MAT 122 Test 2 – Spring, 2018 Name__________________
100 Points NO GRAPHING CALCULATORS!!!

Do all work on separate paper, provided. All I want on this page is your name.

1. (5 pts) Kindness Points. I'm so glad you followed all formatting preferences I've been talking about all semester, like margins, writing clearly and dark enough for me to read. Thanks for circling final answers, and not cramming too much stuff into a small space. Thanks for organizing your work so that it's easy to follow, from the top on down. Thanks for leaving a 1-inch margin in the top left of each page. I thank you and your classmates thank you.

2. (10 pts) Find the values of all six trigonometric functions, given $\tan(u) = \frac{3}{5}$ and $\sin(u) < 0$.

3. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u) = -\frac{\sqrt{5}}{3}$ and $\tan(u) < 0$.
   Assume $0 < u < 2\pi$. Give final answers in simplified radical form.

4. Consider the equation $3\tan^3(x) - 3\tan^2(x) - \tan(x) + 1 = 0$.

   a. (10 pts) Find all solutions x, in radians, to the equation, above, in the interval $[0, 2\pi)$. Give *exact* answers, here. (Hint: Factor by grouping.)

   b. (10 pts) Find *all* real solutions x, in radians.

5. (10 pts) Re-write $\sin\left(\arctan\left(\frac{3x}{2}\right)\right)$ as an algebraic expression.

6. Evaluate $\cos\left(\frac{7\pi}{12}\right)$ in two ways: (Give *exact* answers, in simplified radical form.)

   a. (10 pts) Use a Sum identity.

   b. (10 pts) Use a Half-Angle identity.

7. (10 pts) Re-write $\cos[\arctan(x) - \arcsin(x)]$ as an algebraic expression. You will have a radical expression in the denominator. Leave it that way.

8. (10 pts) Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\sin(u) = -\frac{2}{3}$ and $\tan(u) > 0$. Give *exact* answers, in simplified radical form.

Bonus: Answer *up to three* (3) for up to 15 extra points:

1. A wheel of diameter $d = 20$ cm rolls 300 m. To the nearest full revolution, how many revolutions of the wheel were there? (BEWARE CONFLICTING UNITS!)

2. Build a cosine function that achieves its maximum height of $y = 200$ m at time $x = 5$ seconds and its minimum height of $y = -2$ at $x = 53$ seconds.

3. What is the area of the sector intercepted by an arc of 290^0 in a circle of radius 60 cm? Give an *exact* answer!

4. Sketch the graph of $100\sin\left(\frac{\pi}{16}x + \frac{11\pi}{8}\right) + 20$.

Double-Angle Formulas: $\sin(2u)=2\sin(u)\cos(u)$, $\cos(2u)=\cos^2(u)-\sin^2(u)=2\cos^2(u)-1=1-2\sin^2(u)$,

$\tan(2u)=\dfrac{2\tan(u)}{1-\tan^2(u)}=\dfrac{\sin(2u)}{\cos(2u)}$! You have to determine the "$\pm$" deal by determing the quadrant in which $\frac{u}{2}$ resides.

Half-Angle Formulas: $\sin\left(\frac{u}{2}\right)=\pm\sqrt{\dfrac{1-\cos(u)}{2}}$, $\cos\left(\frac{u}{2}\right)=\pm\sqrt{\dfrac{1+\cos(u)}{2}}$, $\tan\left(\frac{u}{2}\right)=\dfrac{1-\cos(u)}{\sin(u)}=\dfrac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}$!!!

You have to determine the "$\pm$" by determining the quadrant in which $\frac{u}{2}$ resides.

Power-Reducing Formulas: $\sin^2(u)=\dfrac{1-\cos(2u)}{2}$, $\cos^2(u)=\dfrac{1+\cos(2u)}{2}$, $\tan^2(u)=\dfrac{1-\cos(2u)}{1+\cos(2u)}=\dfrac{\sin^2(u)}{\cos^2(u)}$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v)-\cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v)+\cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v)+\sin(u-v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Pythagorean Identities

$\tan^2(x)+1=\sec^2(x)$

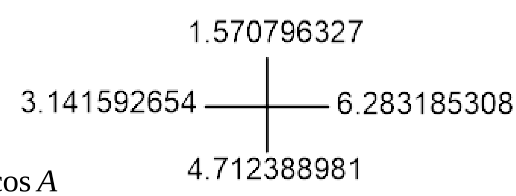
$\cot^2(x)+1=\csc^2(x)$

Angle Sum Formulas

$\sin(u+v)=\sin(u)\cos(v)+\sin(v)\cos(u)$

$\cos(u+v)=\cos(u)\cos(v)-\sin(u)\sin(v)$

Radians without π



Law of Sines $\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{c}$ **Law of Cosines** $a^2=b^2+c^2-2bc\cos A$

Heron's $Area=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$. **Magnitude** $\bar{u}=\langle a,b\rangle \Rightarrow \|\bar{u}\|=\sqrt{a^2+b^2}$

Arc Length: $s=r\theta$, **Area of a Sector:** $A=\dfrac{1}{2}r^2\theta$