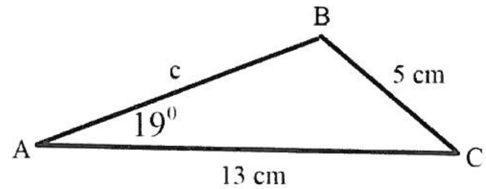
MAT 122 Test 3 – Spring, 2019 Name KEY
100 Points NO GRAPHING CALCULATORS!!!

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

1. Consider the triangle in the figure. Do not use rounded results in your calculations for new results. Only round in the final answer to each of the following:



   a. (10 pts) This triangle is oriented a bit differently than others you've seen for this SSA situation. But you can still show there are 2 solutions to this triangle. Do so.

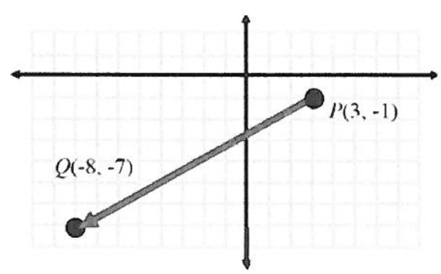
   b. (10 pts) Draw the picture for the case where angle B is acute and angle C is obtuse. You probably have already drawn it, in your answer to the previous question. In the figure, I've given the picture for an obtuse angle B and acute angle C. Don't worry about the length c, yet. That's part e.

   c. (10 pts) Choose the picture where the angle B is obtuse (the one I've drawn), and use the Law of Sines (and pictures and logic) to find (obtuse) angle B, to 4 decimal places.

   d. (10 pts) Use angle A and your (un-rounded) result for angle B to find angle C in the obvious way (subtraction!), to 4 decimal places.

   e. (5 pts) Use your (un-rounded) result for angle C and the Law of Cosines to find the length of side c. You can check your answer using the Law of Sines, but I insist on seeing the Law of Cosines, here. Give the length c rounded to 4 decimal places.

2. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\overline{u}$:



   a. (10 pts) Express the vector $\overline{u} = \overrightarrow{PQ}$ in component form.

   b. (10 pts) Compute the magnitude of $\overline{u}$. Leave your answer in simplified radical form.

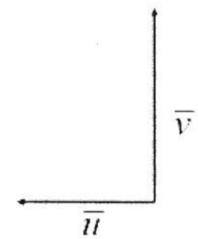
   c. (5 pts) Find the direction angle of $\overline{u}$ (the positive angle measured from the positive x-axis). Use degrees, rounded to 4 places.

3. Let $\overline{u} = \langle 3, -2 \rangle$.

   a. (10 pts) Express $\overline{u}$ as a linear combination of the canonical (standard) unit vectors $\overline{i}$ and $\overline{j}$.

   b. (10 pts) What's another word for the sum of 2 vectors?

4. The current in a river is flowing at 5 miles per hour, due West. $\left(\|\overline{u}\| = 5 \text{ mph} \right)$. A man in a boat points his boat due North to attempt a crossing. His boat's speed is 10 miles per hour $\left(\|\overline{v}\| = 10 \text{ mph} \right)$.



   a. (5 pts) Express $\overline{u}$ and $\overline{v}$ in component form.

   b. (5 pts) How far downstream will the current take the boat, if the river is 1 mile wide?

BONUS Answer up to 4 of the following for up to 20 bonus points.

5. Let $f(x) = 6x^4 - 35x^3 + 70x^2 + 25x - 26$.

   a. (5 pts) Use synthetic division to show that $x = 3 + 2i$ is a solution of the equation $f(x) = 0$.

   b. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

6. Let $z = -4 - 4\sqrt{3}\,i$

   a. (5 pts) Find $z + \overline{z}$ and $z\overline{z}$, where $\overline{z}$ is the complex conjugate of z.

   b. (5 pts) Express z in trigonometric form.

7. Let $z = 16\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right) \right)$.

   a. (5 pts) Express z in standard form.

   b. (5 pts) Find the trigonometric form of the principal 4$^{\text{th}}$ root of z, i.e., find $\sqrt[4]{z}$.

   c. (5 pts) Now, find *all* the 4$^{\text{th}}$ roots of z, in trigonometric form.

   d. (5 pts) Find the trigonometric form of z^2.

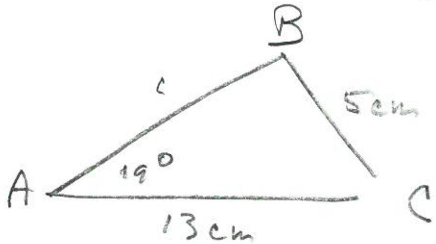
   e. (5 pts) Finally, let $w = 3\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right) \right)$, and find the trigonometric form of the product $z \cdot w$.

