1420

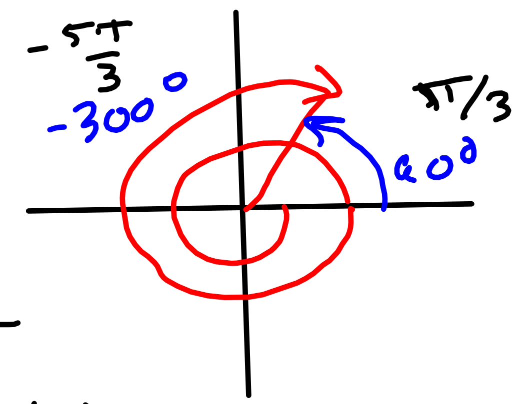
WEEK 2 WRITTEN

H. MILLS

① $\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{12\pi}{3} + \frac{\pi}{3}\right) = \sin\left(-4\pi + \frac{\pi}{3}\right)$

$= \sin\left(-2(2\pi) + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\boxed{\sin\left(-\frac{11\pi}{3}\right) = \frac{\sqrt{3}}{2}}$



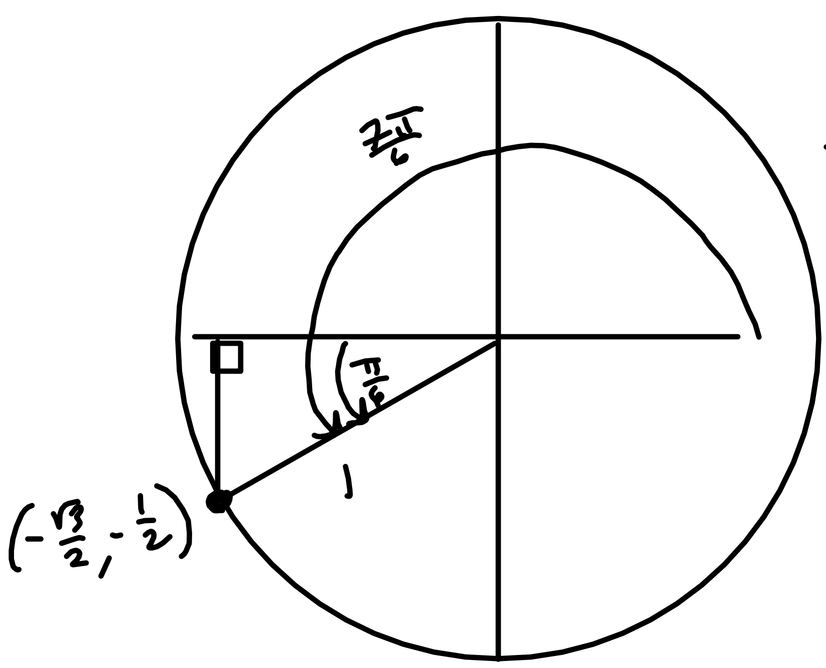
$-\left(\frac{11\pi}{3}\right)\left(\frac{1}{2\pi}\right) = 1.8\overline{3}$

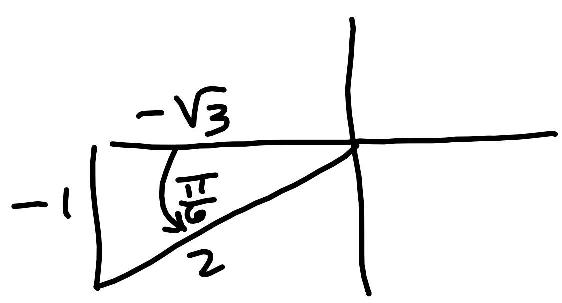
$-(.8\overline{3})(2\pi)$

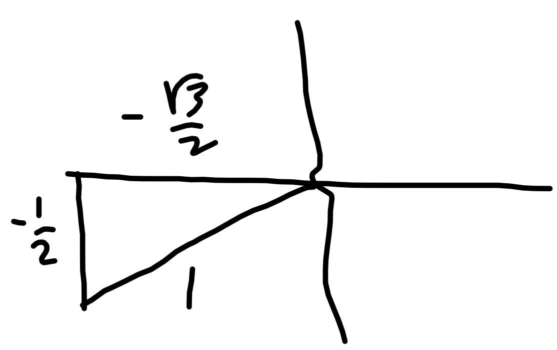
$= -\frac{5}{6}(2\pi) = -\frac{5\pi}{3}$

