}{4}$

want QIV

$\sqrt{5^2+5^2} = \sqrt{2 \cdot 25}$

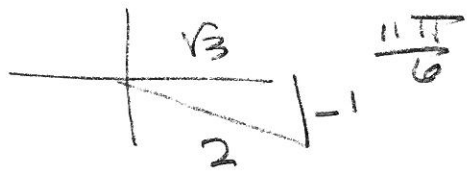
$= 25\sqrt{2}$

$$z = \boxed{5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$$

: 122 T 4

(8) $z = 81\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

(a) (10 pts)



$z = 81\left(\frac{\sqrt{3}}{2} + i\left(\frac{-1}{2}\right)\right)$

$= \boxed{\frac{81\sqrt{3}}{2} - \frac{81}{2}i = z}$

(b) (10 pts) $\boxed{\sqrt[4]{z} = 3\left(\cos\frac{11\pi}{24} + i\sin\frac{11\pi}{24}\right)}$

(c) (10 pts) $\frac{2\pi}{4} = \frac{\pi}{2} = \frac{12\pi}{24}$, $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

$\frac{11+12}{24}\pi = \frac{23\pi}{24}$

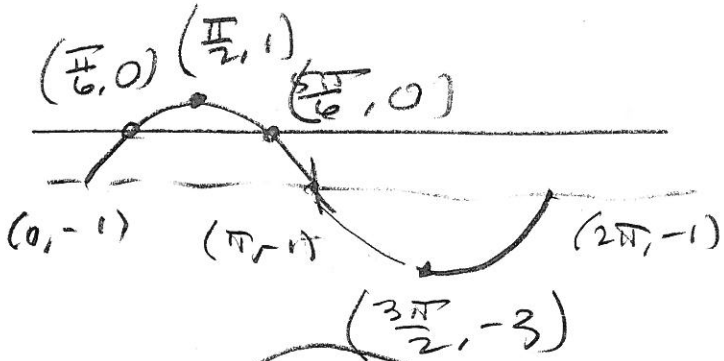
$\frac{23+12}{24}\pi = \frac{35}{24}\pi$

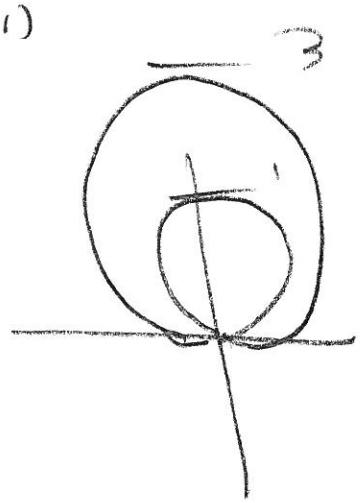
$\frac{35+12}{24}\pi = \frac{47\pi}{24}$

$$\boxed{\begin{array}{l} 3\left(\cos\frac{23\pi}{24} + i\sin\frac{23}{24}\pi\right) \\ 3\left(\cos\left(\frac{35}{24}\pi\right) + i\sin\left(\frac{35}{24}\pi\right)\right) \\ 3\left(\cos\left(\frac{47\pi}{24}\right) + i\sin\left(\frac{47\pi}{24}\right)\right) \end{array}}$$

