MAT 1420-R11 Written Final Test – Fall, 2025 Print Name_____
130 Points Covers Chapters 1 – 4 and parts of Chapter 6 (5 pts)

NOTE TO PROCTORS: The time control is 2 hours. Please encourage students to write on only one side of each sheet of paper. This way they can refer to previous work for current work.

Materials Permitted: Pencil/Pen, Scientific Calculator (not graphing), Straight Edge, 2-page cheat sheet (either one sheet, 2 sided or 2-page, 1-sided). Include student cheat sheets in scans. Place them at the end.

TO STUDENTS: Show all work. That means scratch work goes with the problem, and not on a separate sheet. **Do your own work**. Leave at least ½-inch margins around each sheet. If you're taking the test in Horizon Hall, this has already been done for you, with a border.

You may work up to 4 bonus problems, at your discretion. There's something of everything on this test.

Remember #1 is above #2 is above #3a is above #3b is above #3c, ... If your work is in two columns, with #5 to the right of #3, you won't receive any points for #5. Writing to communicate is a requirement. If I can't read your work, I can't give you a passing grade.

Leave plenty of space. If it's too cramped to efficiently award partial credit, there will be no partial credit. If you get the answer right, but I can't understand what you did, you will get ½-credit, at most. I need to see the support. I need to see all the scratch work for each problem WITH that problem, not on a separate sheet.

Turn in your test sheets, your work, and your cheat sheet. Test sheets on top. Your work, next, and Cheat Sheet at the bottom.

Draw Pictures! Fare well!

1. (5 pts) How fast, in miles per hour, is a tire with a 30-inch diameter rolling, if it's spinning at 100 revolutions per minute?

2. Answer the questions about the equation $\sin(\theta) = -\frac{2\sqrt{10}}{7}$. Assume $\theta \in [0, 2\pi)$.

   a. (5 points) Sketch two triangles on the same set of coordinate axes that satisfy $\sin(\theta) = -\frac{2\sqrt{10}}{7}$.

   b. (5 pts) Given $\tan(\theta) < 0$, in what quadrant does θ lie?

   c. (5 pts) Again, assuming $0 \le \theta < 2\pi$, find the *exact* value of θ, in radians.

   d. (5 pts) Round your previous answer to 3 decimal places in both radians and degrees.

   e. (5 pts) Find the exact value of the other 5 trigonometric functions for θ, based on previous work.
   $\sin(\theta) = -\frac{2\sqrt{10}}{7}$

   f. (**Bonus** 5 pts) Find the exact values of $\sin\left(\frac{\theta}{2}\right)$ and $\cos\left(\frac{\theta}{2}\right)$, in simplified radical form.

g. (5 pts) Find approximate values of $\sin\left(\frac{\theta}{2}\right)$ and $\cos\left(\frac{\theta}{2}\right)$, in radians and degrees, rounded to 4 decimal places.

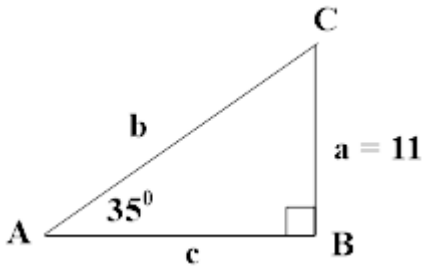
h. (5 pts) Based on your work in part *c*, find *all* solutions to the equation $\sin(\theta) = -\frac{2\sqrt{10}}{7}$, in radians. Give *exact* answers.

3. Let $f(x) = -11\sin\left(\frac{\pi}{10}x - \frac{2\pi}{5}\right) + 9$.

a. (5 pts) Sketch one period of the graph of $f(x)$. Clearly label all highs, lows, and intersections with the midline. I don't care where you start the one period. Just graph the one period correctly.

b. (**Bonus** 5 pts) Solve $f(x) = 0$. Label these points on your graph for part *a*.