$2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$

② $t = \frac{7\pi}{6}$ looks like this on the unit circle







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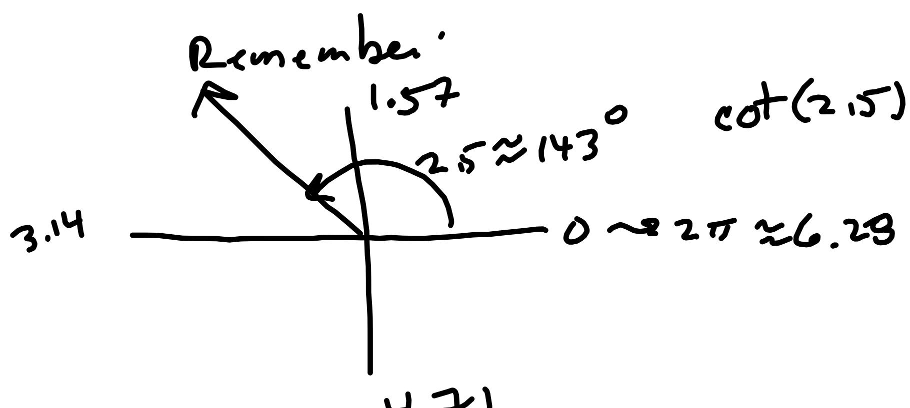
(3) We find $\cot(2.5)$ to 4 places:

$\cot(2.5) \approx -1.3386481283 \approx \boxed{-1.3386 \approx \cot(2.5)}$

$(2.5)\left(\frac{180^\circ}{\pi}\right) = \frac{25}{10}\left(\frac{180}{\pi}\right) = \frac{25(18)}{\pi} = \left(\frac{450}{\pi}\right)^\circ \approx 143.239448783^\circ$

Playing

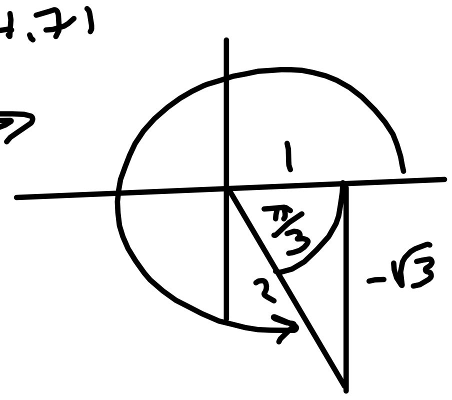
Remember:



