MAT 122 Test 4 – Fall, 2016 Name_____
200 Points NO GRAPHING CALCULATORS!!!

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

1. We convert $(x,y)=(3,-5)$ to polar coordinates, (r,θ).

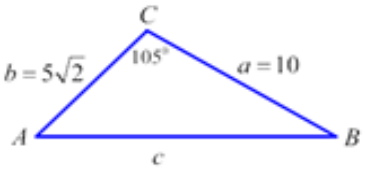
   a. (10 pts) Assume $r > 0$ and $\theta \in [0, 360^0]$. Find the *exact* polar coordinates of the point. Use degrees for angle measures.

   b. (10 pts) Approximate your answer in part a, with 4-decimal-place accuracy.

2. (10 pts) Convert $(r,\theta)=\left(3, -\frac{2\pi}{3}\right)$ to rectangular coordinates. Give an exact answer and a decimal answer, accurate to 4 decimal places.

3. (10 pts) Sketch the graph of $r = 8\cos\theta$.

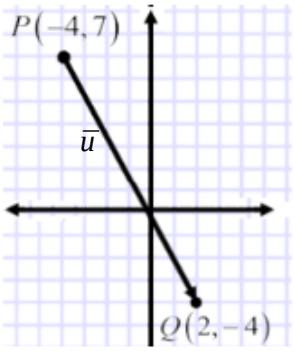
4. Consider the triangle in the figure. Assume lengths are in miles



   a. (10 pts) Use the Law of Cosines to find the length of side *c* in the triangle illustrated on the right. Round your final answer to 4 decimal places, but keep the un-rounded number in your calculator for the next question.

   b. (10 pts) Use the Law of Sines to find the measure of angle *A*, in degrees. (Nice, clean answer, for a change!)

5. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:



   a. (10 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.

   b. (10 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical form.

   c. (10 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.

   d. (10 pts) Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.

6. Let $f(x)=5x^3-22x^2+33x-10$.

   a. (10 pts) Use synthetic division to show that $x = 2+i$ is a solution of the equation $f(x)=0$.

   b. (10 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

7. (10 pts) Express $z = -\sqrt{3} - i$ in trigonometric form.

8. Let $z = 27\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$.

   a. (10 pts) Express z in standard form.

   b. (10 pts) Find the principal 3rd root of z, i.e., find $\sqrt[3]{z}$. Leave z in trigonometric form for this.

   c. (10 pts) Now, find the *other* pair of 3rd roots of z, in trigonometric form.

   d. (10 pts) Finally, let $w = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.

9. (10 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\cos(u) = -\frac{3}{11}$ and $\sin(u) < 0$.

10. (10 pts) Find $\sin\left(\frac{u}{2}\right), \cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = -\frac{3}{11}$ and $\sin(u) < 0$.

11. (10 pts) Build a sine function that achieves its maximum height of $y = 95$ meters at time $x = 10$ seconds and its minimum height of $y = 15$ meters at $x = 18$ seconds.

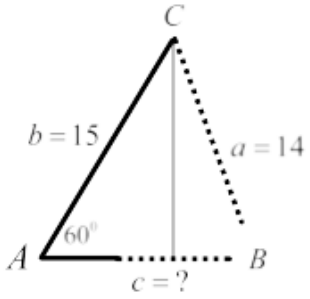
Bonus Section

Bonus 1. (10 pts) Find all solutions of the equation $2\sin(2x) - 1 = 0$ in the interval $[0, 2\pi)$.

Bonus 2. (10 pts) Sketch the graph of $f(\theta) = 11\sin\left(\frac{\pi}{14}\theta - \frac{26\pi}{7}\right) + 4$.

Bonus 3. Consider the triangle described by the following (See figure):

Angle $A = 60^0$, side $b = 15$ and side $a = 14$.



   a. (5 pts) Prove that there are two triangles fitting this description.

   b. (5 pts) Find both possible values of angle *B*.

Bonus 4. (10 pts) Find the *exact* value of side *c* of the triangle in problem #5.