4. (5 pts) Solve the triangle in the figure on the right. That means find all lengths and angles. Give exact answers and then round to 3 decimal places.



5. Training for Trig Substitution in Calculus II

a. (5 pts) Draw the picture and use it to re-write $\tan\left(\arcsin\left(\frac{x}{11}\right)\right)$ as an algebraic expression.

b. (**Bonus** 5 pts) Re-write $\sin\left(\arctan(x) + \arccos(x)\right)$ as an algebraic expression. (Hint: Use Sum identity.)

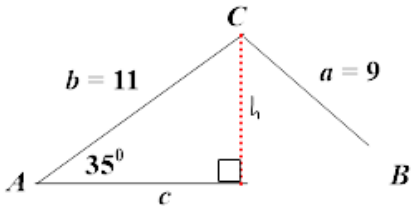
6. Let $z_1 = 81\left(\cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right)\right)$ and $z_2 = 2\left(\cos\left(\frac{\pi}{7}\right) + i\sin\left(\frac{\pi}{7}\right)\right)$.

a. (5 pts) Compute the quotient $\frac{z_1}{z_2}$. Give an exact answer.

b. (5 pts) Find the 4th roots of z_1.

c. (5 pts) Convert z_1 to rectangular form. Round to 3 decimal places.

7. Suppose $a = 9$, $b = 11$ and $A = 35^0$ in triangle ABC. (See figure.)



   a. (5 pts) Show that there are two solutions (two triangles) that satisfy the given information.

   b. (5 pts) Find two answers for angle B. Call them B_1 and B_2. Round your answers to 3 decimal places.

8. (5 pts) Suppose $b = 10$, $c = 5$, and $A = 35^0$ for a triangle ABC. Find the exact value of the 3rd side, whose length we call 'a.' Express your answer in simplified radical form. Then round your answer to 3 decimal places.

9. Let $\overline{u} = \langle -3, 2 \rangle$ and $\overline{v} = \langle 1, 5 \rangle$ be vectors in the plane.

   a. (5 pts) Sketch the vectors $\overline{u}$, $\overline{v}$, and $\text{proj}_{\overline{v}}\overline{u}$.

   b. (5 pts) Find $\overline{u} \bullet \overline{v}$.

   c. (5 pts) Find $\|\overline{u}\|$ and $\|\overline{v}\|$.

   d. (5 pts) Find the angle between $\overline{u}$ and $\overline{v}$, in degrees, rounded to 3 decimal places.

   e. (5 pts) Find $\text{proj}_{\overline{v}}\overline{u}$. Round your answer to 3 decimal places. Does the $\text{proj}_{\overline{v}}\overline{u}$ look about right for what you know it must look like from your sketch in part *a*?

   f. (**Bonus** 5 pts) Find two vectors, $\overline{w}_1$ and $\overline{w}_2$, such that $\overline{u} = \overline{w}_1 + \overline{w}_2$, $\overline{w}_1$ is parallel to $\overline{v}$, and $\overline{w}_2$ is orthogonal to $\overline{v}$.

   g. (**Bonus** 5 pts) What's the name of the algorithm you performed to answer part *f*?

10. Let $r = 2\sin(\theta) + 1$.

   a. (5 pts) Sketch the graph of $r(\theta)$ in rectangular coordinates over the interval $[0, 2\pi]$. Label the high points, low points, and points on the midline. Also find and lable the $\theta-$intercepts.

   b. (5 pts) Sketch the graph of $r(\theta)$ in polar coordinates.

11. (5 pts) Identify the conic section and graph $r(\theta) = \dfrac{16}{4 + 2\sin(\theta)}$. Find and label the center, foci, endpoints of the major axis, in polar coordinates. Give exact answers.

12. (**Bonus** 5 pts) Find the endpoints of the minor axis. You may use rectangular coordinates for these points. Round to 3 decimal places.

13. (**Bonus** 10 pts) A weight of 450 pounds is suspended from two cables, as shown in the figure. Find the tension in both cables. Round to the nearest pound.