(1) $x = 4\cos t,\ y = 3\sin t$

(a) 5pts

$\Longrightarrow \frac{x}{4} = \cos t,\quad \frac{y}{3} = \sin t$

$\Longrightarrow \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = \cos^2 t + \sin^2 t = 1$

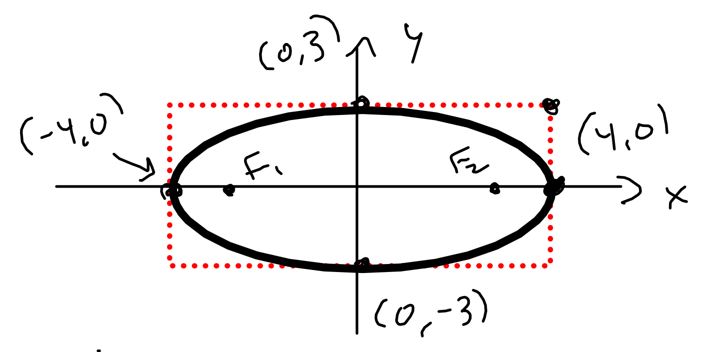
$\Longrightarrow \boxed{\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1}$ ellipse!

(b) 5pts

sketch

$a = 4,\ b = 3,$

Center $= (0,0)$



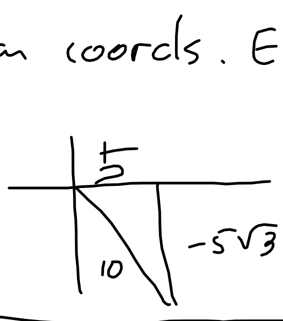
(c) 5pts Find F_1, F_2

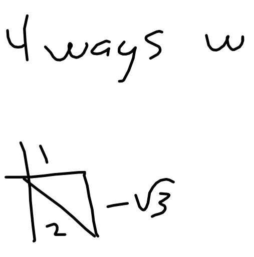
$c^2 = a^2 - b^2 = 4^2 - 3^2 = 7 \longrightarrow c = \sqrt{7} \approx 2.64575$

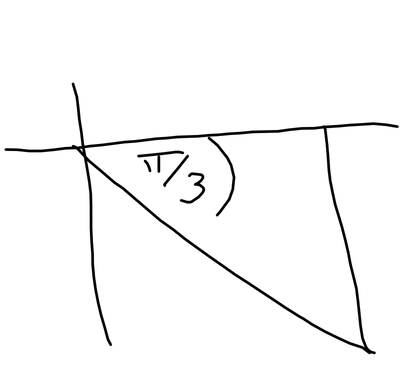
$\boxed{\begin{array}{l} F_1 = (-\sqrt{7}, 0) \\ F_2 = (\sqrt{7}, 0) \end{array}}$

(2) 5pts Convert to polar coords. Express 4 ways with $\theta \in (-2\pi, 2\pi)$

$(5, -5\sqrt{3})$

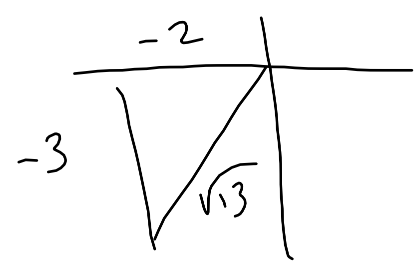






$= \left(10, \frac{5\pi}{3}\right)$ or $\left(10, -\frac{\pi}{3}\right)$
OR $\left(-10, \frac{2\pi}{3}\right)$ OR $\left(-10, -\frac{4\pi}{3}\right)$

(3) 5pts Convert $(-2,-3)$ to polar coords. θ in radians. $0 < \theta < 2\pi$.



$\approx (3.605551275, 4.124386378)$

$\approx \boxed{(3.606, 4.124) \approx (r, \theta)}$

(4) 5pts $\left(\sqrt{13}, \pi + \arctan\left(\frac{3}{2}\right)\right)$

Exact Answer for #3.

OR $\left(\sqrt{13}, \pi + \arccos\left(\frac{2}{\sqrt{13}}\right)\right)$

OR $\left(\sqrt{13}, \pi + \arcsin\left(\frac{3}{\sqrt{13}}\right)\right)$

(5) $r = 2\sin(2\theta) + 1$

ONE METHOD. 2nd Method on Next PAGE.

Strategy: Find zeros ($r = 0$) and intersections with the midline ($r = 1$).

To find the zeros, we want to find all $\theta \in [0, 2\pi] \ni r = 0$

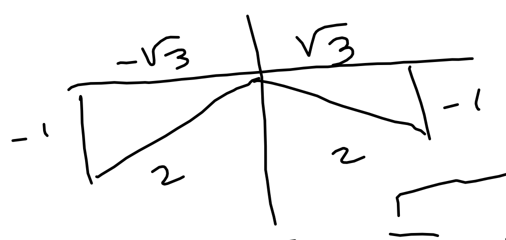
This means we want all $2\theta \in [0, 4\pi] \ni r = 0$.

