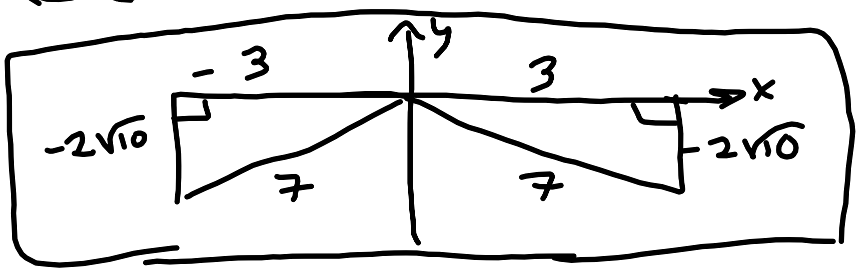
(1) (5 pts) $\left(\frac{100\text{ rev}}{\text{min}}\right)\left(\frac{2\pi\text{ radians}}{1\text{ rev}}\right)(15\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right) =$

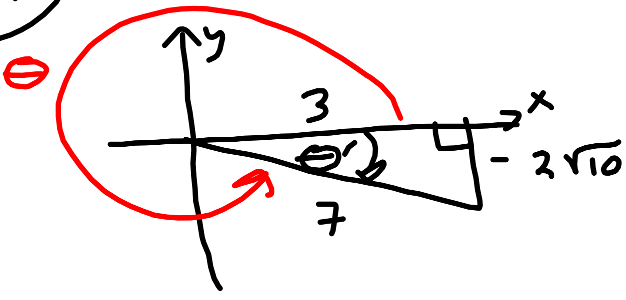
$= \boxed{\frac{125\pi}{44}\ \frac{\text{mi}}{\text{hr}}} \approx 8.924979131\ \frac{\text{mi}}{\text{hr}} \approx 8.925\ \frac{\text{mi}}{\text{hr}}$

Diameter = 30 in $\Longrightarrow$
radius = 15 in.

(2) (a) $\sin\theta = -\frac{2\sqrt{10}}{7}$



(b) (5 pts) $\tan\theta < 0$



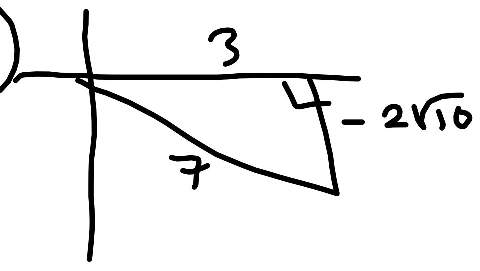
$\boxed{\text{QIV}}$

(c) (5 pts) $\theta' = \arccos\left(\frac{3}{7}\right)$ or $\arcsin\left(-\frac{2\sqrt{10}}{7}\right)$ or $\arctan\left(-\frac{2\sqrt{10}}{3}\right)$

$\boxed{\theta = 2\pi - \arcsin\left(\frac{2\sqrt{10}}{7}\right) \quad \text{OR} \quad 2\pi - \arccos\left(\frac{3}{7}\right)}$

(d) (5 pts) $2\pi - \arcsin\left(\frac{2\sqrt{10}}{7}\right) \approx \boxed{5.155 \approx 295.377^\circ}$ 3 PLACES.

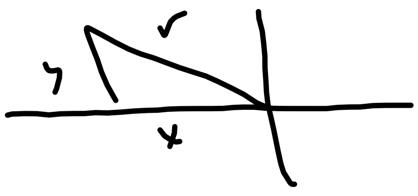
(e)



$$\boxed{\begin{array}{ll} \sin\theta = \frac{-2\sqrt{10}}{7} & \csc\theta = -\frac{7}{2\sqrt{10}} \\ \cos\theta = \frac{3}{7} & \sec\theta = \frac{7}{3} \\ \tan\theta = -\frac{2\sqrt{10}}{3} & \cot\theta = -\frac{3}{2\sqrt{10}} \end{array}}$$

(f) (5 pts) $\frac{3\pi}{2} < \theta\ 2\pi$

$\frac{3\pi}{4} < \theta < \pi \Longrightarrow \frac{\theta}{2} \in \text{QII}$



$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{3}{7}}{2}} = \sqrt{\frac{\frac{7-3}{7}}{2}}$