8. (5 pts) Find the *exact* value of $\sin\left(\frac{u}{2}\right), \cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, if $u = \frac{23\pi}{6}$.

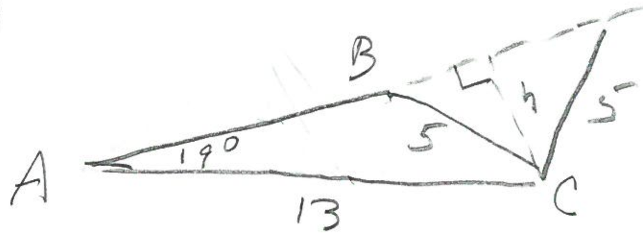
9. (5 pts) In what quadrant does $2u$ lie, if $\cos(u) = -\frac{2}{3}$ and $\sin(u) < 0$?

122 TEST 3 Spring, 2019

(1)



(1a) (10pts)



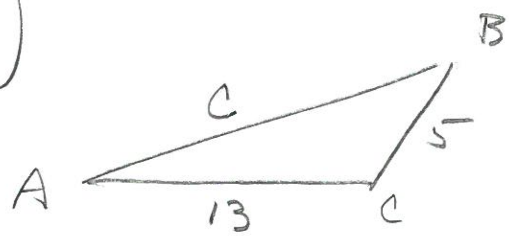
$\frac{h}{13} = \sin 19°$

$h = 13 \sin 19° \approx 4.232386011$

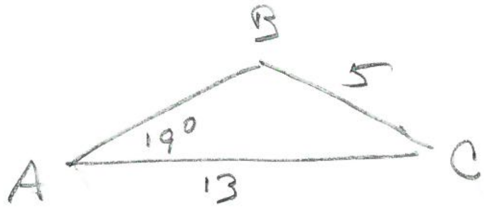
$h < 5 \Rightarrow$ 5 is big enough for 1 soln.

$5 < 13 \Rightarrow$ 5 is small enough for 2 solns

(1b) (10pts)



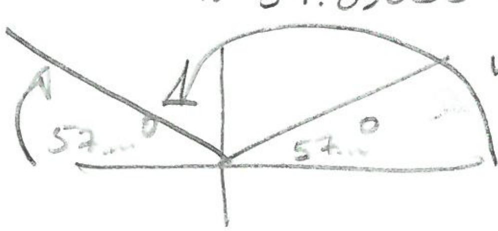
(1c)



$\frac{\sin B}{13} = \frac{\sin A}{5} \Rightarrow \sin B = \frac{13 \sin 19°}{5} \approx .8464772022$

$\sin^{-1}(\sin B) \approx \sin^{-1}(.8464772022)$

$\approx 57.83053298°$ is acute!



want this B:

$180° - 57...°$ 122.1694°

$\approx \boxed{122.169447\,0° \approx B}$

122 T3

1d 10 pts $C = 180^\circ - A - B = 180^\circ - 19^\circ - 57\ldots^\circ$

$\approx 38.8305530^\circ$

$\approx \boxed{38.8306^\circ \approx C}$

1e 5 pts $c^2 = a^2 + b^2 - 2ab\cos C$

$= 5^2 + 13^2 - 2(5)(13)\cos C$

$= 25 + 169 - 130\cos C$

$= 194 - 130\cos(38.\ldots^\circ)$

≈ 92.7295168

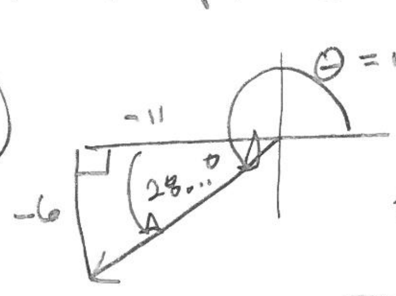
$\Rightarrow c \approx 9.62961649$

$\approx \boxed{9.6296 \text{ cm} \approx c}$

2a 10 pts $\overrightarrow{PQ} = \langle -8-3, -7-(-1) \rangle = \boxed{\langle -11, -6 \rangle = \bar{u}}$

2b 10 pts $\|\bar{u}\| = \sqrt{11^2 + 6^2} = \sqrt{121 + 36} = \boxed{\sqrt{157} = \|\bar{u}\|}$

