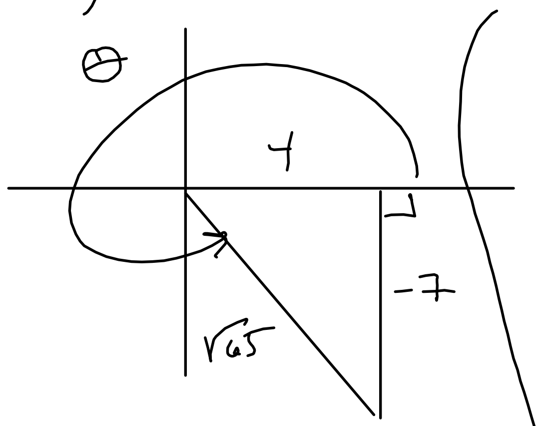
① We find the exact values of the 6 trig functions corresponding to the point $(4,-7)$.

5pts



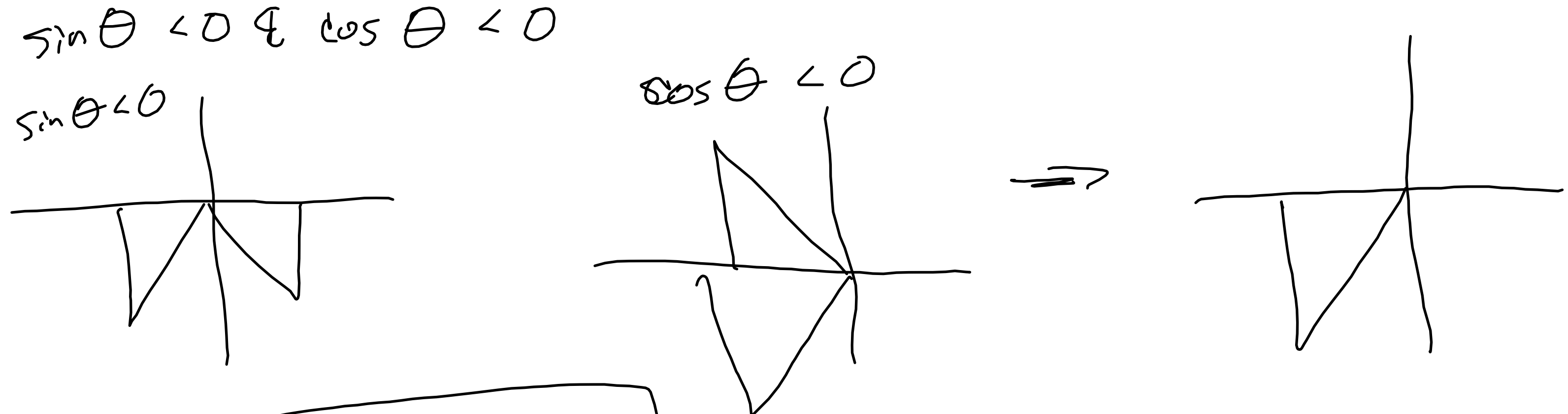
$4^2+7^2 = 16+49 = 65$

$$\sin\theta = -\frac{7}{\sqrt{65}} \qquad \csc\theta = -\frac{\sqrt{65}}{7}$$

$$\cos\theta = \frac{4}{\sqrt{65}} \qquad \sec\theta = \frac{\sqrt{65}}{4}$$

$$\tan\theta = -\frac{7}{4} \qquad \cot\theta = -\frac{4}{7}$$

② $\sin\theta < 0$ & $\cos\theta < 0$

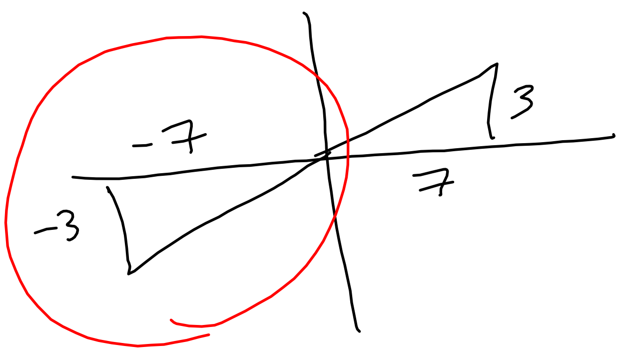


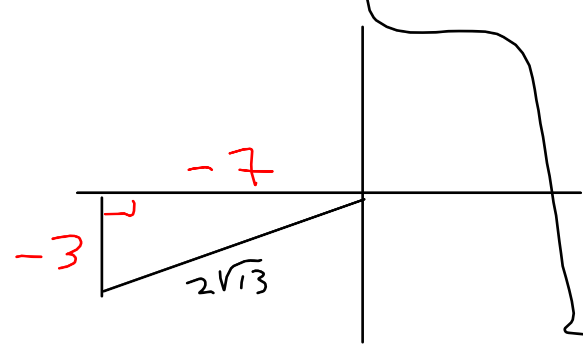
$\Rightarrow \boxed{\theta \in Q\,\text{III}}$

1420

MILLS

(3) 5pts $\tan\theta = \frac{3}{7}$, $\cos\theta < 0 \Rightarrow$





$\sin\theta = \frac{-3}{\sqrt{58}}$ $\csc\theta = -\frac{\sqrt{58}}{3}$