$= \sqrt{\frac{4}{14}} = \sqrt{\frac{2}{7}}$ OR $\boxed{\frac{\sqrt{14}}{7} = \sin\left(\frac{\theta}{2}\right)}$

$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{3}{7}}{2}} = -\sqrt{\frac{\frac{7+3}{7}}{2}} = -\sqrt{\frac{10}{14}} = -\sqrt{\frac{5}{7}}$

OR $\boxed{-\frac{\sqrt{35}}{7} = \cos\left(\frac{\theta}{2}\right)}$

$\boxed{\text{Bonus 5 pts:}}$ Above & beyond.

$\tan\left(\frac{\theta}{2}\right) = \frac{\frac{\sqrt{14}}{7}}{-\frac{\sqrt{35}}{7}} = -\sqrt{\frac{14}{35}} = \boxed{-\sqrt{\frac{2}{5}} \text{ OR } -\frac{\sqrt{10}}{5} = \tan\left(\frac{\theta}{2}\right)}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{3}{7}}{\frac{-2\sqrt{10}}{7}} = \frac{\frac{4}{7}}{\frac{-2\sqrt{10}}{7}} = -\frac{4}{2\sqrt{10}} = -\frac{2}{\sqrt{10}} = -\frac{2\sqrt{10}}{10}$

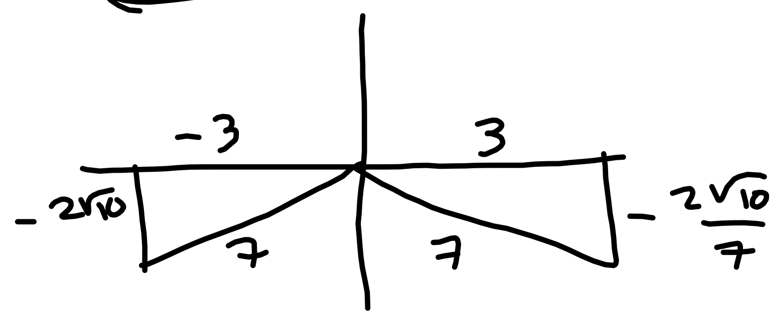
$= -\frac{\sqrt{10}}{5}$ ✓

$= \frac{\sin\theta}{1+\cos\theta} = \frac{-\frac{2\sqrt{10}}{7}}{1+\frac{3}{7}} = -\frac{\frac{2\sqrt{10}}{7}}{\frac{10}{7}} = -2\frac{\sqrt{10}}{10} = -\frac{\sqrt{10}}{5}$ ✓

(g) (5pts) $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{14}}{7} \approx \boxed{.535 \approx \sin\left(\frac{\theta}{2}\right)}$

$\cos\left(\frac{\theta}{2}\right) = -\frac{\sqrt{35}}{7} \approx \boxed{-.845 \approx \cos\left(\frac{\theta}{2}\right)}$

(h) (5pts) All sol'ns of $\sin\theta = -\frac{2\sqrt{10}}{7}$



$$\boxed{\begin{array}{l} \pi + \arcsin\left(\frac{2\sqrt{10}}{7}\right) + 2n\pi, \\ 2\pi - \arcsin\left(\frac{2\sqrt{10}}{7}\right) + 2n\pi,\ n \in \mathbb{Z} \end{array}}$$