$\cot(2.5) \approx -1.3386481283$

$\approx \boxed{-1.3386 \approx \cot(2.5)}$

(4) (a) $t = \frac{5\pi}{3}$

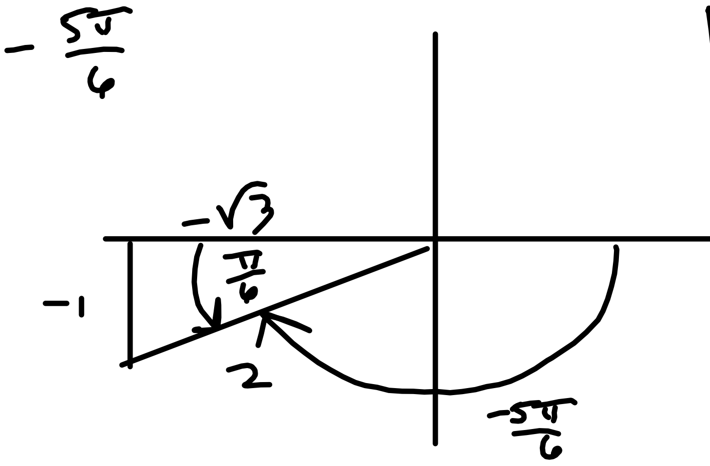


$\sin(t) = \frac{-\sqrt{3}}{2}$ $\quad \csc(t) = -\frac{2}{\sqrt{3}}$

$\cos(t) = \frac{1}{2}$ $\quad \sec(t) = 2$

$\tan(t) = -\sqrt{3}$ $\quad \cot(t) = \frac{1}{-\sqrt{3}}$

(b) $t = -\frac{5\pi}{6}$

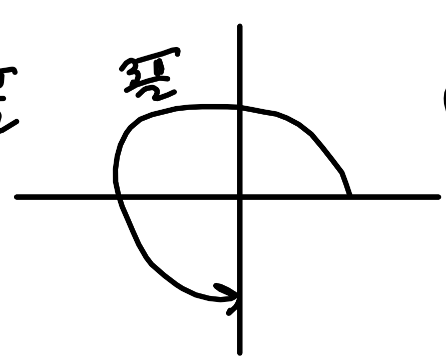


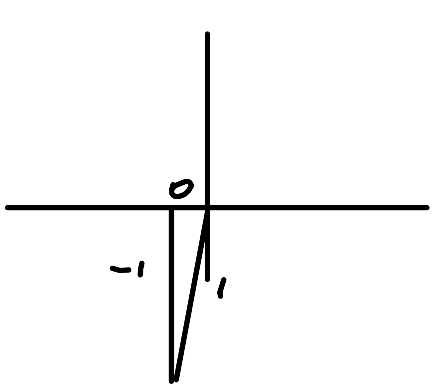
$\sin(t) = -\frac{1}{2}$ $\quad \csc(t) = -2$

$\cos(t) = -\frac{\sqrt{3}}{2}$ $\quad \sec(t) = -\frac{2}{\sqrt{3}}$

$\tan(t) = \frac{1}{\sqrt{3}}$ $\quad \cot(t) = \sqrt{3}$

(c) $t = \frac{3\pi}{2}$





$\sin(t) = \frac{-1}{1} = -1$ $\quad \csc(t) = -1$

$\cos(t) = \frac{0}{1} = 0$ $\quad \sec(t) \nexists$

$\tan(t) = \frac{-1}{0} \nexists$ $\quad \cot(t) = 0$

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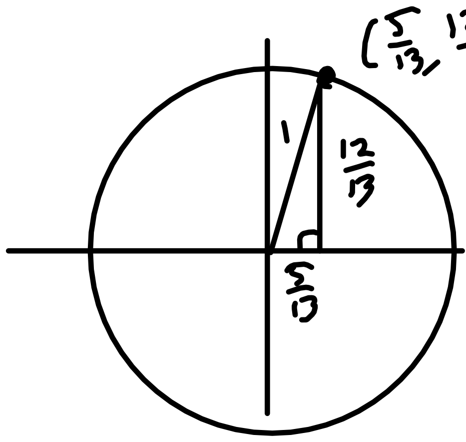
(5) We find the 6 trig values corresponding to $\left(\frac{5}{13}, \frac{12}{13}\right)$ on the unit circle.

Check: $\frac{5^2}{13^2} + \frac{12^2}{13^2} = \frac{25+144}{169} = \frac{169}{169} = 1$. Yep. Unit circle.