$\cos\theta = \frac{-7}{\sqrt{58}}$ $\sec\theta = -\frac{\sqrt{58}}{7}$

$\tan\theta = \frac{3}{7}$ $\cot\theta = \frac{7}{3}$

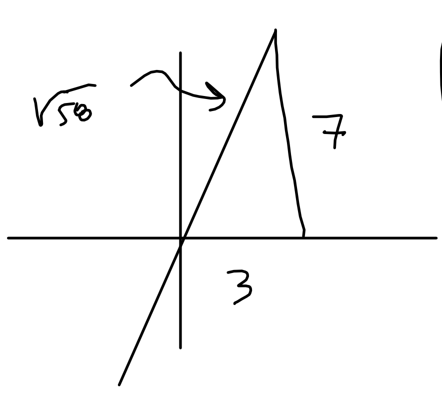
$7^2 + 3^2 = 49 + 9 = 58$

$2\,|\,58$
$2\,|\,26$
13 → No!

$\sqrt{58} = 2\sqrt{13}$ NO!

$58 = 2 \cdot 29$, not $2 \cdot 26$.

(4) 5pts we find the value of the 6 trigs for a point lying on $y = \frac{7}{3}x$ in QI:



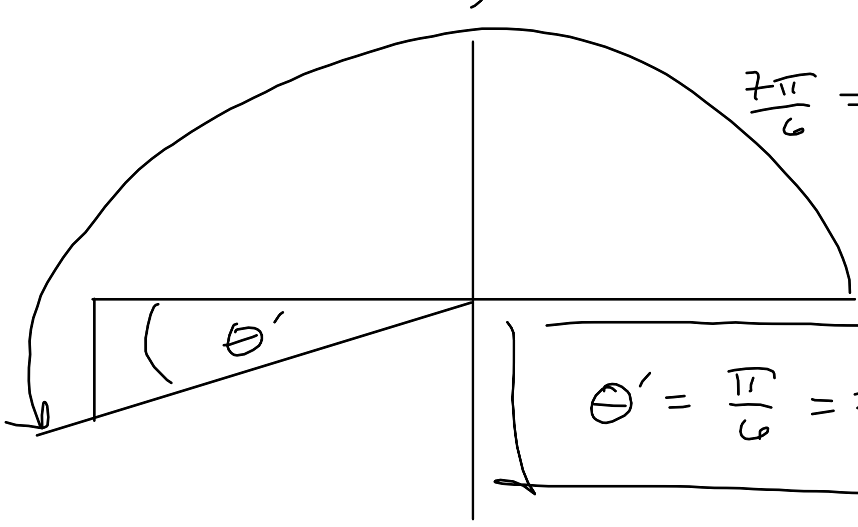
$\sin\theta = \frac{7}{\sqrt{58}}$ $\csc\theta = \frac{\sqrt{58}}{7}$

$\cos\theta = \frac{3}{\sqrt{58}}$ $\sec\theta = \frac{\sqrt{58}}{3}$

$\tan\theta = \frac{7}{3}$ $\cot\theta = \frac{3}{7}$

1420 W#3 MILLS

(5) We sketch $\theta = \frac{7\pi}{6}$ in standard position, and find and label the reference angle θ'