③ 5pts $f(x) = -11\sin\left(\frac{\pi}{10}x - \frac{2\pi}{5}\right) + 9$

$= -11\sin\left(\frac{\pi}{10}(x-4)\right) + 9$

$\frac{\pi}{10}x = 2\pi$

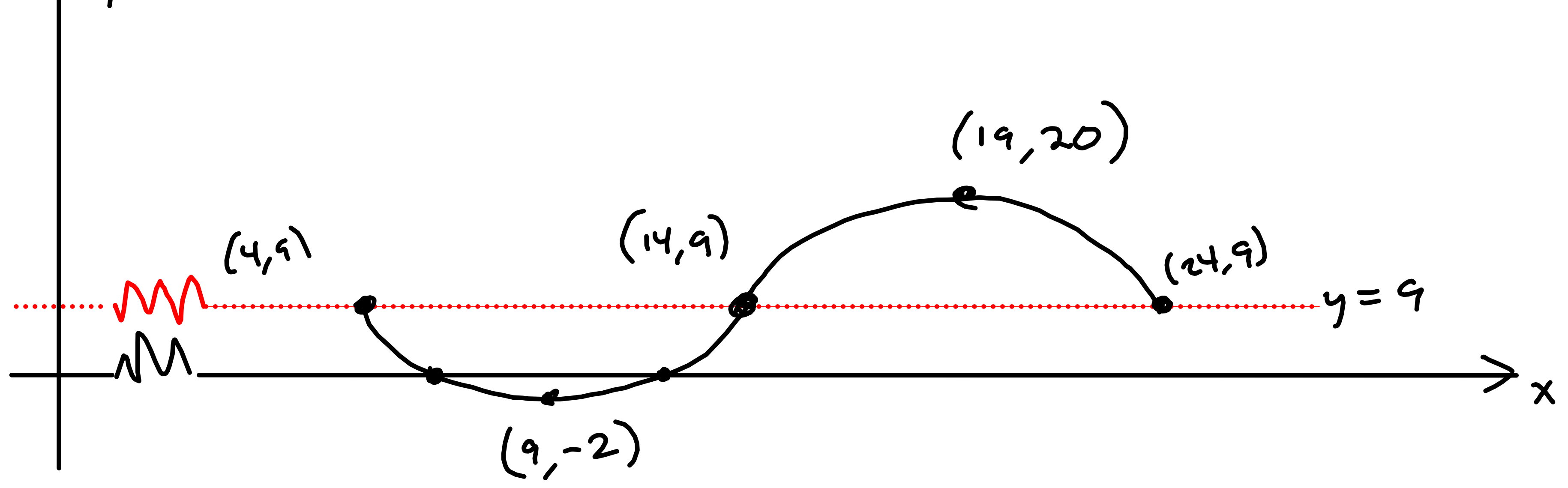
$x = \frac{2\pi}{\frac{\pi}{10}} = 20 = \text{period}$

$\frac{20}{4} = 5 = \text{increment}$

$a = 11$ amp.

$d = 9$ up

$c = 4$ RIGHT



$-11\sin\left(\frac{\pi}{10}(x-4)\right) + 9 = 0$

$-11\sin\left(\frac{\pi}{10}(x-4)\right) = -9$

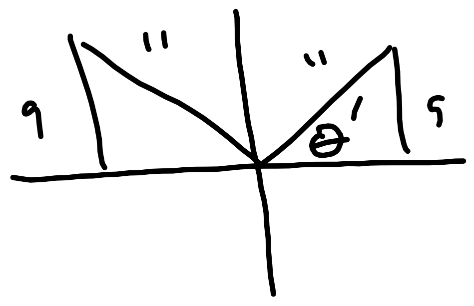
$\sin\left(\frac{\pi}{10}(x-4)\right) = \frac{9}{11}$