$r = 0$

$2\sin(2\theta) + 1 = 0$

$2\sin(2\theta) = -1$

$\sin(2\theta) = -\frac{1}{2}$



$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$, $+2\pi$, $+2\pi$ | $\frac{31\pi}{6}$ is too far.

$\Rightarrow \theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$\boxed{A = (\frac{7\pi}{12}, 0),\ B = (\frac{11\pi}{12}, 0),\ C = (\frac{19\pi}{12}, 0),\ D = (\frac{23\pi}{12}, 0)}$

Low Points are $\frac{1}{2}$-way between intercepts

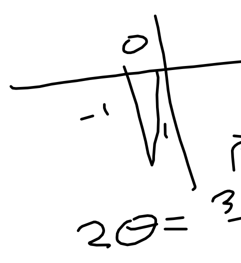
$\frac{\frac{7\pi}{12} + \frac{11\pi}{12}}{2} = \frac{18\pi}{24} = \frac{3\pi}{4}$, $\frac{\frac{19\pi}{12} + \frac{23\pi}{12}}{2} = \frac{42\pi}{24} = \frac{7\pi}{4}$

$\boxed{E = (\frac{3\pi}{4}, -1),\ F = (\frac{7\pi}{4}, -1)}$

Directly: $2\sin(2\theta) + 1 = -1$

$2\sin(2\theta) = -2$

$\sin(2\theta) = -1$



$p + 2\pi$

$2\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

To find midline points:

Logic: They divide 2π into 4 equal parts:

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\boxed{G = (0, 1),\ H = (\frac{\pi}{2}, 1),\ I = (\pi, 1),\ J = (\frac{3\pi}{2}, 1),\ K = (2\pi, 1)}$

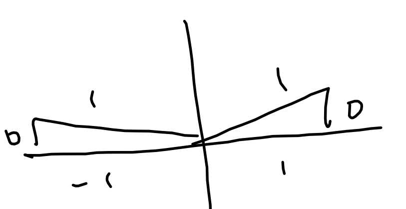
Directly: $2\sin(2\theta) + 1 = 1$

$2\sin(2\theta) = 0$

$\sin(2\theta) = 0$

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

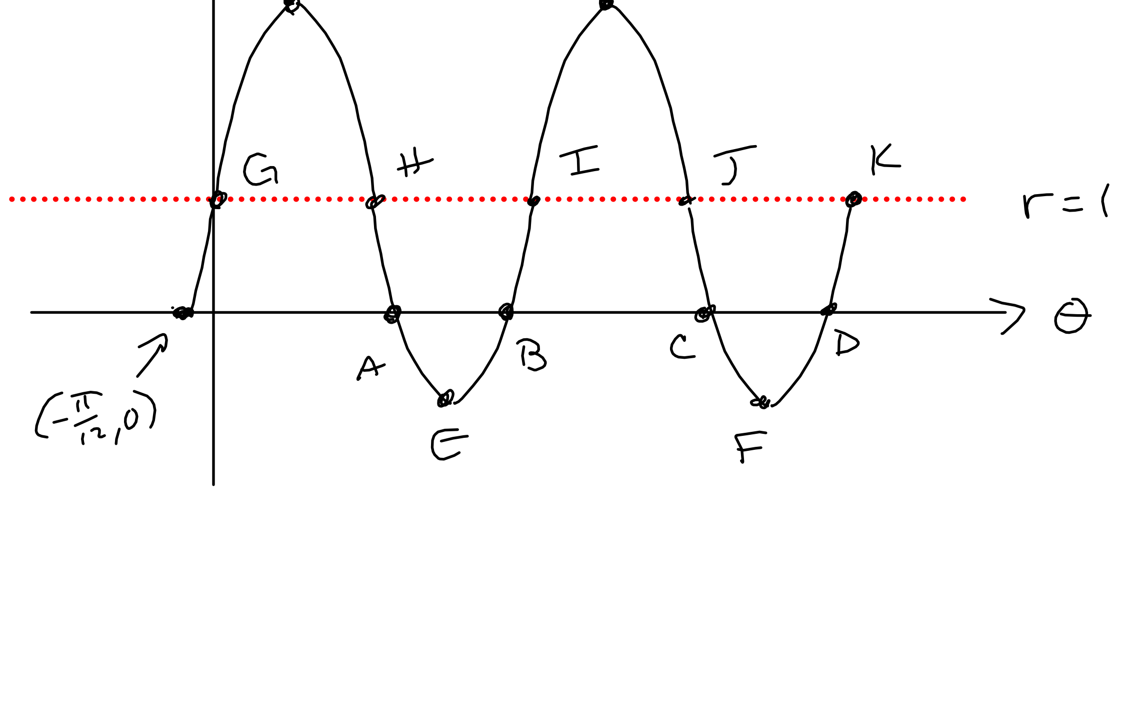
$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.