2c 5 pts



$\arctan\left(\frac{6}{11}\right)$

$\approx 28.61045966^\circ$

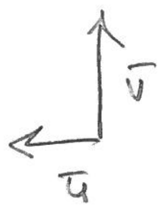
$\Rightarrow \theta = 180^\circ + 28\ldots^\circ \approx \boxed{208.6105^\circ \approx \theta}$

122 T3

3 a 10 pts $\bar{u} = \langle 3, -2 \rangle = \boxed{3\bar{i} - 2\bar{j} = \bar{u}}$

b 10 pts RESULTANT

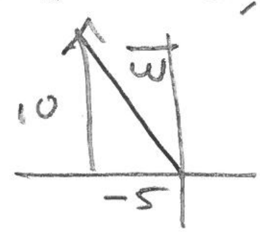
4 a 5 pts



$\bar{u} = \langle -5, 0 \rangle$

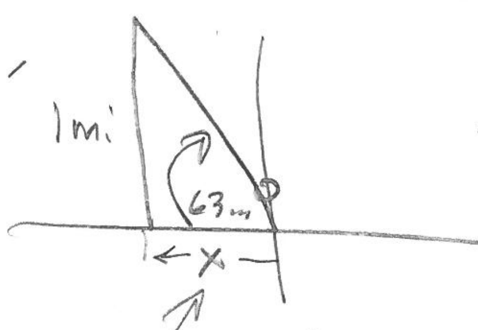
$\bar{v} = \langle 0, 10 \rangle$

b 5 pts $\bar{u} + \bar{v} = \langle -5, 10 \rangle = \bar{w}$



$\arctan(-2) \approx -63.43494883°$

so,



You could also just notice $-5 = -\frac{10}{2}$

so $-\frac{1}{2} = -.5$

What we want!

$\frac{1}{x} = \tan 63...° \Longrightarrow$

$\cot(63...°) = \frac{1}{\tan 63...°} = x \approx -.499999997$

$\Longrightarrow \approx \boxed{-.5000 \text{ miles!}}$

122 T3

B5 5pts $f(x) = 6x^4 - 35x^3 + 70x^2 + 25x - 26$

(a)

$3+2i$	6	-35	70	25	-26
		$18+12i$	① $-75+2i$	② $-19-4i$	③ 26
$3-2i$	6	$-17+12i$	$-5+2i$	$6-4i$	0 ← $3+2i$ is a zero
		$18-12i$	$3-2i$	$-6+4i$	
	6	1	-2	0	

(b) 5pts

$6x^2+x-2 = 6x^2+4x-3x-2 = 2x(3x+2)-1(3x+2) = (3x+2)(2x-1)$

① $(3+2i)(-17+12i) = -51+36i-34i-24 = -75+2i$

② $(3+2i)(-5+2i) = -15+6i-10i-4 = -19-4i$

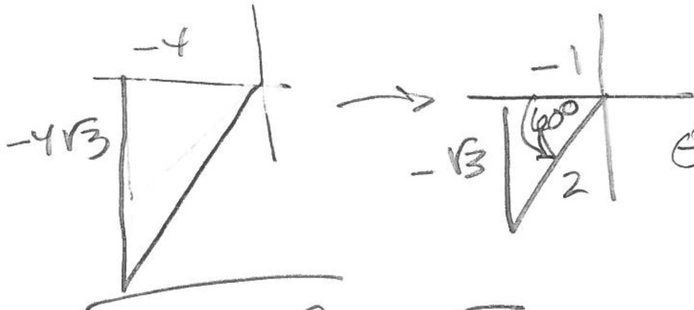
③ $(3+2i)(6-4i) = 2(3+2i)(3-2i) = 2(3^2+2^2) = 2(13) = 26$

So, $\boxed{f(x) = (3x+2)(2x-1)(x-(3+2i))(x-(3-2i))}$

B6 (a) 5pts $z = -4-4\sqrt{3}\,i \implies \bar{z} = -4+4\sqrt{3}\,i$

$\longrightarrow \boxed{z+\bar{z} = -8}$ & $z\bar{z} = 4^2 + 16\cdot 3 \boxed{= 64 = z\bar{z}}$

(b) 5pts



$\theta = 180° + 60° = 240°$
$= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$|\bar{z}| = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{64} = 8$

$\longrightarrow \boxed{z = 8\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)}$