$\frac{\pi}{10}(x-4) = \arcsin\left(\frac{9}{11}\right)$

$x - 4 = \frac{10}{\pi}\arcsin\left(\frac{9}{11}\right)$

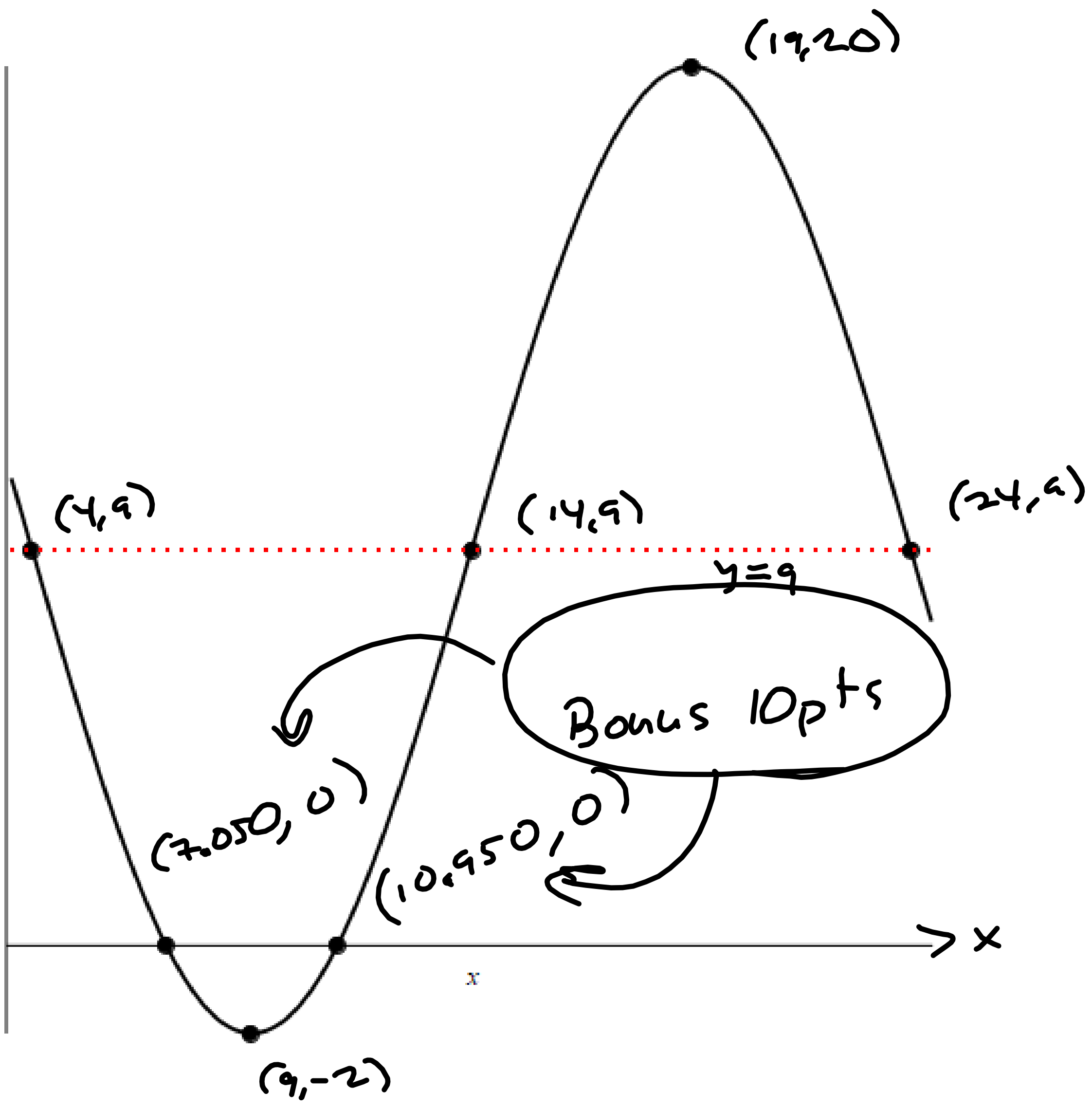
$x = \frac{10}{\pi}\arcsin\left(\frac{9}{11}\right) + 4 \approx 7.050177708$

$x = \frac{10}{\pi}\left(\pi - \arcsin\left(\frac{9}{11}\right)\right) + 4 \approx 10.94982229$



$\theta = \theta' + 2\pi n$

$\theta = \pi - \theta' + 2\pi n$



x-ints:

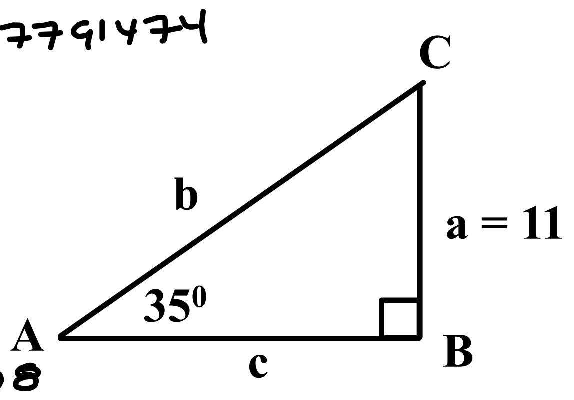
$x = \frac{10}{\pi}\arcsin\left(\frac{9}{11}\right) + 4 + 20n,\ n \in \mathbb{Z}.$