High points: Halfway between midline pts:

$\theta = \frac{0 + \frac{\pi}{2}}{2} = \frac{\pi}{4}$, $\frac{\pi + \frac{3\pi}{2}}{2} = \frac{5\pi}{4}$

$L = (\frac{\pi}{4}, 3),\ M = (\frac{5\pi}{4}, 3)$



(5) $r = 2\sin(2\theta) + 1$

$r = 2\sin(2\theta) + 1$

Amplitude $a = 2$

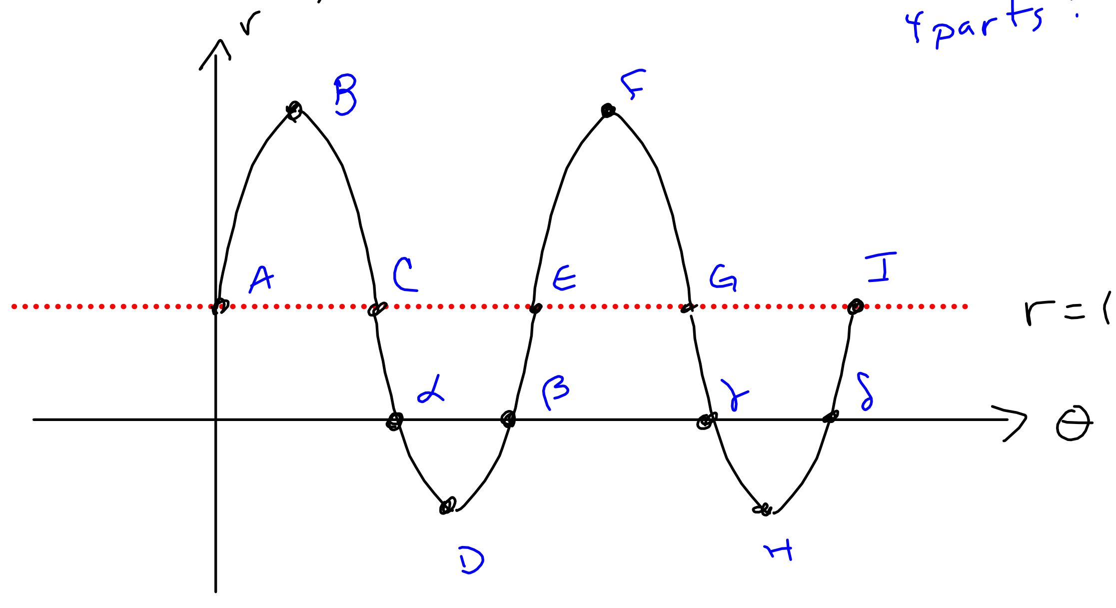
starts @ $\theta = 0$

$r = 1$ midline

Period: $2\theta = 2\pi$ when $\theta = \pi$ = Period.

Graphing over $[0, 2\pi]$: 1st Graph 1 period (π). then Repeat.

4 parts: $\frac{\pi}{4}$ = increment



$A = (0, 1)$
$B = (\frac{\pi}{4}, 3)$
$C = (\frac{\pi}{2}, 1)$
$D = (\frac{3\pi}{4}, -1)$
$E = (\pi, 1)$
$F = (\frac{5\pi}{4}, 3)$
$G = (\frac{3\pi}{2}, 1)$
$H = (\frac{7\pi}{4}, -1)$
$I = (2\pi, 1)$

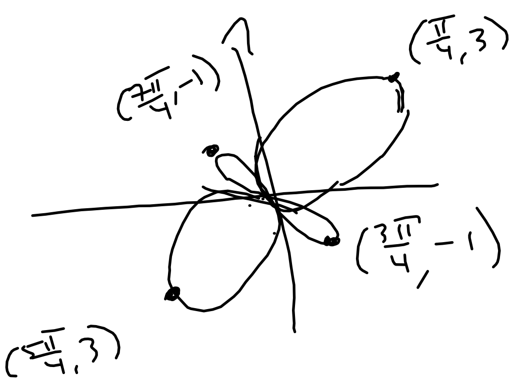
Now, fill in the zeros:

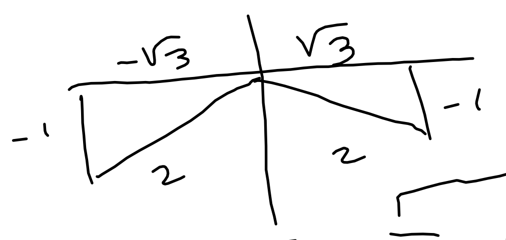
$r = 0$

$2\sin(2\theta) + 1 = 0$

$2\sin(2\theta) = -1$

$\sin(2\theta) = -\frac{1}{2}$





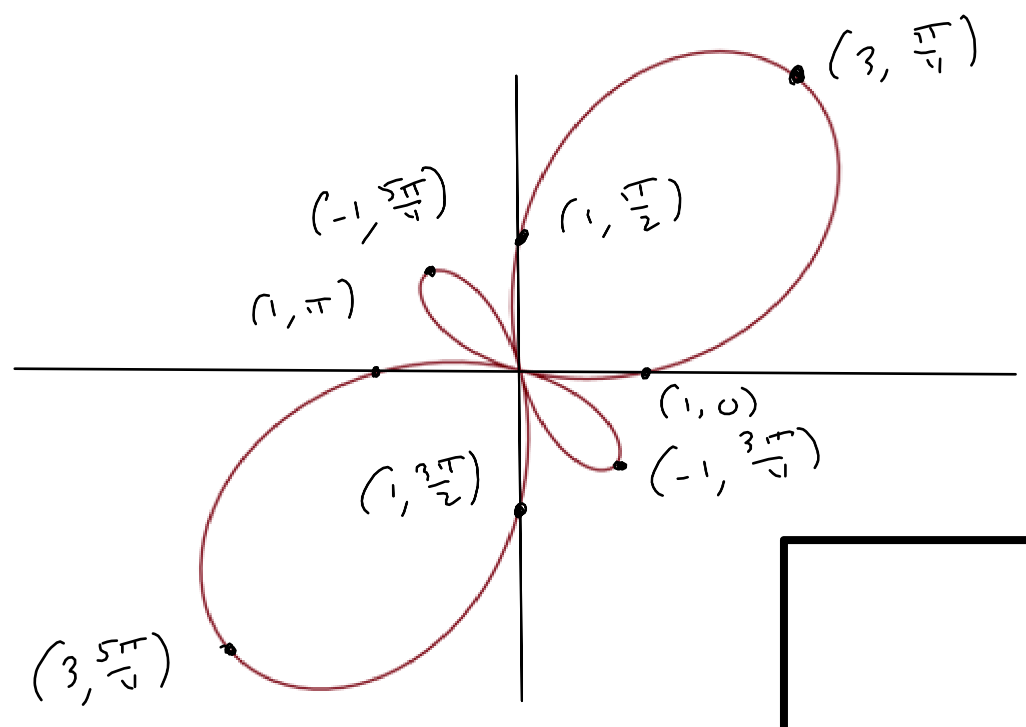
$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$, $\boxed{\frac{31\pi}{6} \text{ is too far.}}$

+2π

+2π $= \frac{12\pi}{6}$

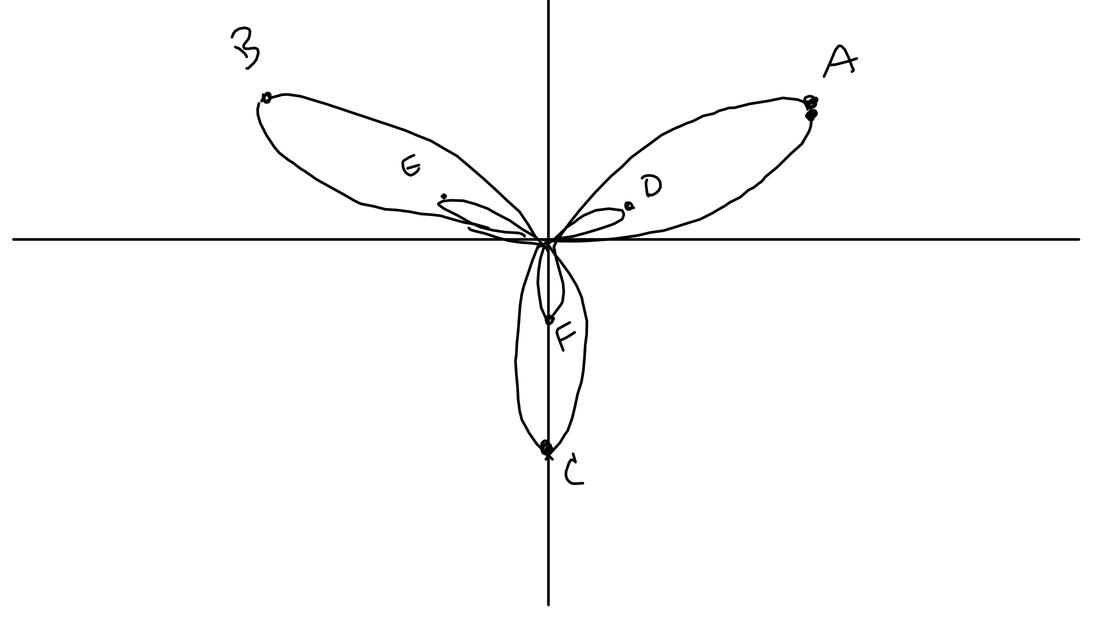
$\Rightarrow \theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$\boxed{\alpha = (\frac{7\pi}{12}, 0), \beta = (\frac{11\pi}{12}, 0), \gamma = (\frac{19\pi}{12}, 0), \delta = (\frac{23\pi}{12}, 0)}$