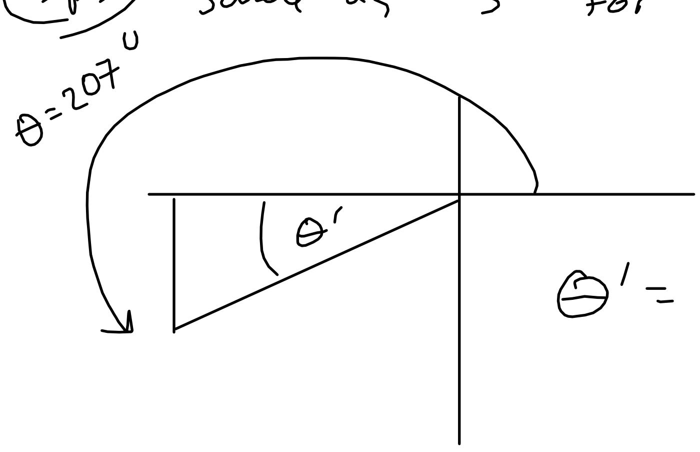
$\frac{7\pi}{6} = \theta$

Check: $\frac{7\pi}{6} = \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi}$

$= 7(30^\circ) = 210^\circ$

$\boxed{\theta' = \frac{\pi}{6} = 30^\circ}$ via $\frac{7\pi}{6} - \pi$ or $210^\circ - 180^\circ = 30^\circ$

(6) 5pts Same as #5 for $\theta = 207^\circ$



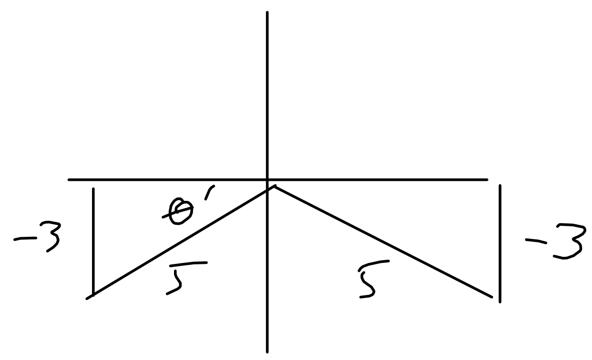
$\theta' = 207^\circ - 180^\circ = \boxed{27^\circ = \theta'}$

1420 W#3 MILLS

(7) We sketch and find 2 sol'ns to each of the following. We round to 4 places.

(a) 5pts $\sin\theta = -\frac{3}{5}$

$\arcsin\left(-\frac{3}{5}\right) \approx \boxed{-36.8698976458^\circ \approx \theta_1}$



My work is cramped and a bit disorganized.

The angle terminating in Q III will be

$180^\circ - \arcsin(-3/5) \approx 180 + 36.8698976458^\circ$

$= 216.8698976458^\circ$

b/c the angle's negative, to add θ'. $\approx \boxed{216.8699^\circ \approx \theta_2}$

$\boxed{\approx 3.78509376239 \approx 3.7851}$

Other answers are possible:

$\theta_2 = -180^\circ + \arcsin(-3/5)$

$\approx -143.130102354^\circ$

$\approx \boxed{-143.1301^\circ} \approx \theta_2$

$\approx -2.49809154479 \approx \boxed{-2.4981 \approx \theta_2}$

$\theta_1 = 360^\circ + \arcsin(-3/5)$

$\approx 323.130102354^\circ$

$\approx \boxed{323.1301^\circ}$ ≈ 5.63968419838

$\boxed{\approx 5.6397 \approx \theta_1}$

I should've specified that the solutions must lie between 0 and 360 degrees, but I didn't. Woe is me.

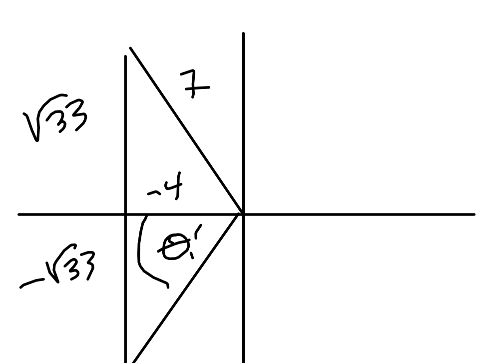
Radians answers:

$\theta_1 \approx -36.8698976458^\circ \approx -0.643501108793 \approx \boxed{-.6435}$

$\theta_2 \approx 323.130102354^\circ \approx 1.7951672353 \approx \boxed{1.7952}$

(b) 5pts Let's see if we can clean this up.

