MAT 122
100 Points

Test 2 – Fall, 2017

Name KEY
NO GRAPHING CALCULATORS!!!

Do all work on separate paper, provided. All I want on this page is your name.

1. (5 pts) Kindness Points. I'm so glad you followed all formatting preferences I've been talking about all semester, like margins, writing clearly and dark enough for me to read. Thanks for circling final answers, and not cramming too much stuff into a small space. Thanks for organizing your work so that it's easy to follow, from the top on down. Thanks for leaving a 1-inch margin in the top left of each page. I thank you and your classmates thank you.

2. (10 pts) Find the values of all six trigonometric functions, given $\tan(u)=-\frac{3}{8}$ and $\sin(u)<0$.

3. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u)=-\frac{2}{3}$ and $\tan(u)<0$. Give final answers in radical form. You do not need to rationalize denominators I do not want any use of calculators on this problem.

4. Consider the equation $3\csc^3(x)-6\csc^2(x)-4\csc(x)+8=0$.

   a. (10 pts) Find all solutions x, in radians, to the equation, above, in the interval $[0,2\pi)$. Give *exact* answers, here. (Hint: Factor by grouping.)

   b. (10 pts) Find *all* real solutions x, in radians.

5. (10 pts) Re-write $\tan\left(\arccos\left(\frac{3x}{\sqrt{9x^2+25}}\right)\right)$ as an algebraic expression.

6. (10 pts) Solve $\csc^2(x)-4\csc(x)=-4$. Find all solutions in $[0,2\pi)$. You may certainly use degrees to "see" things, better, but I expect (require) an exact answer, in radians, as your final answer.

7. Evaluate $\sin\left(\frac{19\pi}{12}\right)$ in two ways: (Give *exact* answers, in simplified radical form.)

   a. (10 pts) Use a Sum identity.

   b. (10 pts) Use a Half-Angle identity.

8. (10 pts) Re-write $\sin[\arccos(x)+\arctan(x)]$ as an algebraic expression. (Hint: Use Sum identity.)

9. (10 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\sin(u) = \frac{2}{3}$ and $\cos(u) < 0$. Give *exact* answers, in simplified radical form.

Bonus: Answer *up to three* (3) for up to 15 extra points:

1. A wheel of diameter $d = 10$ cm rolls 500 m. To the nearest full revolution, how many revolutions of the wheel were there?

2. Build a cosine function that achieves its maximum height of $y = 110$ m at time $x = 4$ seconds and its minimum height of $y = -2$ m at $x = 120$ seconds.

3. What is the area of the sector intercepted by an arc of 330^0 in a circle of radius 60 cm? Give an *exact* answer!

4. Sketch the graph of $100\sin\left(\frac{\pi}{13}x + \frac{11\pi}{13}\right) + 20$.

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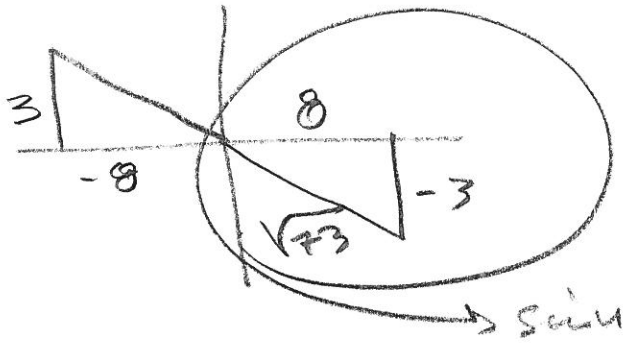
(1) Kindness (5pts)

(2) $\tan u = -\frac{3}{8}$, $\sin u < 0$



$9+64=73$

$\sin u < 0$

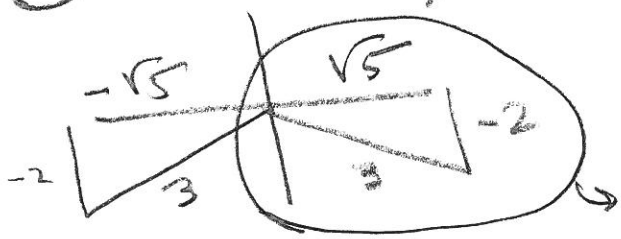
(10pts)