$\left(\frac{5}{13}, \frac{12}{13}\right) = (\cos\theta, \sin\theta)$



$\sin\theta = \frac{12}{13}$ $\csc\theta = \frac{13}{12}$

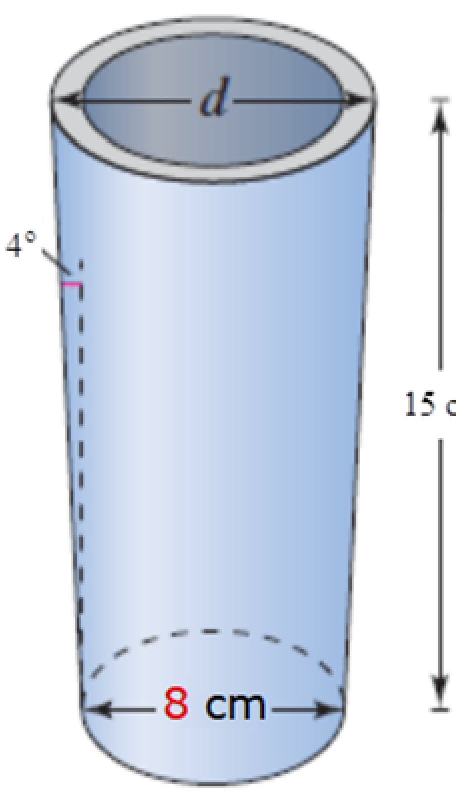
$\cos\theta = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$ $\cot\theta = \frac{5}{12}$

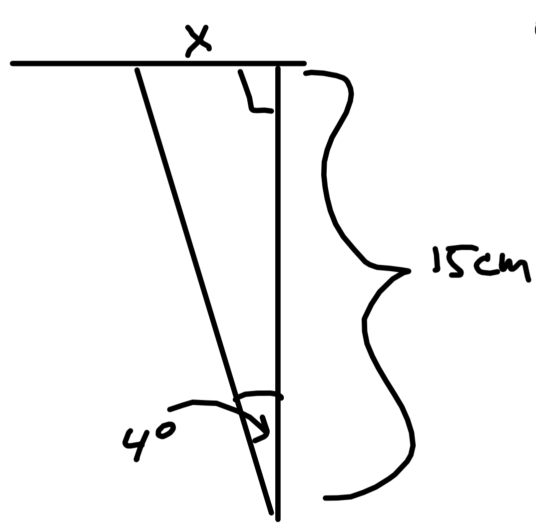
(6) f is odd $\Longleftrightarrow$ $f(-t) = -f(t)$

f is even $\Longleftrightarrow$ $f(-t) = f(t)$

(7) A tapered shaft is 8 cm diameter at the bottom, ~~and~~ a taper of 4°, and a length of 15 cm.

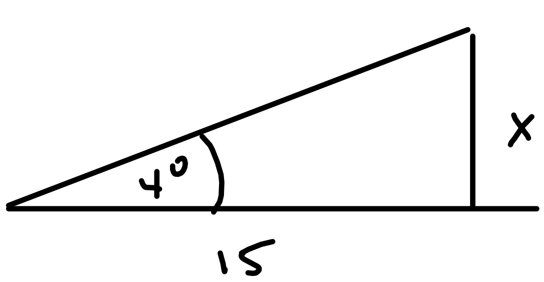


Find diameter d of the (wider) top of the shaft.



radius of the bottom is $\frac{8}{2} = 4$ cm.

radius of the top is $4 + x$, where x is the opposite side of a right triangle of 4° and an adjacent side of 15 cm