$\sec\theta = -\frac{7}{4} \quad \left(\cos\theta = -\frac{4}{7}\right)$



$\theta_1 = \arccos\left(-\frac{4}{7}\right) \approx 124.849904579^\circ \approx 2.1790419057$

$\boxed{\theta_1 \approx 124.8499^\circ \text{ OR } 2.1790}$

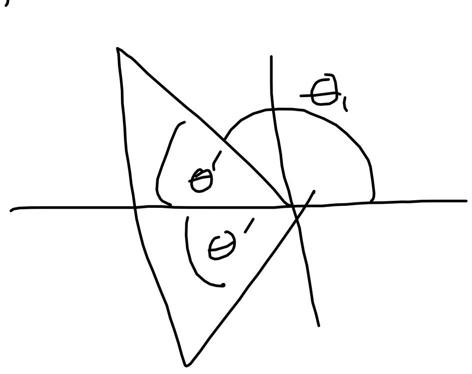
$7^2 - 4^2 = 49 - 16 = 33$

$\theta_1' = 180^\circ - \theta_1 \approx 55.150095421^\circ$

$\theta_2 = 180^\circ + \theta' = 180^\circ + (180^\circ - \theta_1) = 360^\circ - \theta_1$

$\approx 235.150095421^\circ \approx 4.10414340148$

$\boxed{\theta_2 \approx 235.1501^\circ \approx 4.1041}$



Notice I didn't use the rounded degrees to find the radians. I found the radians from the raw degrees and *then* rounded.

$\theta_1' = 180^\circ - \theta_1$

1420 W#3 MILLS

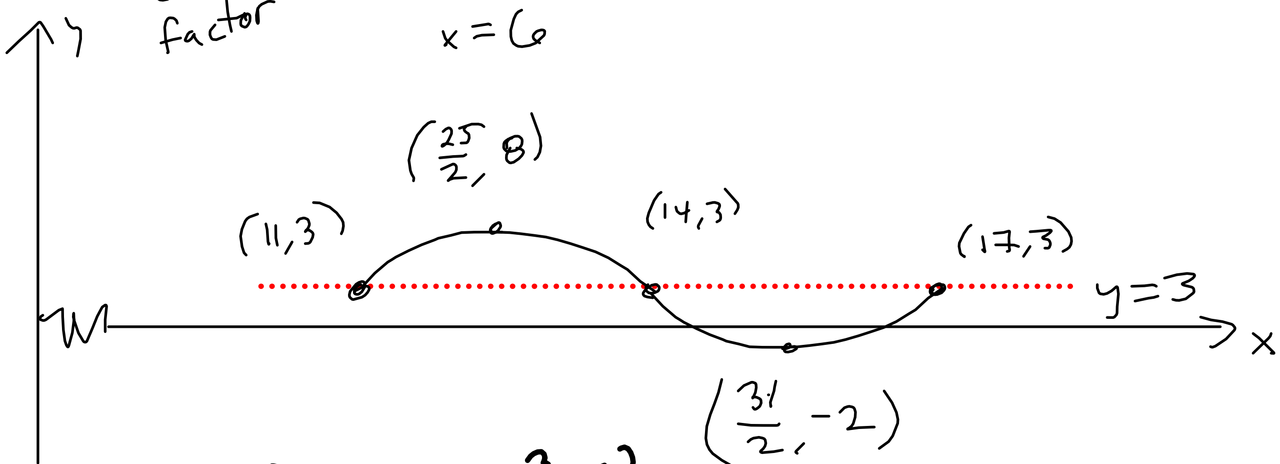
(8) (5 pts) We sketch one period of $f(x) = 5\sin\left(\frac{\pi}{3}x - \frac{11\pi}{3}\right) + 3$