122 TY

B1 $r = 2\sin(x) - 1$





122 T4

B2 10 pts

$2\sin(2x) - 1 = 0$

$\sin(2x) = \frac{1}{2}$

$2x = \frac{\pi}{6},\ x = \frac{\pi}{12}$

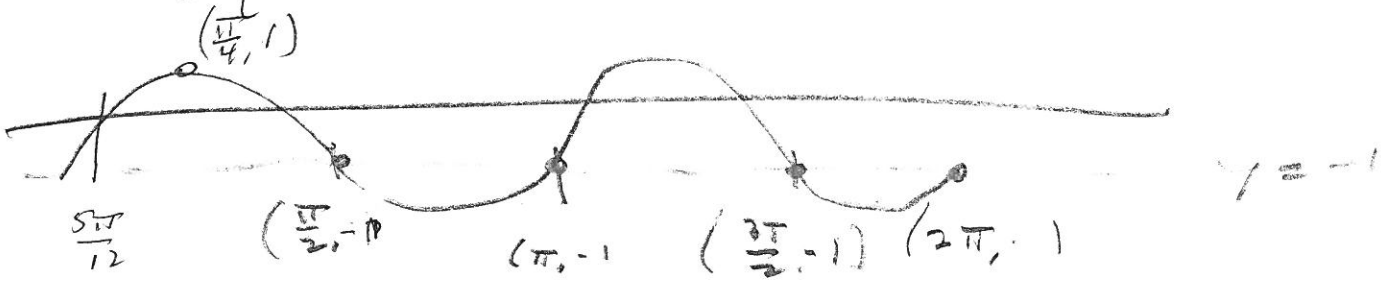
$y = -1$ = mid.

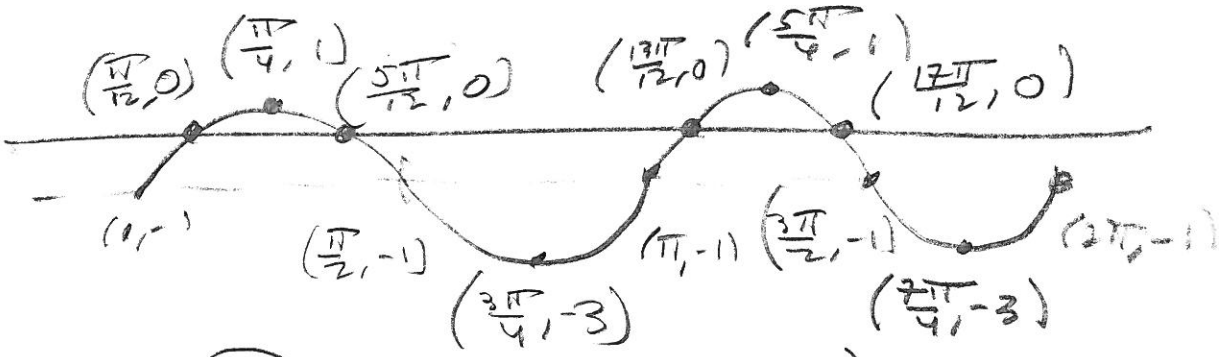
Period $= \pi$ Need 2 periods for polar graph.

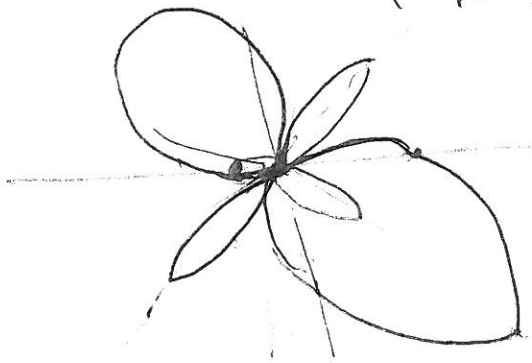
$\sin(2x) = \frac{1}{2}$

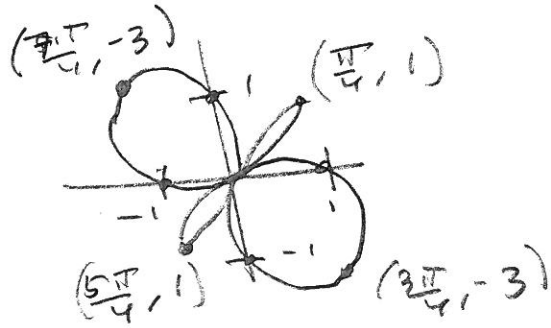
$2x = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}$

$x = \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$









122 TY

B3 (10 pts) $f(\theta) = 20\cos\left(\frac{\pi}{30}\theta + \frac{\pi}{3}\right) + 40$