$\tan(4^\circ) = \frac{x}{15} \Rightarrow$

$x = 15\tan(4^\circ) \approx 1.04890217915$

$\Rightarrow r + x = 4 + x \approx 5.04890217915 \Rightarrow$

Diameter $= d = 2r \approx 10.0978043583$

$\Rightarrow$ $\boxed{d \approx 10.0978043583 \text{ cm}}$

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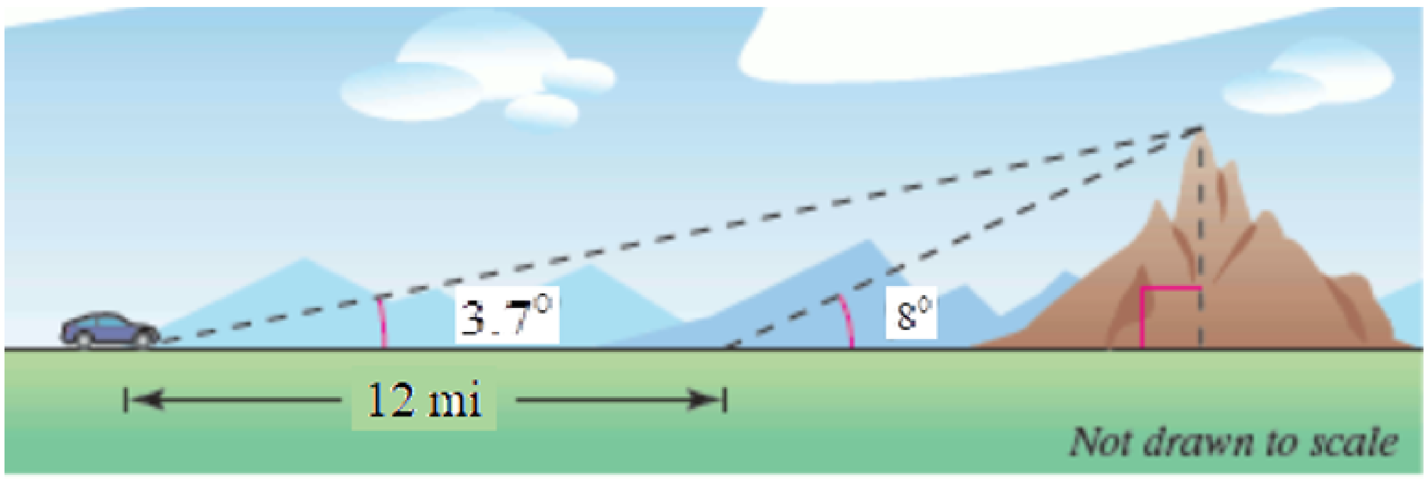
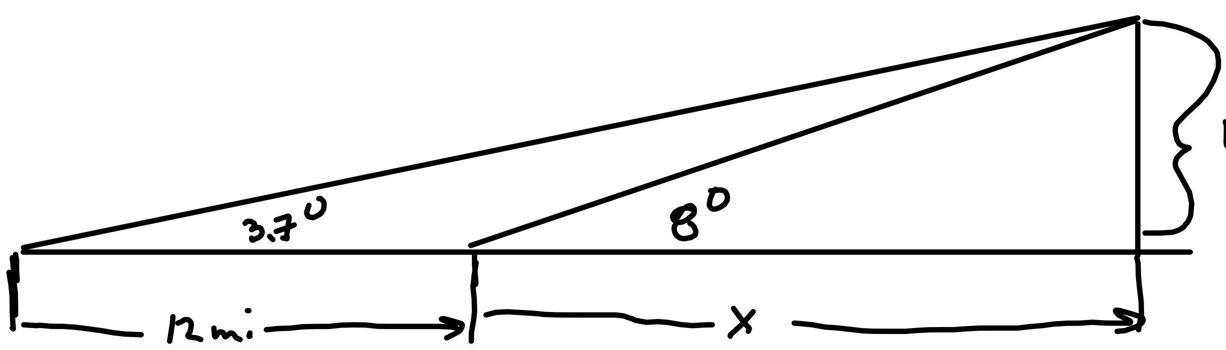
8. In traveling across flat land, you see a mountain directly in front of you. Its angle of elevation (to the peack) is 3.7^0. After you drive 12 miles closer to the mountain, the angle of elevation is 8^0. Approximate the height of the mountain above the plain. Give your answer in feet, to the nearest foot. See figure, below:





Let h = height of the mountain, in feet.