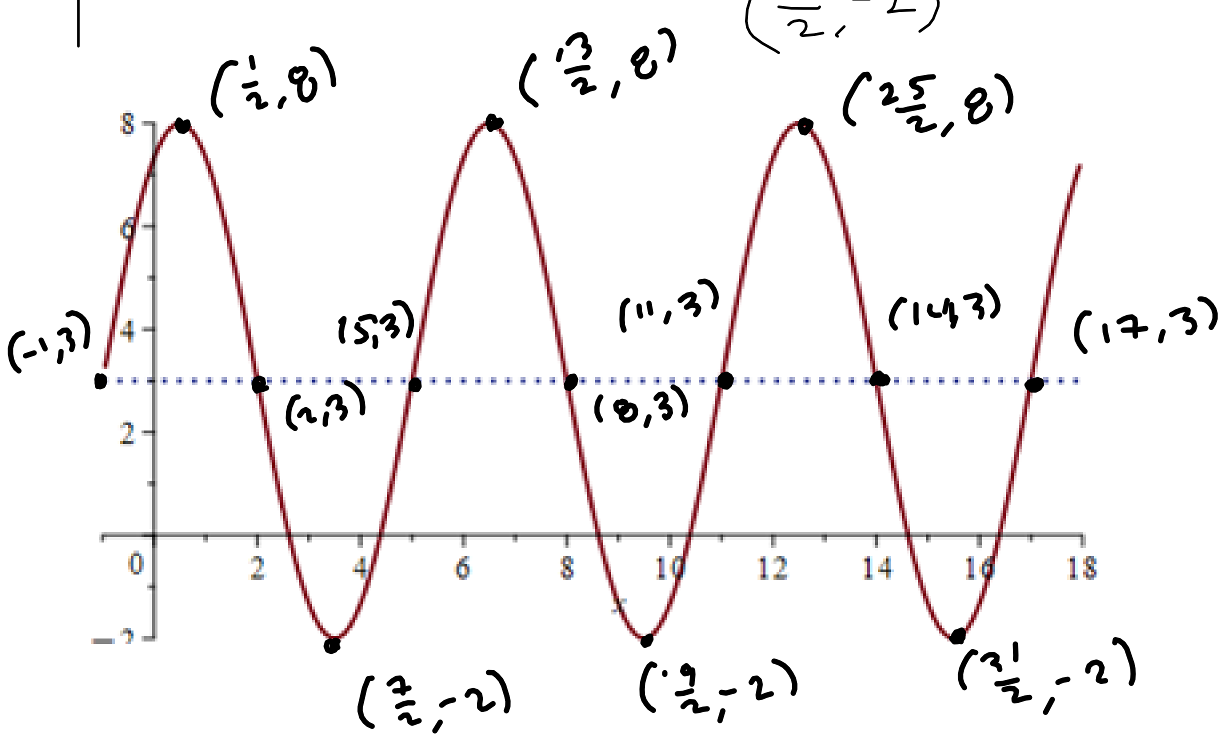
$= 5\sin\left(\frac{\pi}{3}(x-11)\right) + 3$

up & down 5 factor

$\frac{\pi}{3}x = 2\pi$

$x = 6$





(9) NOTHING ASKED?!

1420 10 5pts

w #3

MILLS

On the longest day of the year, there are 22 hours of daylight. On the shortest day of the year, there are 2 hours of daylight. We model y = the # of hours of daylight as a function of x = the day of the year (as a number, starting with x = 1 corresponding to January 1st.

$a\cos(b(x-c))+d$

HIGH = 22, LOW = 2 } Midline: $y = 12 = d$

Amplitude $= \frac{22-2}{2} = \frac{20}{2} = 10$

Period = 365

Start on the high point

$bx = 2\pi$

$365b = 2\pi \Rightarrow b = \frac{2\pi}{365}$

The longest day of the year occurs on the summer solstice, which, in the northern hemisphere is June 21st.

J F M A M J

31 28 31 30 31 21

June 21^{st} = Day 172 $\Rightarrow$ x = 172 is max height, so

a) $\boxed{y = f(x) = 10\cos\left(\frac{2\pi}{365}(x-172)\right) + 12}$ $\approx$ $10\cos\left(\frac{2\pi}{365}x - 2.960843488\right)$

Midline & growing longer:

Mar 21^{st}:

J F M

31 + 28 + 21 = 80

Building Sine

$y = f(x) = 10\sin\left(\frac{2\pi}{365}(x-80)\right) + 12$

.0172142063 2 x − 2.960843488.