A $a = 20$ ↕ mid: $y = 40$

$\frac{\pi}{30}\theta\ (\theta + 10)$ START: $x = -10$

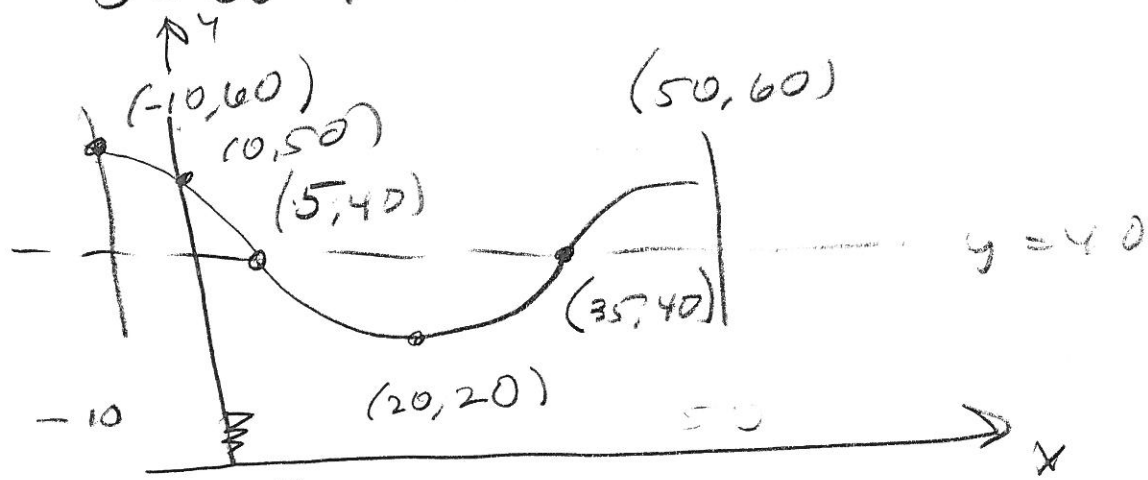
$\frac{\frac{\pi}{3}}{\frac{\pi}{30}} = \frac{\pi}{3} \cdot \frac{30}{\pi} = 10$

$40 + 20 = 60$ HIGH

$40 - 20 = 20$ LOW

$\frac{\pi}{30}\theta = 2\pi$

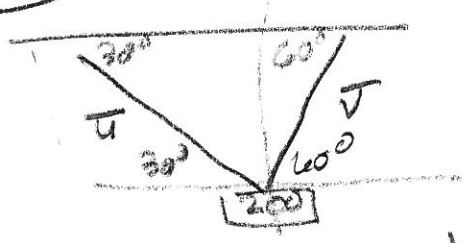
$\theta = 60$ = period



$f(0) = 20\cos\left(\frac{\pi}{3}\right) + 40$

$= 20\left(\frac{1}{2}\right) + 40 = 10 + 40 = 50$

B4 (10 pts) $\langle 0, -200 \rangle$ = weight vector in pounds



$\|\bar{u}\| = a$, $\|\bar{v}\| = b$

$a\langle \cos(150°), \sin(150°)\rangle$

$+ b\langle \cos(60°), \sin(60°)\rangle$

$= \langle 0, 200 \rangle$

a<

: 122 T4

B4 cont'd

$$a\langle -\tfrac{\sqrt{3}}{2}, \tfrac{1}{2}\rangle + b\langle \tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\rangle \overset{\text{SET}}{=} \langle 0, 200\rangle$$

$$-\tfrac{\sqrt{3}}{2}a + \tfrac{1}{2}b = 0 \longrightarrow -\tfrac{\sqrt{3}}{2}a = -\tfrac{1}{2}b$$

$$\longrightarrow a = \tfrac{1}{\sqrt{3}}b$$

$$\tfrac{1}{2}a + \tfrac{\sqrt{3}}{2}b = 200$$

$$\longrightarrow \tfrac{1}{2}\left(\tfrac{1}{\sqrt{3}}\right)b + \tfrac{\sqrt{3}}{2}b = 200$$

$$\left(\tfrac{1}{2\sqrt{3}} + \tfrac{\sqrt{3}}{2}\right)b = 200 \qquad \tfrac{\sqrt{3}}{6} + \tfrac{3\sqrt{3}}{6}$$

$$\tfrac{2\sqrt{3}}{3} = \tfrac{4\sqrt{3}}{6}b = 200 \longrightarrow b = \tfrac{600}{2\sqrt{3}} = \tfrac{600\sqrt{3}}{6}$$

$$\|\bar{v}\| = \boxed{100\sqrt{3} \text{ lbs} = b}$$

$$\|\bar{u}\| = a = \tfrac{1}{\sqrt{3}}b = \tfrac{1}{\sqrt{3}}\cdot 100\sqrt{3} \boxed{= 100 = a = \|\bar{u}\| \text{ lbs}}$$