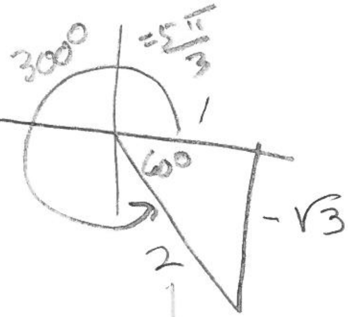
122 J3

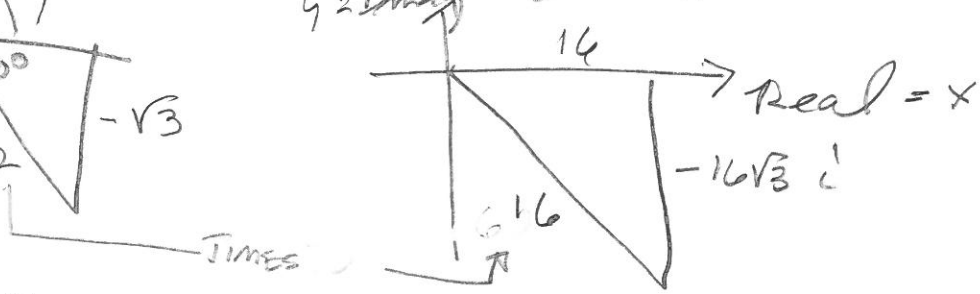
37 $z = 16\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

a

5pts

$\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$





$1 \cdot 16 = 16, \ -\sqrt{3}\cdot 16 = -16\sqrt{3}$

$\boxed{z = 16 - 16\sqrt{3}}$

b 5pts $\sqrt[4]{z} = \sqrt[4]{16}\left(\cos\frac{\frac{5\pi}{3}}{4} + i\sin\frac{5\pi}{12}\right)$

$= \boxed{2\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = \sqrt[4]{z}}$

c 5pts The other (3) 4th roots

Increment $= \frac{2\pi}{4} = \frac{\pi}{2}$

$\frac{5\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12} + \frac{6\pi}{12} = \frac{11\pi}{12}$

$\frac{11\pi}{12} + \frac{6\pi}{12} = \frac{17\pi}{12}$

$\frac{17\pi}{12} + \frac{6\pi}{12} = \frac{23\pi}{12}$

$$\boxed{\begin{array}{l} 2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right), \\ 2\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right), \\ 2\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right) \end{array}}$$

4th power of ALL of these is the original z!

12 2 T3

B7 (d) 5 pts $z^2 = 16^2\left(\cos\left(2\left(\frac{5\pi}{3}\right)\right) + i\sin\frac{10\pi}{3}\right)$

$= \boxed{256\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right) = z^2}$

(e) 5 pts $w = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \Longrightarrow$

$zw = 16\cdot 3\left(\cos\left(\frac{5\pi}{3} + \frac{\pi}{6}\right) + i\sin\left(\frac{5\pi}{3}\cdot\frac{2}{2} + \frac{\pi}{6}\right)\right)$

$= \boxed{48\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = zw}$

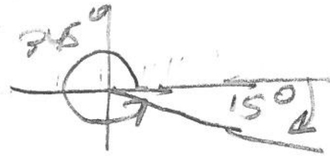
B8 5 pts $u = \frac{23\pi}{6} \Longrightarrow \frac{u}{2} = \frac{23\pi}{12}$

M1 $\frac{23\pi}{12}$ is just $\frac{\pi}{12}$ away from $\frac{24\pi}{12} = 2\pi$

$\Longrightarrow$ QIV $\Longrightarrow$

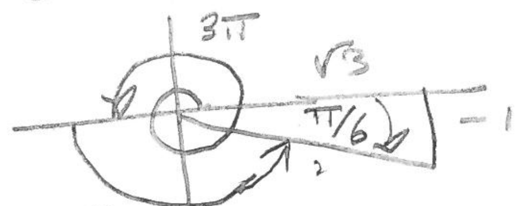
M2 $\frac{23\pi}{\cancel{12}_{1}} \cdot \frac{\cancel{180}^{15}}{\pi} = 23\cdot 15^\circ$

$= 345^\circ$



$\sin\frac{u}{2} = -\sqrt{\dfrac{1-\cos\frac{23\pi}{6}}{2}}$

$\frac{23\pi}{6} = \frac{18\pi}{6} + \frac{5\pi}{6} = 3\pi + \frac{5\pi}{6}$



$= -\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\dfrac{2-\sqrt{3}}{4}}$

$\boxed{= -\dfrac{\sqrt{2-\sqrt{3}}}{2} = \sin\frac{u}{2}}$

$\boxed{\cos\frac{u}{2} = \dfrac{\sqrt{2+\sqrt{3}}}{2}}$ $\tan\frac{u}{2} = \sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}}\cdot\sqrt{\dfrac{2-\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\dfrac{4-4\sqrt{3}+3}{4+3}}$