$x = \frac{10}{\pi}\left(\pi - \arcsin\left(\frac{9}{11}\right)\right) + 4 + 20n,\ n \in \mathbb{Z}.$

(4) (5pts) $\sin 35^\circ = \frac{11}{b} \longrightarrow$

$$b = \frac{11}{\sin 35^\circ} \approx \frac{11}{0.5735764364} \approx 19.17791474$$

$$\boxed{b = \frac{11}{\sin 35^\circ} \approx 19.178}$$



$\tan 35^\circ = \frac{11}{c} \longrightarrow$

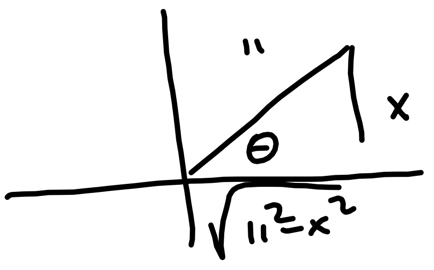
$$c = \frac{11}{\tan 35^\circ} \approx \frac{11}{0.7002075381} \approx 15.70962808$$

$$\boxed{c = \frac{11}{\tan 55^\circ} \approx 15.710}$$

$$C = 90^\circ - 35^\circ = \boxed{55^\circ = C}$$

$$\boxed{B = 90^\circ}$$

(5) (a) (5pts) $\tan\left(\arcsin\left(\frac{x}{11}\right)\right) = \boxed{\tan\theta = \frac{x}{\sqrt{121 - x^2}}}$

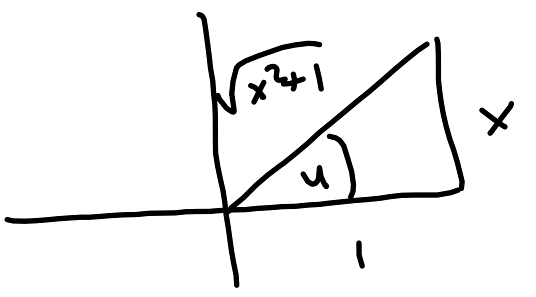


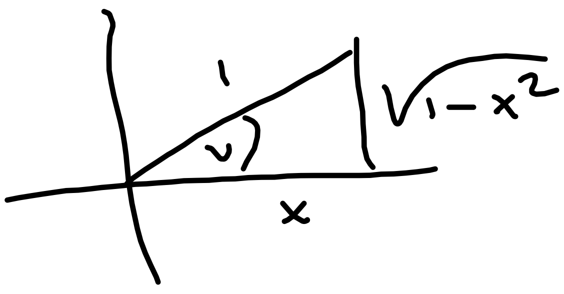
(b) Bonus 5pts $\sin(\arctan(x) + \arccos(x)) = \sin(u + v)$

$$= \sin u \cos v + \sin v \cos u = \frac{x}{\sqrt{x^2+1}} \cdot \frac{x}{1} + \frac{\sqrt{1-x^2}}{1} \cdot \frac{1}{\sqrt{x^2+1}}$$

$$= \boxed{\frac{x^2 + \sqrt{1-x^2}}{\sqrt{x^2+1}} = \sin(u+v)}$$

$u = \arctan(x)$ $\qquad$ $v = \arccos(x)$





⑥ $z_1 = 81\left(\cos\frac{4\pi}{9} + i\sin\frac{4\pi}{9}\right)$, $z_2 = 2\left(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}\right)$

ⓐ 5pts $\boxed{\dfrac{z_1}{z_2} = \dfrac{81}{2}\left(\cos\left(\dfrac{19\pi}{63}\right) + i\sin\left(\dfrac{19\pi}{63}\right)\right)}$

$\frac{4\pi}{9} - \frac{\pi}{7} = \frac{28\pi - 9\pi}{63} = \frac{19\pi}{63}$

ⓑ 5pts 4^{th} roots of z_1

$\frac{4\pi}{9} \div 4 = \frac{\pi}{9}$

$\sqrt[4]{81} = 3$

increment $= \frac{2\pi}{4} = \frac{\pi}{2}$. $\frac{\pi}{9} + \frac{\pi}{2} = \frac{2\pi}{18} + \frac{9\pi}{18} = \frac{11\pi}{18}$, $\frac{20\pi}{18} = \frac{10\pi}{9}$, $\frac{29\pi}{18}$, $\frac{38\pi}{18}$