$r = 2\sin(3\theta) + 1$

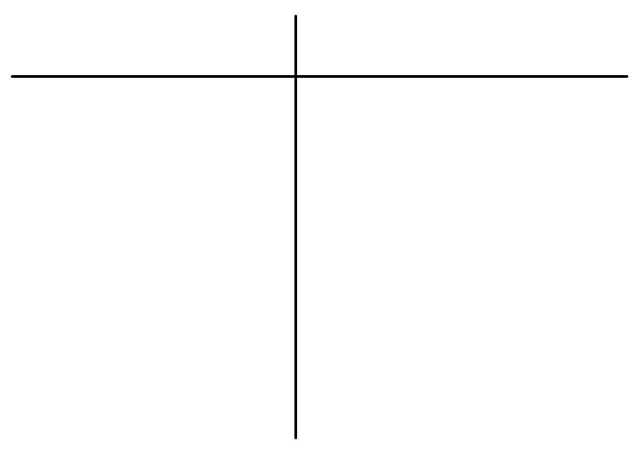
Not assigned

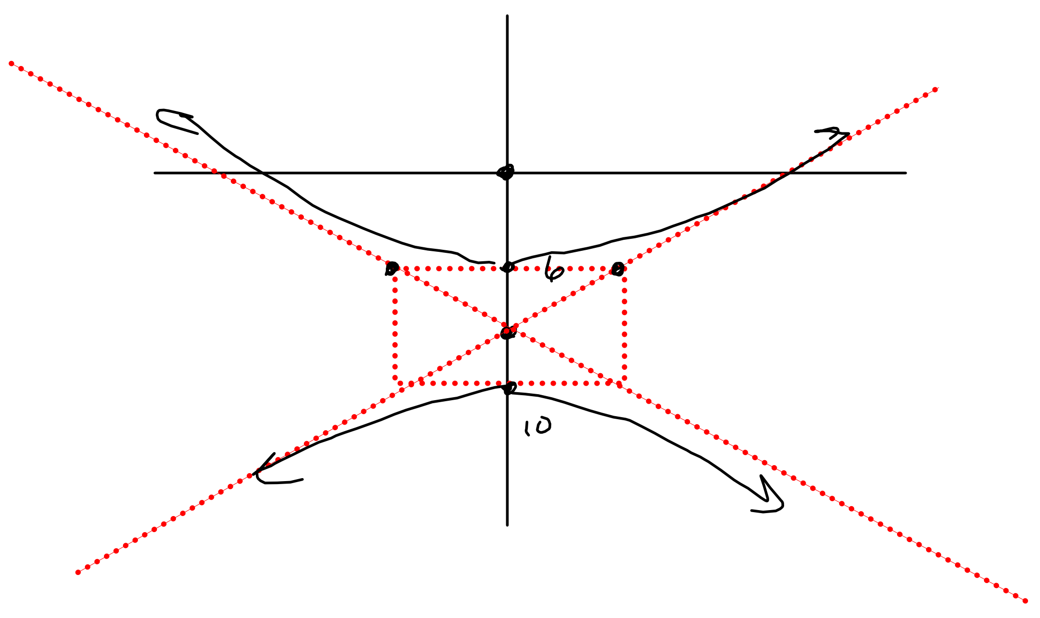


A=

(6) Identify & sketch.

(a) (5pts)





$\frac{x^2}{64} +$

$c=10,\ a=6,\ b=8$

$e=\frac{c}{a}=\frac{10}{6}=\frac{5}{3}$

$r(\frac{\pi}{2}) = \frac{\frac{5}{3}p}{1-\frac{5}{3}\sin\theta} \quad -14$

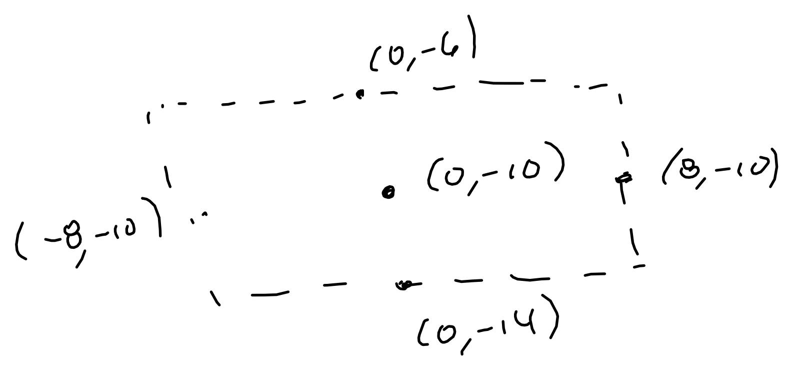
$= \frac{\frac{5}{3}p}{-\frac{2}{3}} = -\frac{5}{2}p = -14 \Rightarrow p = \frac{-28}{-5} = \frac{28}{5}$

$\frac{(y+10)^2}{36} - \frac{x^2}{64} = 1$

$r = \frac{\frac{5}{3}(\frac{28}{5})}{1-\frac{5}{3}\sin\theta} = \frac{\frac{28}{3}}{1-\frac{5}{3}\sin\theta} = \frac{28}{3-5\sin\theta}$

$r(\frac{\pi}{2}) = \frac{28}{3-5} = \frac{28}{-2} = -14$

$r(\frac{3\pi}{2}) = \frac{28}{3+5} = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$

