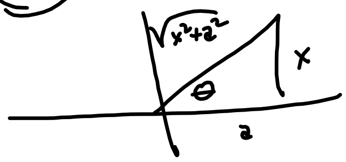
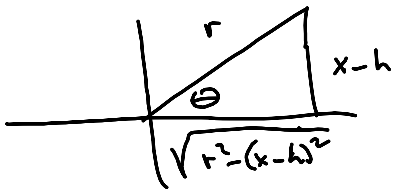
1420 WEEK 06 SOL'NS H. MILLS

(1) We write an algebraic expression equivalent to the given trigonometric expression.

(a) 5pts $\csc\left(\arctan\left(\frac{x}{a}\right)\right) = \csc\theta = \boxed{\frac{\sqrt{x^2+a^2}}{x}}$



(b) 5pts $\cos\left(\arcsin\left(\frac{x-h}{r}\right)\right) = \cos\theta = \boxed{\frac{\sqrt{r^2-(x-h)^2}}{r}}$



(2) Verify the identity algebraically.

(a) 5pts $\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1 = \csc\theta$

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1 = \frac{\cos\theta\cot\theta - (1-\sin\theta)}{1-\sin\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin\theta} + (\sin\theta - 1)\frac{\sin\theta}{\sin\theta}}{1-\sin\theta} = \frac{\cos^2\theta + \sin^2\theta - \sin\theta}{\sin\theta(1-\sin\theta)} = \frac{1-\sin\theta}{\cos\theta(1-\sin\theta)}$$

$$= \csc\theta \checkmark$$

(b) 5pts $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$?! I don't think so.

$$\sqrt{\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\left(\frac{1+\cos\theta}{1+\cos\theta}\right)} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \frac{|1+\cos\theta|}{\sqrt{\sin^2\theta}}$$

$$= \frac{1+\cos\theta}{|\sin\theta|}$$, b/c $1+\cos\theta \geq 0$ ($\cos\theta \leq 1$)

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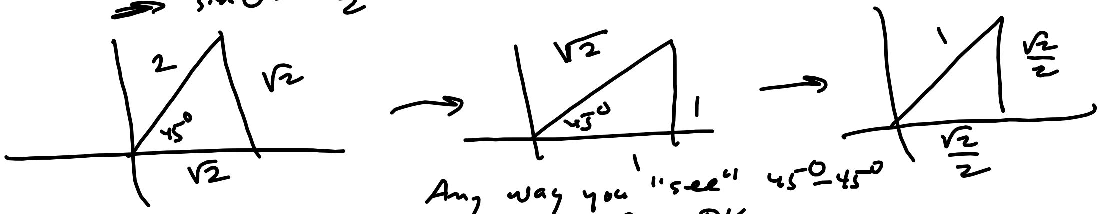
#3 Solve the trig equations:

a 5pts $\frac{\sqrt{2}}{2}\csc\theta - 1 = 0 \longrightarrow$

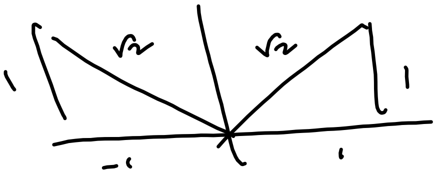
$\csc\theta = \frac{2}{\sqrt{2}}$

$\Rightarrow \sin\theta = \frac{\sqrt{2}}{2}$



Any way you "see" 45°–45° right triangle is OK.

So, $\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}$ on $[0, 2\pi)$. In general,

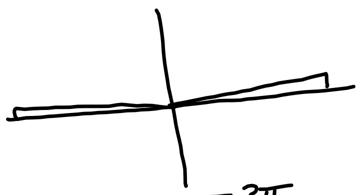
$\boxed{x = \frac{\pi}{4} + 2n\pi \text{ or } \frac{3\pi}{4} + 2n\pi,\ n \in \mathbb{Z}}$

b 5pts $\sin(2x)(2\sin(x)+1) = 0 \longrightarrow$

$\sin(2x) = 0$ or $2\sin(x) + 1 = 0$

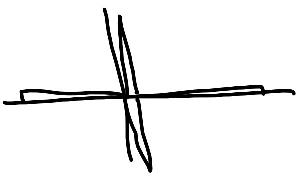
To find all $x \in [0, 2\pi)$, find all $2x \in [0, 4\pi)$

$\sin(2x) = 0$



$2x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

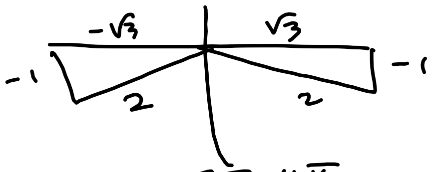


All $\frac{\pi}{2}$ apart, so

$x = \frac{n\pi}{2}$

$2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$



$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ on $[0, 2\pi)$

In general,

$\boxed{x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi,\ n \in \mathbb{Z}}$

$\boxed{x = \frac{\pi}{2} + \frac{n\pi}{2},\ n \in \mathbb{Z}}$

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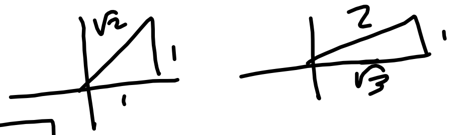
(4) Find $\sin\theta$, $\cos\theta$, $\tan\theta$ (exact).