1^{st} $\sqrt[4]{z_1} = 3\left(\cos\left(\frac{\pi}{9}\right) + i\sin\left(\frac{\pi}{9}\right)\right)$

2^{nd} $3\left(\cos\left(\frac{11\pi}{18}\right) + i\sin\left(\frac{11\pi}{18}\right)\right)$

$3\left(\cos\left(\frac{10\pi}{9}\right) + i\sin\left(\frac{10\pi}{9}\right)\right)$

$3\left(\cos\left(\frac{29\pi}{18}\right) + i\sin\left(\frac{29\pi}{18}\right)\right)$

$\frac{38\pi}{18} = \frac{36\pi}{18} + \frac{2\pi}{18} = 2\pi + \frac{\pi}{9}$ is coterminal with $\frac{\pi}{9}$ ✓

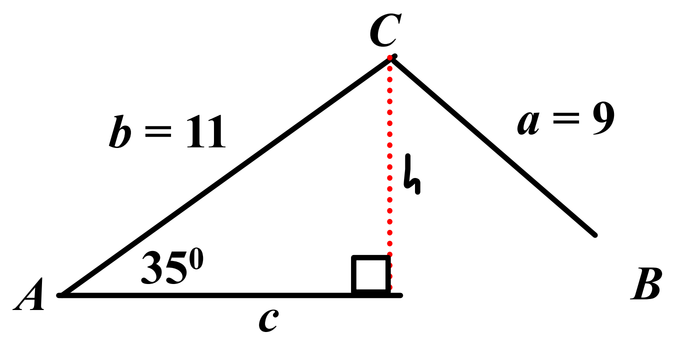
ⓒ 5pts $z_1 = 81\left(\cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right)\right)$

$\approx 81\,(0.1736481773 + 0.9848077531\, i)$

$\approx 14.06550236 + 79.76942800\, i$

$\approx \boxed{14.066 + 79.769\, i}$

7 $a=9,\ b=11,\ A=35^\circ$



a) 5pts $h = 11\sin 35^\circ \approx 6.309340799$

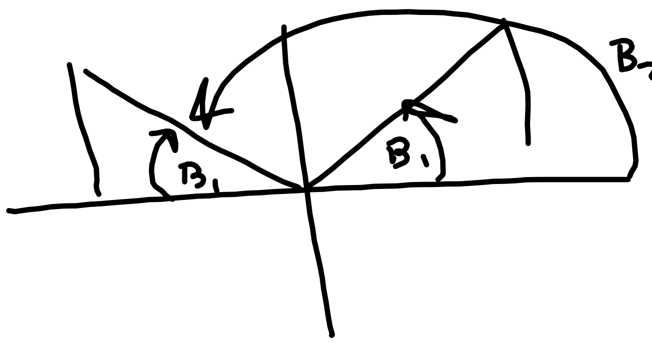
$h \approx 6.3 < 9 = a < b = 11$

CASE 1 : B1 is acute:

$$\frac{\sin A}{a} = \frac{\sin B_1}{b}$$

$$\sin B_1 = \frac{b\sin A}{a} = \frac{11\sin 35^\circ}{9} \longrightarrow$$

$$B_1 = \arcsin\left(\frac{11\sin 35^\circ}{9}\right) \approx 44.51033158^\circ \approx \boxed{\begin{array}{c}44.510^\circ \\ \approx B_1\end{array}}$$



$B_2 = 180^\circ - B_1 = 180^\circ - 44.510\ldots^\circ$

$\approx$

$$B_2 = 180^\circ - B_1 \approx 135.4896684^\circ \approx \boxed{135.490^\circ \approx B_2}$$

8 $b=10,\ c=5,\ A=35^\circ \longrightarrow$

5pts

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 100 + 25 - 2(10)(5)\cos 35^\circ$$