$\sin u = -\frac{3}{\sqrt{73}}$ $\quad \csc u = -\frac{\sqrt{73}}{3}$

$\cos u = \frac{8}{\sqrt{73}}$ $\quad \sec u = \frac{\sqrt{73}}{8}$

$\tan u = -\frac{3}{8}$ $\quad \cot u = -\frac{8}{3}$

(3) $\sin u = -\frac{2}{3}$, $\tan u < 0$



$9-4=5$

$\tan u < 0$

So, $\frac{3\pi}{2} < u < 2\pi$

$\Rightarrow \frac{3\pi}{4} < \frac{u}{2} < \pi$

$\Rightarrow$ QII $\sin\frac{u}{2} > 0$, $\cos\frac{u}{2} < 0$

$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{\sqrt{5}}{3}}{2}}$

$= \sqrt{\frac{3-\sqrt{5}}{6}} = \frac{\sqrt{18-6\sqrt{5}}}{6}$

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(3) cont'd

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{\sqrt{5}}{3}}{2}} = -\sqrt{\frac{3+\sqrt{5}}{6}}$$

$$= -\frac{\sqrt{18+6\sqrt{5}}}{6} = \cos\frac{u}{2}$$

10 pts

$$\Rightarrow \tan\frac{u}{2} = \frac{\sin\frac{u}{2}}{\cos\frac{u}{2}} = \frac{\sqrt{\frac{18-6\sqrt{5}}{6}}}{-\frac{\sqrt{18+6\sqrt{5}}}{6}} \cdot \frac{-6}{\sqrt{18+6\sqrt{5}}}$$

$$= -\sqrt{\frac{18-6\sqrt{5}}{18+6\sqrt{5}}} = -\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \tan\frac{u}{2}$$

(4) $3\csc^3 x - 6\csc^2 x - 4\csc x + 8 = 0$

$3u^3 - 6u^2 - 4u + 8 = 0$

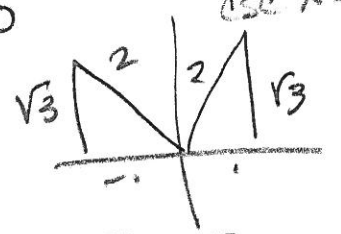
$3u^2(u-2) - 4(u-2) = 0$

$(u-2)(3u^2-4) = 0$

$(u-2)(\sqrt{3}u-2)(\sqrt{3}u+2) = 0$

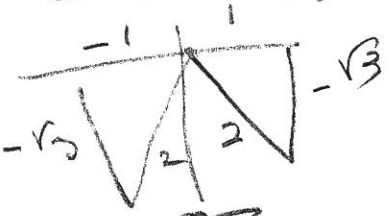
$u = 2, \pm\frac{2}{\sqrt{3}} = \csc x$

$\csc x = \frac{2}{\sqrt{3}}$



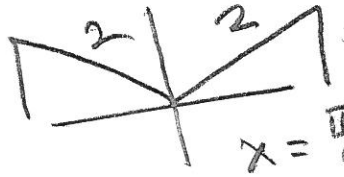
$\frac{\pi}{3}, \frac{2\pi}{3}$

$\csc x = -\frac{2}{\sqrt{3}}$



$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\csc x = 2$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

10 pts

$x \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$

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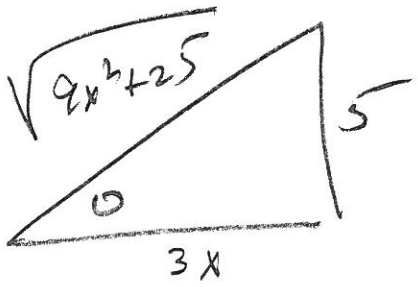
4b (10 pts)

$$x \in \left\{ \frac{\pi}{3}+2n\pi,\ \frac{2\pi}{3}+2n\pi,\ \frac{4\pi}{3}+2n\pi,\ \frac{5\pi}{3}+2n\pi,\ \frac{\pi}{6}+2n\pi,\ \frac{5\pi}{6}+2n\pi \;\middle|\; n \in \mathbb{Z} \right\}$$