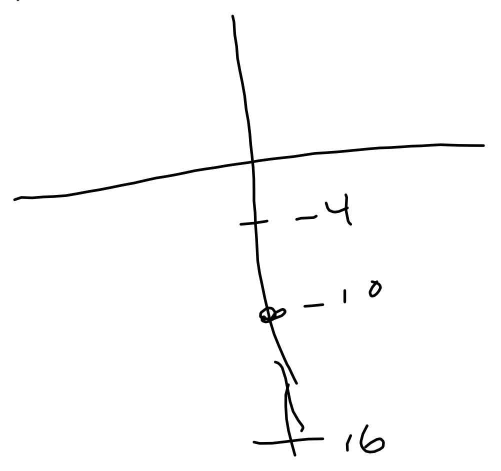


$\frac{(y+10)^2}{36} - \frac{x^2}{64} = 1$

$\frac{(y+10)^2}{36} = 1 + \frac{x^2}{64} = \frac{x^2+64}{}$

$y+10$

$a=6$



$\cancel{e}\quad a^2+b^2 = 10^2 = c^2$

$e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

$r,\ e = \frac{10}{6} = \frac{5}{3},\ \cancel{p=10}$

$r = \frac{ep}{1-e\sin\theta} = \frac{\frac{5}{3}(\ )}{1-\frac{5}{3}\sin\theta} = \frac{5p}{3-5\sin\theta}$

$r(\frac{\pi}{2}) = \frac{5p}{3-5} = \frac{5p}{-2} = -16 \Rightarrow$

$5p=32$

$p=\frac{32}{5}$

$r = \frac{5(\frac{32}{5})}{3-5\sin\theta} = \frac{32}{3-5\sin\theta}$

$r(\frac{3\pi}{2}) = \frac{5p}{3+5} = \frac{5p}{8} = 4 \Rightarrow$

$5p=32$

$p=\frac{32}{5}$

$\frac{(y+10)^2}{36} - \frac{x^2}{64} = 1$

$64(y+10)^2 - 36x^2 = (36)(64)$

$64(y+10)^2 = 36x^2 + 36(64) = 36(x^2+64)$

$(y+10)^2 = \frac{36(x^2+64)}{64}$

$y+10 = \pm\frac{6\sqrt{x^2+64}}{8} \Rightarrow y = -10 \pm \frac{3\sqrt{x^2+64}}{4}$

(6) Identify the conic section & graph

(a) Sols $r = \dfrac{32}{3-5\sin\theta} = \dfrac{\frac{32}{3}}{1-\frac{5}{3}\sin\theta} = \dfrac{ep}{1-e\sin\theta}$

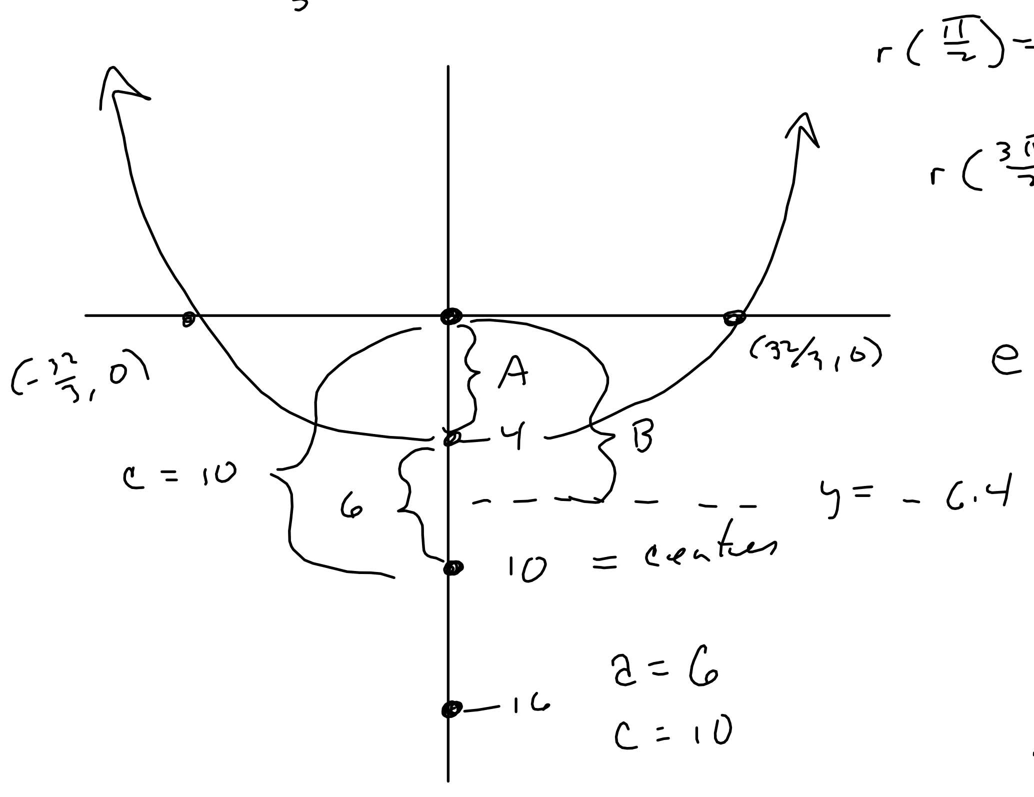
$e = \frac{5}{3}$, directrix horizontal, below the pole.