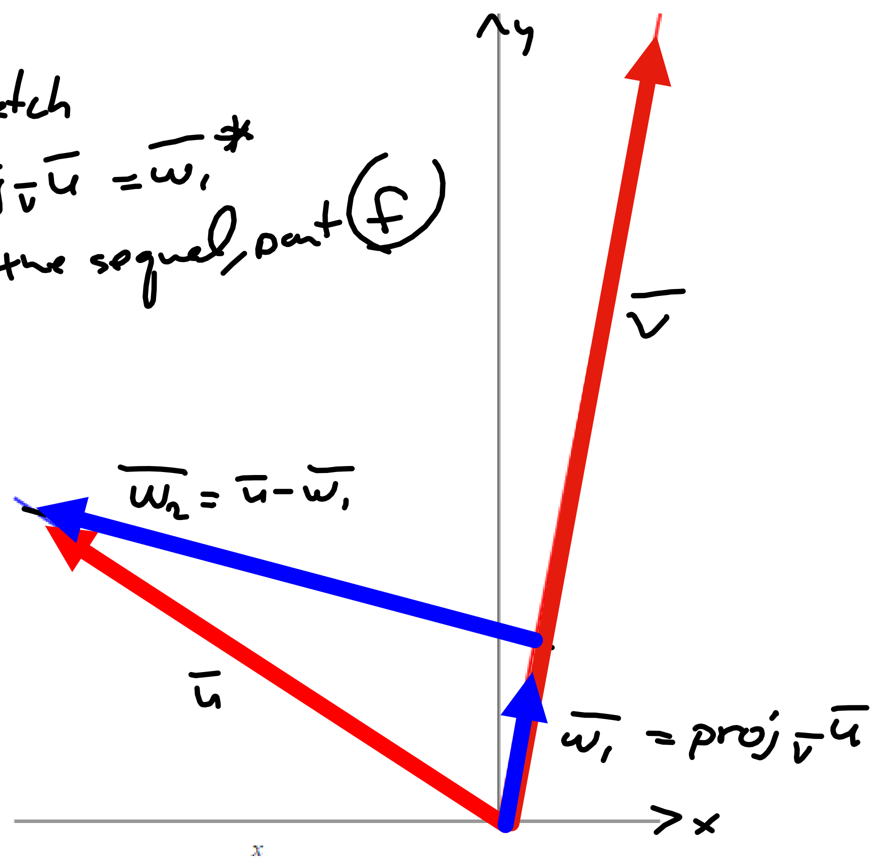
$$= 125 - 100\cos 35^\circ \approx 43.08479555$$

$$\longrightarrow a \approx 6.563900940 \approx \boxed{6.564 \approx a}$$

9 $\bar{u} = \langle -3,2 \rangle,\ \bar{v} = \langle 1,5 \rangle$

a (5pts) Sketch $\bar{u}, \bar{v}, \text{proj}_{\bar{v}}\bar{u} = \bar{w}_1$ *

*in the sequel, part (f)



b (5pts) $\bar{u} \cdot \bar{v} = \langle -3,2 \rangle \cdot \langle 1,5 \rangle = (-3)(1) + (2)(5) = -3 + 10 = 7$

$\boxed{\bar{u} \cdot \bar{v} = 7}$

c (5pts) $\|\bar{u}\| = \sqrt{\bar{u} \cdot \bar{u}} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \boxed{\sqrt{13} = \|\bar{u}\|}$