5) $\tan\left(\arccos\left(\frac{3x}{\sqrt{9x^2+25}}\right)\right) = \tan\theta$

$$= \frac{5}{3x}$$

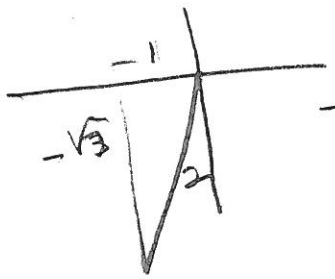


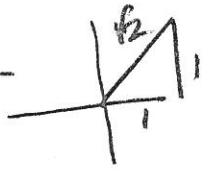
10 pts

7) $\sin\frac{19\pi}{12}$ 8) $\frac{19\pi}{12} = \frac{18\pi}{12} + \frac{\pi}{12} = \frac{(17+2)\pi}{12}$

$$= \frac{(16+3)\pi}{12} = \frac{16\pi}{12} + \frac{3\pi}{12} = \frac{4\pi}{3} + \frac{\pi}{4} \longrightarrow$$

$$\sin\left(\frac{19\pi}{12}\right) = \sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{4\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{4\pi}{3}$$





$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right)$$

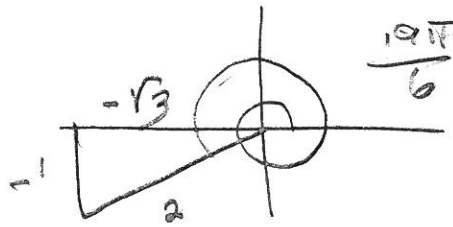
$$= \frac{-\sqrt{3}-1}{2\sqrt{2}}$$

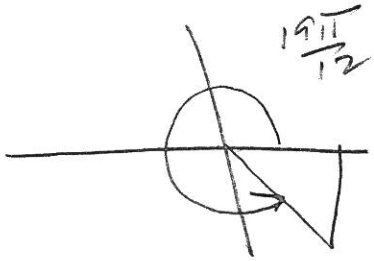
122 T2

(b) (10 pts) $\frac{19\pi}{12} = \frac{1}{2}\left(\frac{19\pi}{6}\right)$

$\frac{19\pi}{6}$ p/c.

$\frac{19\pi}{6} = 3\pi + \frac{\pi}{6}$





$\sin\frac{19\pi}{12} < 0$

$\frac{8\pi}{12} = \frac{2\pi}{3} = 120^\circ$ $\frac{19\pi}{12} = \pi + \frac{7\pi}{12}$

$\frac{7\pi}{12} = 120^\circ - 15^\circ$ =

$= 105^\circ$

So add 105° to $180^\circ \Rightarrow 285^\circ$

$$\sin\frac{19\pi}{12} = -\sqrt{\frac{1-\cos\frac{19\pi}{6}}{2}} = -\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2} = \sin\frac{19\pi}{12}$$

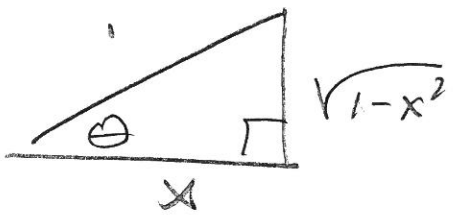
122 T2

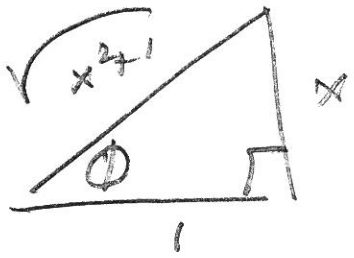
(a) $\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta$

