① Simplify the products & quotients

(a) (5pts) $(3+2i)(5-7i) = 15 - 21i + 10i - 14i^2 = \boxed{29 - 11i}$

(b) (5pts) $(3+2i)(3-2i) = 3^2 + 2^2 = \boxed{13}$

(c) (5pts) $\frac{3+2i}{3-2i} = \frac{(3+2i)(3+2i)}{(3-2i)(3+2i)} = \frac{9 + 2(6i) + (2i)^2}{3^2+2^2} = \frac{9+12i-4}{13}$

$= \boxed{\frac{5}{13} + \frac{12}{13}i}$

(d) (5pts) $\left(5\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)\right)\left(7\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)\right)$

r_3: $\sqrt{2}$, -1

$= 35\left(\cos\left(\frac{2\pi}{3}\cdot\frac{4}{4} + \frac{\pi}{4}\cdot\frac{3}{3}\right) + i\sin\left(\frac{11\pi}{12}\right)\right) = \boxed{35\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)}$

$= 35\left(\left(\cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}\right) + i\left(\sin\left(\frac{2\pi}{3}\right)\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{2\pi}{3}\right)\right)$

$= 35\left(\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + i\left(\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)\right)\right)$

$= 35\left(-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} + i\left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right)\right) =$

② $f(x) = x^4 + 2x^3 - 18x^2 + 106x - 91$

(a) (5pts) Use $f(2-3i) = 0$

$(x-2-3i)(x-2+3i) = x^2 - 2x + 3ix - 2x + 4 - 6i - 3ix + 6i - 9i^2$

$= x^2 - 4x + 13$

$x^2 + 6x - 7 = (x-7)(x+1)$

$x^2 - 4x + 13 \;\overline{)\; x^4 + 2x^3 - 18x^2 + 106x - 91}$

$-(x^4 - 4x^3 + 13x^2)$

$6x^3 - 31x^2 + 106x - 91$

$-(6x^3 - 24x^2 + 78x)$

$-7x^2 + 28x - 91$

$-(-7x^2 + 28x - 91)$

0

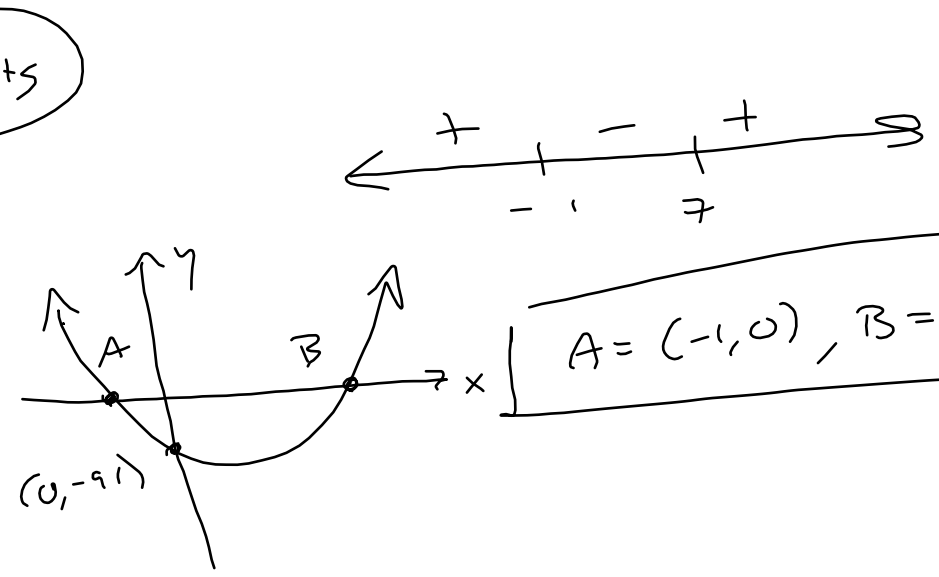
NOT ASKED.

Zeros

$x = 2 \pm 3i, -1, 7$

$\longrightarrow \boxed{f(x) = (x-2+3i)(x-2-3i)(x-7)(x+1)}$

(b) (5pts)



$\boxed{A = (-1, 0),\ B = (7, 0)}$

(3) (5pts) Solve $x^5 - 32 = 0$

$x^5 = 32$

$x = \sqrt[5]{32} = \boxed{2 = x}$

Increment: $\frac{2\pi}{5}$

z_0 $2 = 2(\cos(0) + i\sin(0)) = 2 = z_0$

$z_1 : 2\left(\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)\right) \approx .618 + 1.902i \approx z_1$

$z_2 : 2\left(\cos\left(\frac{4\pi}{5}\right) + i\sin\left(\frac{4\pi}{5}\right)\right) \approx -1.618 + 1.176i \approx z_2$

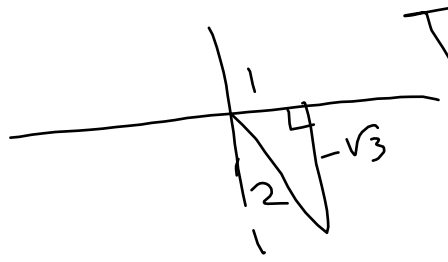
$z_3 : 2\left(\cos\left(\frac{6\pi}{5}\right) + i\sin\left(\frac{6\pi}{5}\right)\right) \approx -1.618 - 1.176i \approx z_3$

$z_4 : 2\left(\cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)\right) \approx .618 - 1.902i \approx z_4$

$z_0 : 2\left(\cos\left(\frac{10\pi}{5}\right) + i\sin\left(\frac{10\pi}{5}\right)\right) = 2 = z_0$ ✓

4 (2) (5 pts) Write $1-\sqrt{3}i$ in trig form

$$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$$



$$z = 2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$$

b) 5 pts Write 3 cube roots of z

$$\frac{\frac{5\pi}{3}}{3} = \frac{5\pi}{9}, \quad \text{Incr.} = \frac{2\pi}{3} = \frac{6\pi}{9}$$

$$z_1 = \sqrt[3]{2}\left(\cos\left(\frac{5\pi}{9}\right) + i\sin\left(\frac{5\pi}{9}\right)\right)$$

$$z_2 = \sqrt[3]{2}\left(\cos\left(\frac{11\pi}{9}\right) + i\sin\left(\frac{11\pi}{9}\right)\right)$$

$$z_3 = \sqrt[3]{2}\left(\cos\left(\frac{17\pi}{9}\right) + i\sin\left(\frac{17\pi}{9}\right)\right)$$

$$z_4 = \sqrt[3]{2}\left(\cos\left(\frac{23\pi}{9}\right) + i\sin\left(\frac{23\pi}{9}\right)\right)$$

$$= \sqrt[3]{2}\left(\cos\left(\frac{18\pi}{9} + \frac{5\pi}{9}\right) + i\sin\left(2\pi + \frac{5\pi}{9}\right)\right)$$

$$= \sqrt[3]{2}\left(\cos\left(\frac{5\pi}{9}\right) + i\sin\left(\frac{5\pi}{9}\right)\right) = z_1 \checkmark$$

c) Bonus 5 pts