Let x = distance from 2nd sighting to the base of mountain, in miles.

We know: $\frac{h}{x} = \tan(8^0)$, $\frac{h}{x+12} = \tan(3.7^0)$

$\Rightarrow h = x\tan(8^0)$

$\Rightarrow \frac{h}{x+12} = \frac{x\tan(8^0)}{x+12} = \tan(3.7^0) = \frac{ax}{x+12} = b$,

where $a = \tan(8^0)$ & $b = \tan(3.7^0)$

$\Rightarrow ax = bx + 12b$

$ax - bx = 12b$

$x(a-b) = 12b$

$x = \frac{12b}{a-b} = \frac{12\tan(3.7^0)}{\tan(8^0) - \tan(3.7^0)} \approx 10.2275864587$ mi.

Now, $h = x\tan(8^0) \approx 10.2275864587\tan(8^0)$

≈ 1.4373935379 mi

$\approx (1.4373935379 \text{ mi})\left(\frac{5280 \text{ ft}}{\text{mi}}\right)$

≈ 7589.43788011

$\boxed{\approx 7589 \text{ ft} \approx \text{the height of the mountain above the plain}}$

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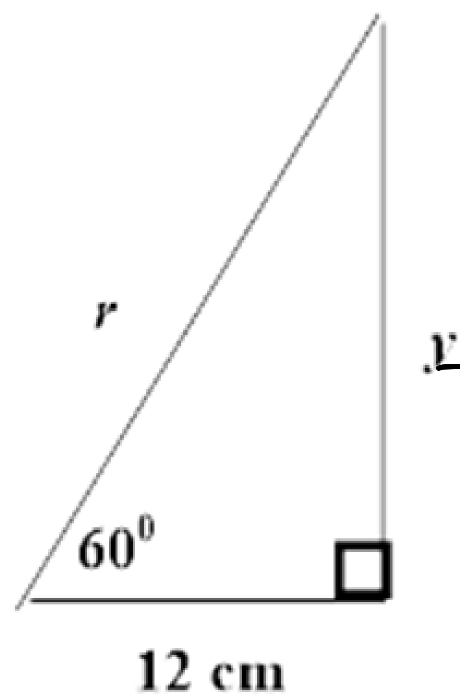
(9) we find y & r:



$\frac{y}{12} = \tan(60^\circ)$

$y = 12\tan(60^\circ)$

$= 12(\sqrt{3}) = \boxed{12\sqrt{3}\text{ cm} = y}$

Find r w/ Pythagorus

$r^2 = 12^2 + (12\sqrt{3})^2 = 144 + 144 \cdot 3$

$= 144(1+3) = 144(4) \Rightarrow$

$r = \sqrt{144(4)} = 12 \cdot 2 = \boxed{24\text{cm} = r}$

by Pythagorus.

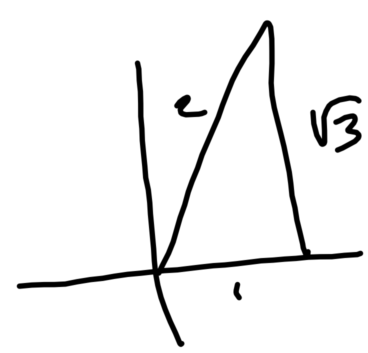
Find r

By trig: $\frac{12}{r} = \cos 60^\circ$

$12 = r\cos 60^\circ$

$\Rightarrow r = \frac{12}{\cos 60^\circ} = \frac{12}{\frac{1}{2}}$

$= 12\left(\frac{2}{1}\right) = \boxed{24\text{cm} = r}$ ✓



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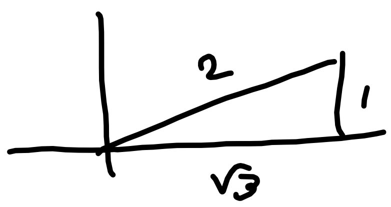
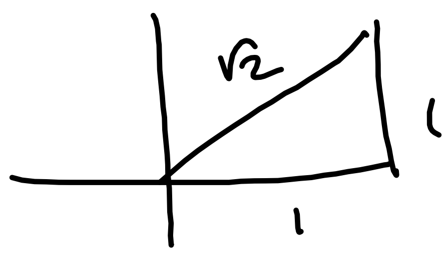
(10) we find θ in degrees & radians w/o calculator. Assume θ's in QI