$= \sin(\arccos(x))\cos(\arctan(x))$

10pts $+ \sin(\arctan(x))\cos(\arccos(x))$

$\theta = \arccos(x)$ $\qquad$ $\phi = \arctan(x)$



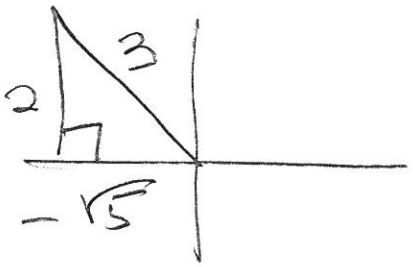


$$= \left(\sqrt{1-x^2}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) + \left(\frac{x}{\sqrt{x^2+1}}\right)x$$

$$= \frac{x^2+\sqrt{1-x^2}}{\sqrt{x^2+1}}$$

122 T2

(9) (10 pts) $\sin u = \frac{2}{3}$, $\cos u < 0$



$\sin(2u) = 2\sin u \cos u$

$= 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$

$= -\frac{4\sqrt{5}}{9} = \sin(2u)$

$\cos(2u) = \cos^2 u - \sin^2 u = \left(-\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2$

$= \frac{5-4}{9} = \frac{1}{9} = \cos(2u)$ →

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = -\frac{4\sqrt{5}}{9} \cdot \frac{9}{1} = -4\sqrt{5} = \tan(2u)$

122 T2

(a) diameter = 10 $\Rightarrow$ $r = 5$

$s = r\theta \rightarrow \theta = \frac{s}{r} = \frac{500}{5} = 100$ radians

$= (100 \text{ radians})\left(\frac{1 \text{ revolution}}{2\pi \text{ radians}}\right) = \frac{100}{2\pi}$

OOPS! $r \neq$ cm !

$s \neq$ m !

Need to convert one of the other:

$\left(\frac{500 \text{ m}}{5 \text{ cm}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right) = \frac{50000}{5} = 10{,}000$ radians

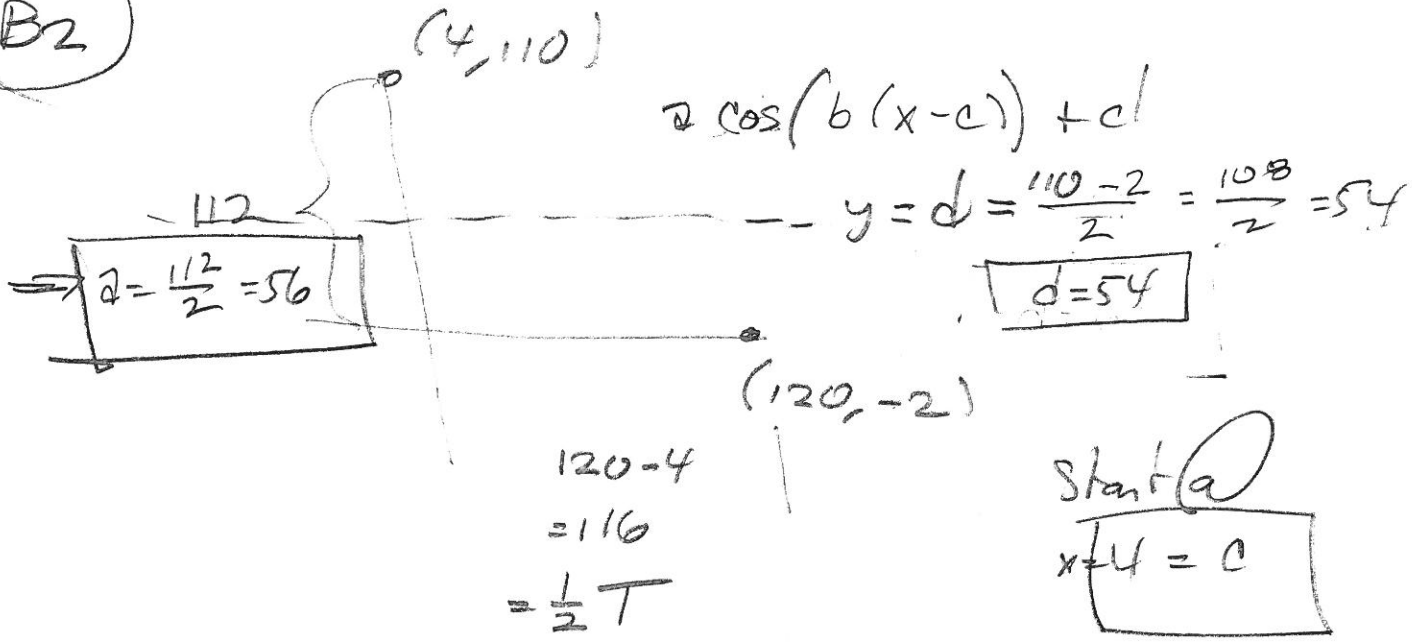
$\rightarrow (10000 \text{ radians})\left(\frac{1 \text{ revolution}}{2\pi \text{ radians}}\right)$