(a) (5pts) $\theta = 75^\circ = 45^\circ + 30^\circ$

$$\sin\theta = \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}} \text{ OR } \frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\cos\theta = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

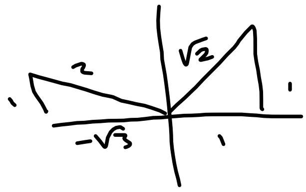
$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}} \text{ OR } \frac{\sqrt{6}-\sqrt{2}}{4}}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\cdot\frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ OR } \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}}$$

(b) (5pts) $\theta = \frac{13\pi}{12} = \pi + \frac{\pi}{12} = \frac{2\pi}{12} + \frac{11\pi}{12} = \frac{3\pi}{12} + \frac{10\pi}{12} = \frac{\pi}{4} + \frac{5\pi}{6}$

$$\sin(u+v) = \sin\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) + \sin\frac{5\pi}{6}\cos\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{-\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}} \text{ OR } \frac{\sqrt{2}-\sqrt{6}}{4}}$$



$$\cos(u+v) = \cos\frac{\pi}{4}\cos\frac{5\pi}{6} - \sin\frac{\pi}{4}\sin\frac{5\pi}{6} =$$

$$= \frac{1}{\sqrt{2}}\cdot\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) = \boxed{-\frac{\sqrt{3}+1}{2\sqrt{2}} \text{ OR } -\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{3}}\right)} = \boxed{\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{2}\sqrt{3}}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+1} \text{ OR } \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}}$$

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(5) $\sin(u) = -\frac{3}{5}$, $\cos(u) < 0$ and $\cos(v) = \frac{14}{17}$, $\tan(v) < 0$

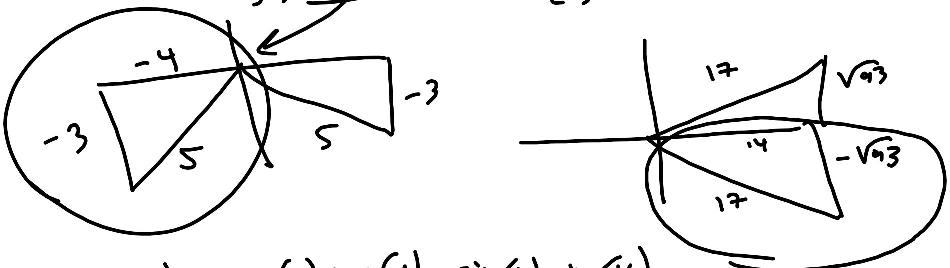
(5pts) Find $\cos(u+v)$

Let's find the quadrant for u & v

$\sin(u) = -\frac{3}{5}$, $\cos(u) < 0$ $\quad$ $\cos(v) = \frac{14}{17}$, $\tan(v) < 0$

$\sqrt{17^2 - 14^2} = \sqrt{289 - 196}$

$\sqrt{93} = \sqrt{93}$



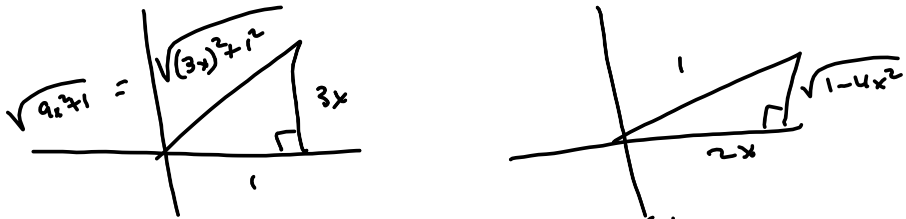
$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$= \left(-\frac{4}{5}\right)\left(\frac{14}{17}\right) - \left(-\frac{3}{5}\right)\left(-\frac{\sqrt{93}}{17}\right)$

$= \boxed{\frac{-56 - 3\sqrt{93}}{85}}$

(6) (5pts) $\sin(\arctan(3x) - \arccos(2x)) = \sin(u - v)$.

$u = \arctan(3x)$ $\quad$ $v = \arccos(2x)$



$\sin(u-v) = \sin(u)\cos(-v) + \sin(-v)\cos(u)$

$= \sin(u)\cos(v) - \sin(v)\cos(u)$

$= \left(\frac{3x}{\sqrt{9x^2+1}}\right)\left(\frac{2x}{1}\right) - \left(\frac{\sqrt{1-4x^2}}{1}\right)\left(\frac{1}{\sqrt{9x^2+1}}\right)$

$= \boxed{\frac{6x^2 - \sqrt{1-4x^2}}{\sqrt{9x^2+1}}}$