$\frac{5}{3} > 1 \longrightarrow$ hyperbola.

$\frac{32}{3} = ep = \frac{5}{3}p \longrightarrow |p| = \frac{32}{3}\cdot\frac{3}{5} = \frac{32}{5} = 6.4$

$r(\frac{\pi}{2}) = \dfrac{32}{3-5\sin(\frac{\pi}{2})} = \dfrac{32}{3-5} = \dfrac{32}{-2} = -16$

$r(\frac{3\pi}{2}) = \dfrac{32}{3+5} = \dfrac{32}{8} = 4$



$e = \frac{c}{a} = \frac{5}{3} = \frac{10}{6}$ ✓

$\frac{4}{2.4} = \frac{40}{24} = \frac{10}{6} = \frac{5}{3}$ ✓

$\cancel{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{(y+10)^2}{6^2} - \frac{x^2}{8^2} = 1$

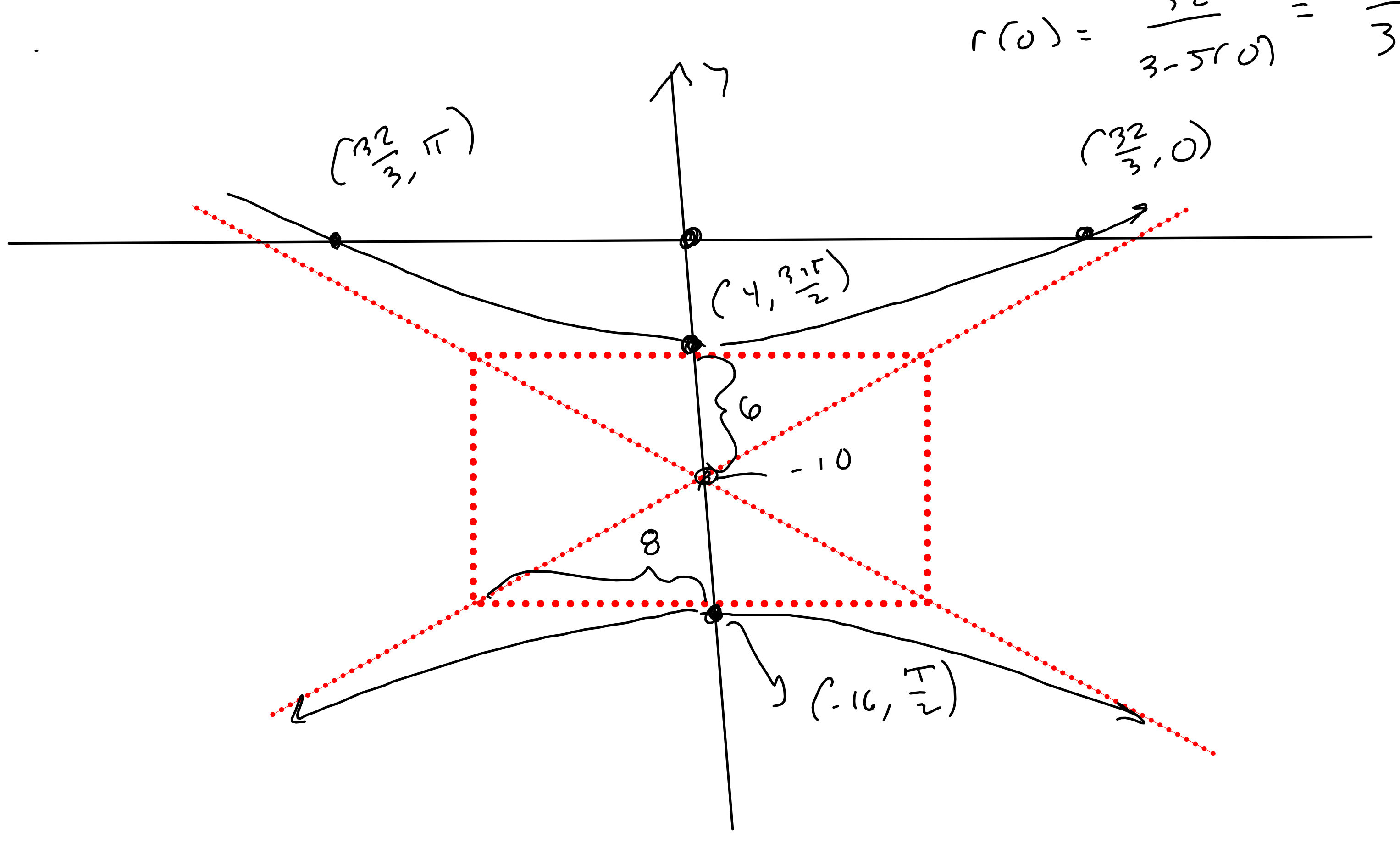
$b^2 = c^2 - a^2 = 100 - 36 = 64$

$r(0) = \frac{32}{3}$

$r(0) = \dfrac{32}{3-5(0)} = \dfrac{32}{3}$

$r(\pi) = \frac{32}{3}$ ✓

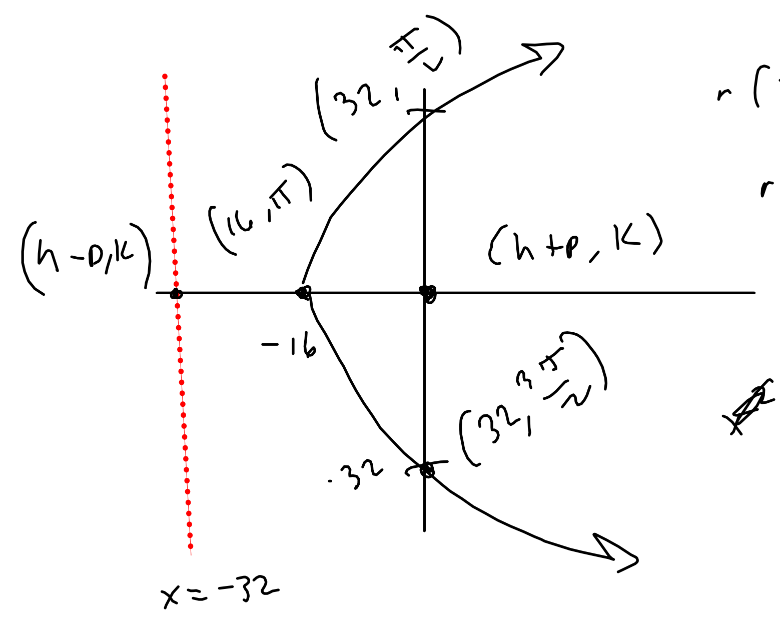
$r(0) = \dfrac{32}{3-5(0)} = \dfrac{32}{3}$



6b

$\frac{32}{1-\cos\theta}$ Polar

Parabola



$r(\pi) = \frac{32}{2} = 16$

$r(\frac{\pi}{2}) = 32$