(a) $\cos\theta = \frac{\sqrt{3}}{2}$



$\Rightarrow \boxed{\theta = 30^\circ = \frac{\pi}{6}}$

(b) $\tan\theta = 1$



$\Rightarrow \boxed{\theta = 45^\circ = \frac{\pi}{4}}$

(11) we show $\tan\theta + \cot\theta = \csc\theta\sec\theta$

Pf

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{\csc\theta}\cdot\frac{1}{\sec\theta}} = \csc\theta\sec\theta \quad \blacksquare$$

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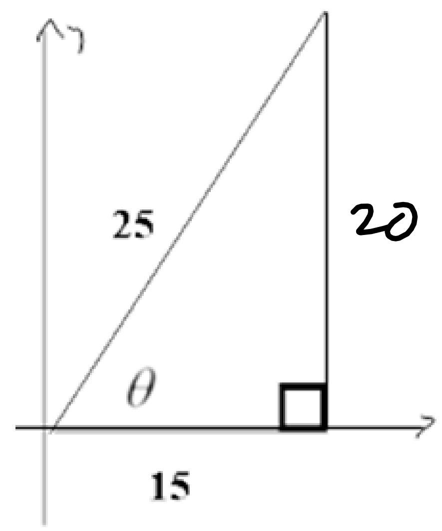
(12) we find the exact values of the 6 trigs for the triangle on the right



$25^2 - 15^2 = 625 - 225 = 400$

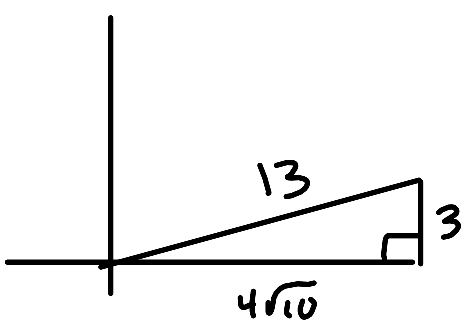
$\sqrt{400} = 20$

$\sin\theta = \frac{20}{25} = \frac{4}{5} = \sin\theta$ $\quad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{15}{25} = \frac{3}{5} = \cos\theta$ $\quad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{20}{15} = \frac{4}{3} = \tan\theta$ $\quad \cot\theta = \frac{3}{4}$

(13) θ is acute $\left(0 < \theta < \frac{\pi}{2}\right)$ & $\sin\theta = \frac{3}{13} \Longrightarrow$



$13^2 - 3^2 = 169 - 9 = 160$

2 | 160
2 | 80
2 | 40
2 | 20
2 | 10
  5

$4\sqrt{10} = \sqrt{160}$

$\sin\theta = \frac{3}{13}$ $\quad \csc\theta = \frac{13}{3}$

$\cos\theta = \frac{4\sqrt{10}}{13}$ $\quad \sec\theta = \frac{13}{4\sqrt{10}}$

$\tan\theta = \frac{3}{4\sqrt{10}}$ $\quad \cot\theta = \frac{4\sqrt{10}}{3}$

(14) $\sin(31^\circ 23' 31'')$

$= \sin\left(\left(31 + \frac{23}{60} + \frac{31}{3600}\right)^\circ\right) \approx 0.520889620891$

$\begin{array}{r} 60 \\ 60 \\ \hline 3600 \end{array}$

≈ 0.5209

$\approx \sin(31^\circ 23' 31'')$