$= \frac{5000}{\pi}$ rev. ≈ 1591.549431 revs

≈ 1592 revolutions

122 T2

B2



$a \cos(b(x-c)) + d$

$y = d = \frac{110-2}{2} = \frac{108}{2} = 54$

$d = 54$

$a = \frac{112}{2} = 56$

$120 - 4$

$= 116$

$= \frac{1}{2} T$

Start @ $x - 4 = 0$

$bx = 2\pi$ when

$x = 232, 50$

$b = \frac{2\pi}{232} = \frac{\pi}{116}$

$$f(x) = 56 \cos\left(\frac{\pi}{116}(x-4)\right) + 54$$

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B3 $A = \frac{1}{2} r^2 \theta = \left(\frac{1}{2}\right)(60)^2 (330^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$= \frac{1}{2}(3600)\left(\frac{330\pi}{180}\right) = \frac{(1800)(330\pi)}{180}$

$= \boxed{3300\pi \text{ cm}^2}$

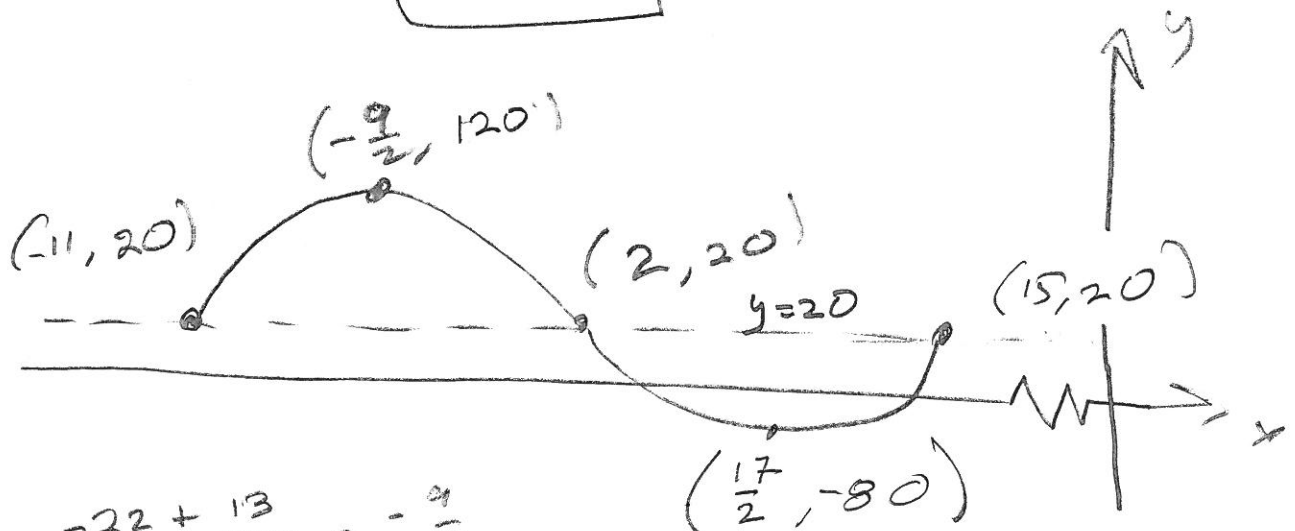
B4 $100 \sin\left(\frac{\pi}{13}(x+11)\right) + 20$ → $y = 20$ mid.

Amp = 100

$\frac{\pi}{13} x = 2\pi$

$\Rightarrow x = 26 = T$

start @ $x = -11$



$\frac{-22 + 13}{2} = -\frac{9}{2}$

$\frac{4 + 13}{2} = \frac{17}{2}$