$r(\frac{3\pi}{2}) = 32$

rectangular

$y^2 = 4px$

$y^2 = 4 \cdot 16x = 64x$

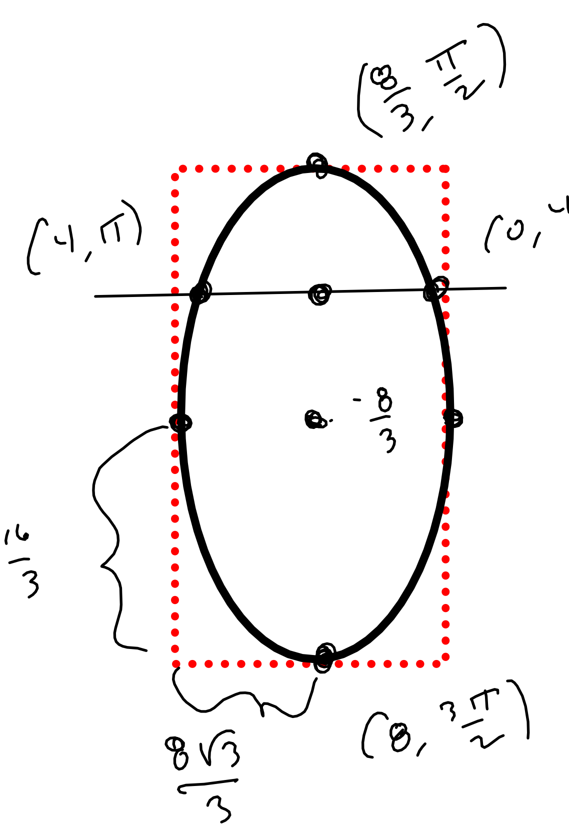
p = distance from focus to Vertex

6C $r = \dfrac{16}{4+2\sin\theta}$ Ellipse. $e = \frac{1}{2} \Rightarrow$ Ellipse.

Directrix ABOVE the pole.

$$\left.\begin{aligned} r(0) &= \frac{16}{4} = 4 \\ r(\pi) &= \frac{16}{4} = 4 \end{aligned}\right\} \text{Polar axis - intercepts}$$

$$\left.\begin{aligned} r\left(\tfrac{\pi}{2}\right) &= \frac{16}{6} = \frac{8}{3} \\ r\left(\tfrac{3\pi}{2}\right) &= \frac{16}{2} = 8 \end{aligned}\right\} \theta = \frac{\pi}{2}\text{ - intercepts}$$



$$a = \frac{8 + \frac{8}{3}}{2} = \frac{\frac{24+8}{3}}{2} = \frac{\frac{32}{3}}{2} = \frac{32}{6} = \frac{16}{3} = a$$

$$a^2 - c^2 = \left(\frac{16}{3}\right)^2 - \left(\frac{8}{3}\right)^2 = \frac{2^2 \cdot 8^2 - 8^2}{3^2} = \frac{3 \cdot 8^2}{3^2} = b^2$$

$$\Rightarrow \boxed{b = \frac{8\sqrt{3}}{3}}$$