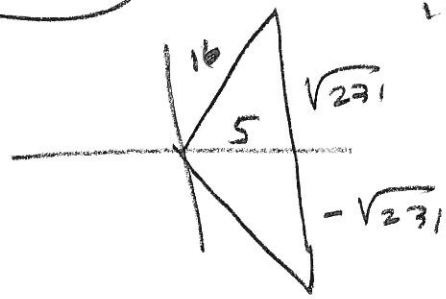
$$\bar{u} = 100\langle -\tfrac{\sqrt{3}}{2}, \tfrac{1}{2}\rangle$$

$$\boxed{\bar{u} = \langle -50\sqrt{3}, 50\rangle}$$

$$\bar{v} = 100\sqrt{3}\langle \tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\rangle = \boxed{\langle 50\sqrt{3}, 150\rangle = \bar{v}}$$

122 T4

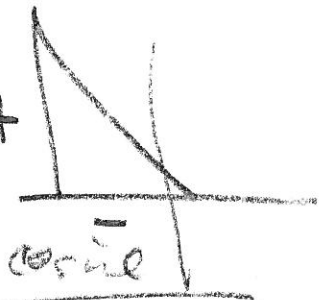
B5 (10 pts) $\cos(u) = \frac{5}{16}$ & $\tan(u) < 0$



$16^2 - 5^2 = 256 - 25$
$= 231$

$\frac{3\pi}{2} < u < 2\pi$

$\frac{3\pi}{4} < \frac{u}{2} < \pi \longrightarrow \frac{u}{2} \in Q\,II$



$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \frac{5}{16}}{2}} = \sqrt{\frac{11}{32}} = \frac{\sqrt{11}}{4\sqrt{2}} = \boxed{\frac{\sqrt{22}}{8} = \sin\left(\frac{u}{2}\right)}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1 + \frac{5}{16}}{2}} = -\sqrt{\frac{21}{32}} = -\frac{\sqrt{21}}{4\sqrt{2}} = \boxed{-\frac{\sqrt{42}}{8} = \cos\left(\frac{u}{2}\right)}$$

$$\tan\left(\frac{u}{2}\right) = -\frac{\sqrt{22}}{8} \cdot \frac{8}{\sqrt{42}} = -\sqrt{\frac{22}{42}}$$

$$= -\frac{\sqrt{11}}{\sqrt{21}} = \frac{-\sqrt{11}\,\sqrt{21}}{21} = \frac{-\sqrt{231}}{21} = \boxed{-\frac{\sqrt{231}}{21}}$$

$$\begin{array}{r} 210 \\ 21 \\ \hline 231 \end{array}$$

122 T4

B6 10pts

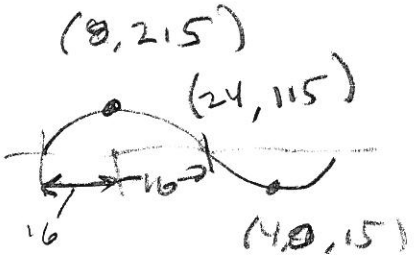
MAX : (8, 215)

MIN : (40, 15)

$32 = \frac{T}{2} \Rightarrow T = 64, \frac{\pi}{32}$

$bx = 2\pi$ when $x = 64$

$64b = 2\pi \rightarrow \boxed{b = \frac{\pi}{32}}$



$215 - 15 = 2a$

$200 = 2a$

$\boxed{100 = a}$

$24 - 8 = 16$

$8 - 16 = -8 = \text{start}$

$\frac{215 + 15}{2} = \frac{230}{2} = 115$

$\boxed{d = 115 = \text{mid}}$

$a \sin(b(x - c)) + d$

$= \boxed{100 \sin\left(\frac{\pi}{32}(x - (-8))\right) + 115 = f(x)}$