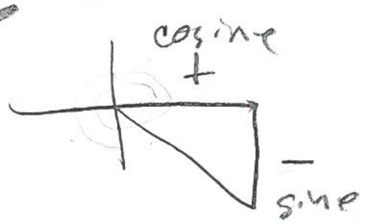
$= \boxed{\sqrt{\dfrac{7-4\sqrt{3}}{7}}} = \boxed{\dfrac{\sqrt{49-28\sqrt{3}}}{7}}$

122 T3

38 5pts $u = \frac{23\pi}{6} \Rightarrow \frac{u}{2} = \frac{23\pi}{12}, \quad \frac{23\pi}{12} \cdot \frac{180^\circ}{\pi} = 330^\circ$

$\frac{u}{2} \in Q\,IV$

$\frac{23\pi}{6} \in Q\,IV \quad \frac{23\pi}{6} \cdot \frac{180^\circ}{\pi} = 23 \cdot 30^\circ = 690^\circ$



$\frac{7\pi}{2} < \frac{23\pi}{6} < 4\pi$

$\frac{7\pi}{4} < \frac{23\pi}{12} < 2\pi$

So $\frac{23\pi}{12} \in Q\,IV$

$\sin\left(\frac{u}{2}\right) = -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2} = \sin\left(\frac{u}{2}\right)}$

$\boxed{\cos\left(\frac{u}{2}\right) = \frac{\sqrt{2+\sqrt{3}}}{2}}$

$\tan\left(\frac{u}{2}\right) = -\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = -\frac{\sqrt{(2-\sqrt{3})(2+\sqrt{3})}}{2+\sqrt{3}} = \frac{-\sqrt{4-3}}{2+\sqrt{3}} = \frac{-1}{2+\sqrt{3}}$

$= -\frac{(2-\sqrt{3})}{4-3} = -(2-\sqrt{3}) \boxed{= \sqrt{3} - 2 = \tan\left(\frac{u}{2}\right)}$

My crappy way?

$\frac{-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = -\sqrt{\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)} = \frac{-(2-\sqrt{3})}{\sqrt{4-3}} = \boxed{\sqrt{3} - 2}$

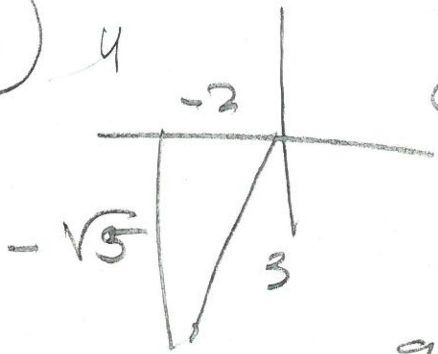
Oh. It's BETTER, once I see that $\sqrt{(2-\sqrt{3})(2-\sqrt{3})} = 2-\sqrt{3}$, instead of $\sqrt{4-2\sqrt{3}+3}$ (also true, but dumb).

122 T3

B9 5pts

$\cos u = -\frac{2}{3}$ & $\sin(u) < 0$

M1



Q III

$\pi < u < \frac{3\pi}{2} \Longrightarrow$

$2\pi < 2u < 3\pi$ doesn't give us the quadrant.

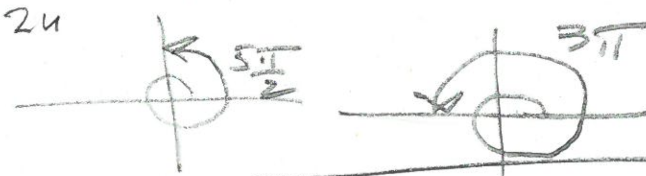
Need to fine-tune it:

$\sqrt{5} > 2$, so we're between 225° & 270°

$\frac{5\pi}{4}$ & $\frac{3\pi}{2}$

$\Longrightarrow \frac{5\pi}{4} < u < \frac{3\pi}{2}$

$\Longrightarrow \frac{5\pi}{2} < 2u < 3\pi$ Squarely in Q II



$\boxed{2u \in \text{Q II}}$

M2 $\arccos\left(-\frac{2}{3}\right) \approx 131.8103148957 8^\circ$

$180^\circ - 131...^\circ \approx 48.1896851042^\circ$

Since $u \in$ Q III, that means

$u \approx 180^\circ + 48...^\circ \approx 228.189685^\circ$

$\Longrightarrow 2u = 2 \cdot 228...^\circ \approx 456.37937^\circ$

Now, $456...^\circ - 360^\circ \approx 96.37937^\circ$ $\boxed{\in \text{Q II}}$ $\boxed{2u \in}$