(b) (5pts) $y = f(x) = 10\cos\left(\frac{2\pi}{365}(x-172)\right) + 12$

Easiest way to write this as a sine is to use the Cofunction Identity:

$\cos(u) = \sin\left(\frac{\pi}{2} - u\right) = -\sin\left(u - \frac{\pi}{2}\right) \longrightarrow$

$f(x) = 10\cos\left(\frac{2\pi}{365}(x-172)\right) + 12$

$\boxed{= -10\sin\left(\frac{2\pi}{365}(x-172) - \frac{\pi}{2}\right) + 12}$

Scratch: $\frac{2\pi(x-172)}{365} \cdot \frac{2}{2} - \frac{\pi}{2} \cdot \frac{365}{365}$

$2 \cdot 172 \cdot 4 = 688$

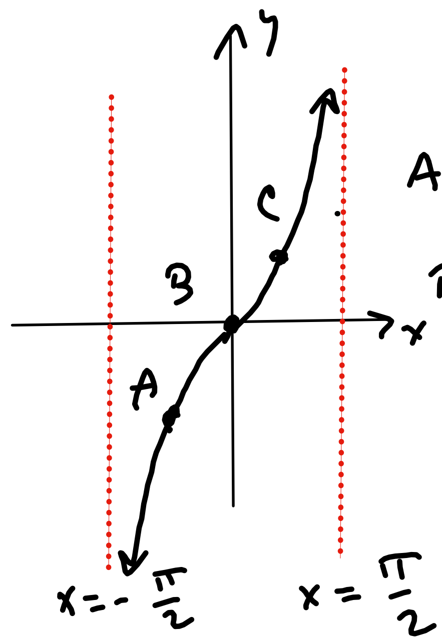
$= \frac{4\pi(x-172)}{2 \cdot 365} - \frac{365\pi}{2 \cdot 365} = \frac{4\pi x - 688\pi - 365\pi}{2 \cdot 365} -$

$= \frac{4\pi x - 1053\pi}{2 \cdot 365} = \frac{2\pi}{365}x - \frac{1053\pi}{2 \cdot 365} = \frac{2\pi}{365}x - \frac{1053\pi}{730}$

$\approx .0172142063 x - 4.531639813$

(1) 5pts Graph $y = 10\tan\left(\frac{\pi}{6}x + \frac{11\pi}{3}\right) + 3$

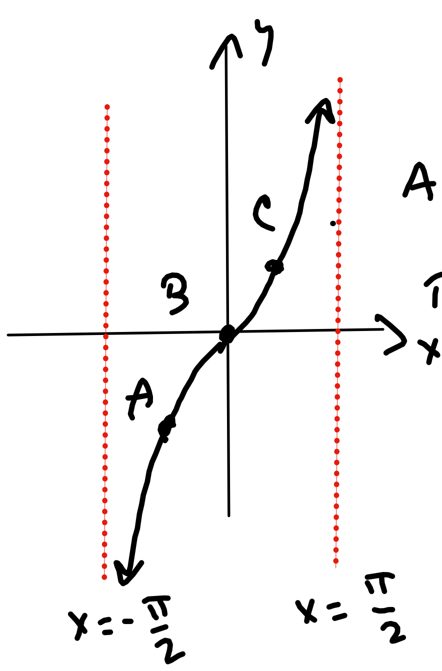
$$\frac{\pi}{6}\left(x + \frac{11\pi}{3}\cdot\frac{6}{\pi}\right) = \frac{\pi}{6}(x + 22)$$



$A = \left(-\frac{\pi}{4}, -1\right)$

$B = (0, 0)$ $y = \tan(x)$

$C = \left(\frac{\pi}{4}, 1\right)$

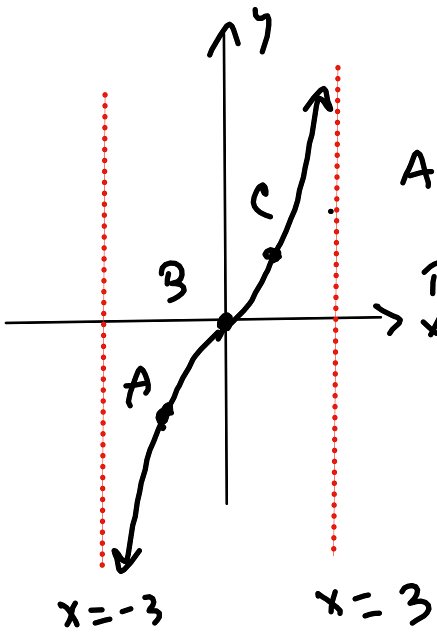


$A = \left(-\frac{\pi}{4}, 10\right)$

$B = (0, 0)$ $y = 10\tan(x)$

$C = \left(\frac{\pi}{4}, 10\right)$

$\frac{6}{\pi}$ horizontal stretch.



$A = \left(-\frac{3}{2}, -10\right)$

$B = (0, 0)$

$C = \left(\frac{3}{2}, 10\right)$

$\frac{\pi}{2} \cdot \frac{6}{\pi} = 3$

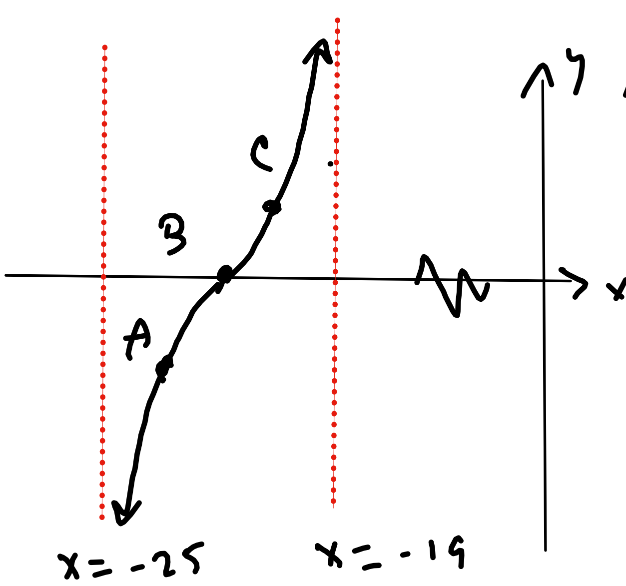
$y = 10\tan\left(\frac{\pi}{6}x\right)$

$x \longmapsto \frac{6}{\pi}x$

$-\frac{\pi}{4} \cdot \frac{6}{\pi} = -\frac{3}{2}$

$\frac{\pi}{4} \cdot \frac{6}{\pi} = \frac{3}{2}$

Left 22



$A = \left(-\frac{47}{2}, -10\right)$

$B = (-22, 0)$

$C = \left(-\frac{41}{2}, 10\right)$

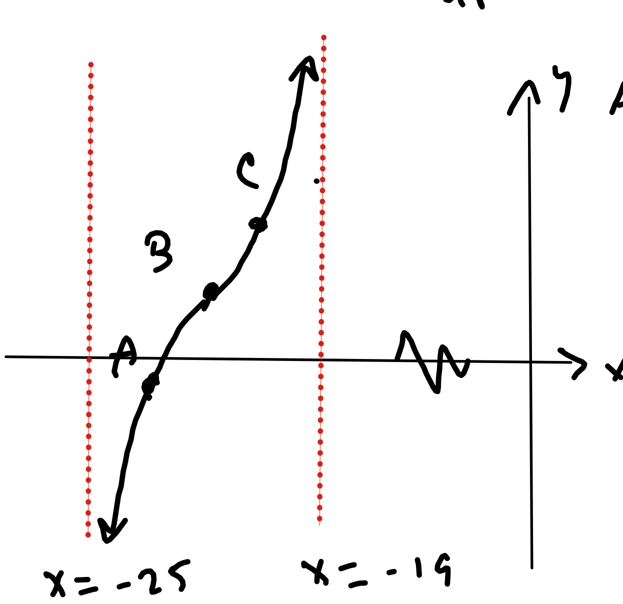
$y = 10\tan\left(\frac{\pi}{6}(x + 22)\right)$

$x \longmapsto x - 22$

$-\frac{3}{2} - \frac{22}{2} \cdot \frac{2}{2} = -\frac{49}{2}$

$\frac{3}{2} - \frac{22}{2} \cdot \frac{2}{2} = -\frac{41}{2}$

up 3



$A = \left(-\frac{47}{2}, -7\right)$

$B = (-22, 3)$

$C = \left(-\frac{41}{2}, 13\right)$

$y = 10\tan\left(\frac{\pi}{6}(x + 22)\right)$

$x \longmapsto x - 22$

$-\frac{3}{2} - \frac{22}{2} \cdot \frac{2}{2} = -\frac{49}{2}$

$\frac{3}{2} - \frac{22}{2} \cdot \frac{2}{2} = -\frac{41}{2}$