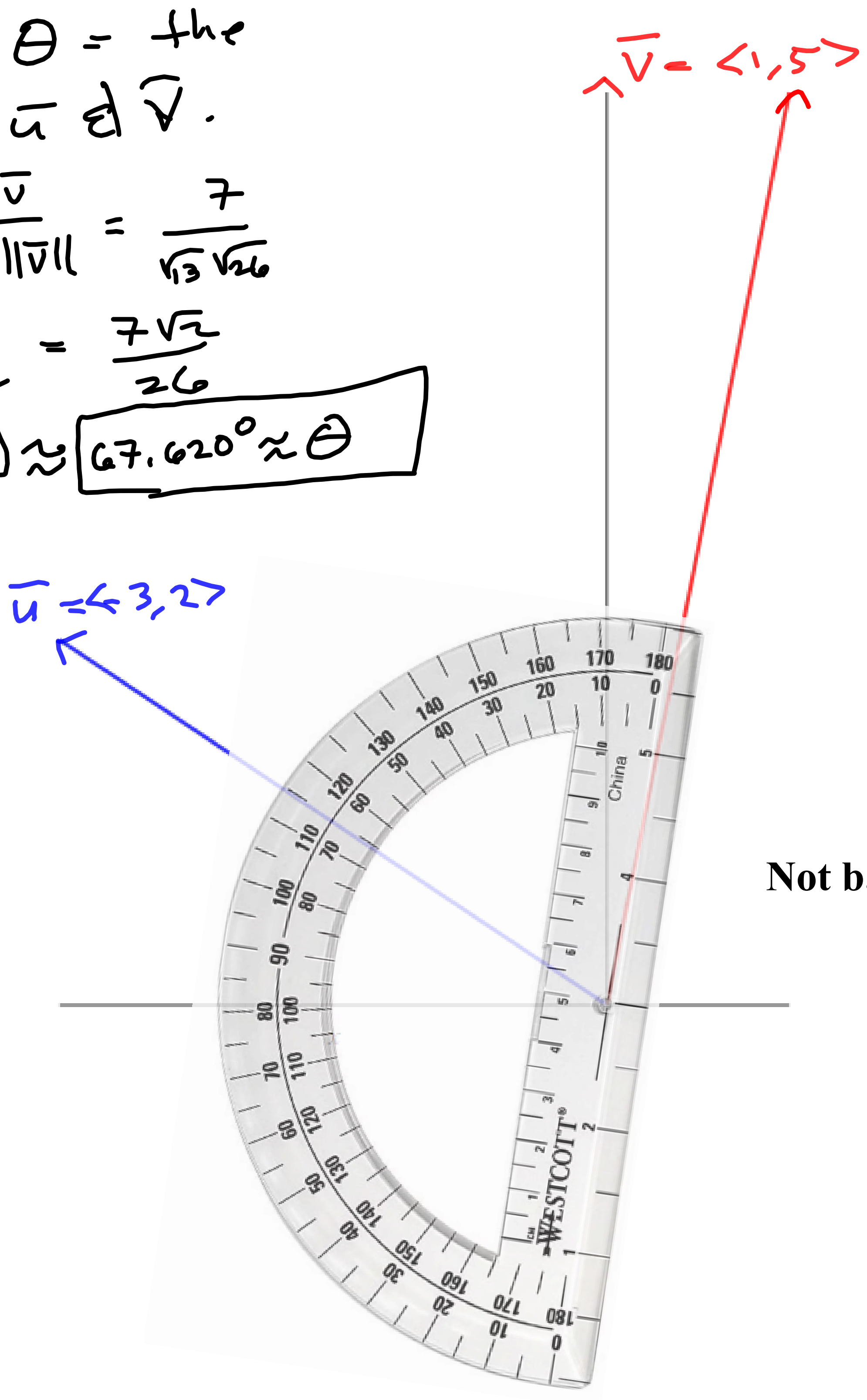
$\|\bar{v}\| = \sqrt{\bar{v} \cdot \bar{v}} = \sqrt{1^2 + 5^2} = \boxed{\sqrt{26} = \|\bar{v}\|}$

d (5pts) Let θ = the angle between $\bar{u}$ & $\bar{v}$.

Then $\cos\theta = \dfrac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{7}{\sqrt{13}\sqrt{26}}$

$= \dfrac{7}{\sqrt{13}\sqrt{13}\sqrt{2}} = \dfrac{7}{13\sqrt{2}} = \dfrac{7\sqrt{2}}{26}$

$\Rightarrow \theta = \arccos\left(\dfrac{7\sqrt{2}}{26}\right) \approx \boxed{67.620^\circ \approx \theta}$



Not bad. Looks pretty close.

e 5 pts $w_1 = \text{proj}_{\bar{v}} \bar{u} = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v} = \dfrac{7}{(\sqrt{26})^2}\langle 1,5\rangle$

$= \boxed{\dfrac{7}{26}\langle 1,5\rangle = \bar{w}_1} \approx \langle .269, 1.346\rangle$

$\left\langle \dfrac{7}{26}, \dfrac{35}{26}\right\rangle$

f Bonus 5 pts $\boxed{\bar{w}_2 = \bar{u} - \bar{w}_1} = \langle -3, 2\rangle$

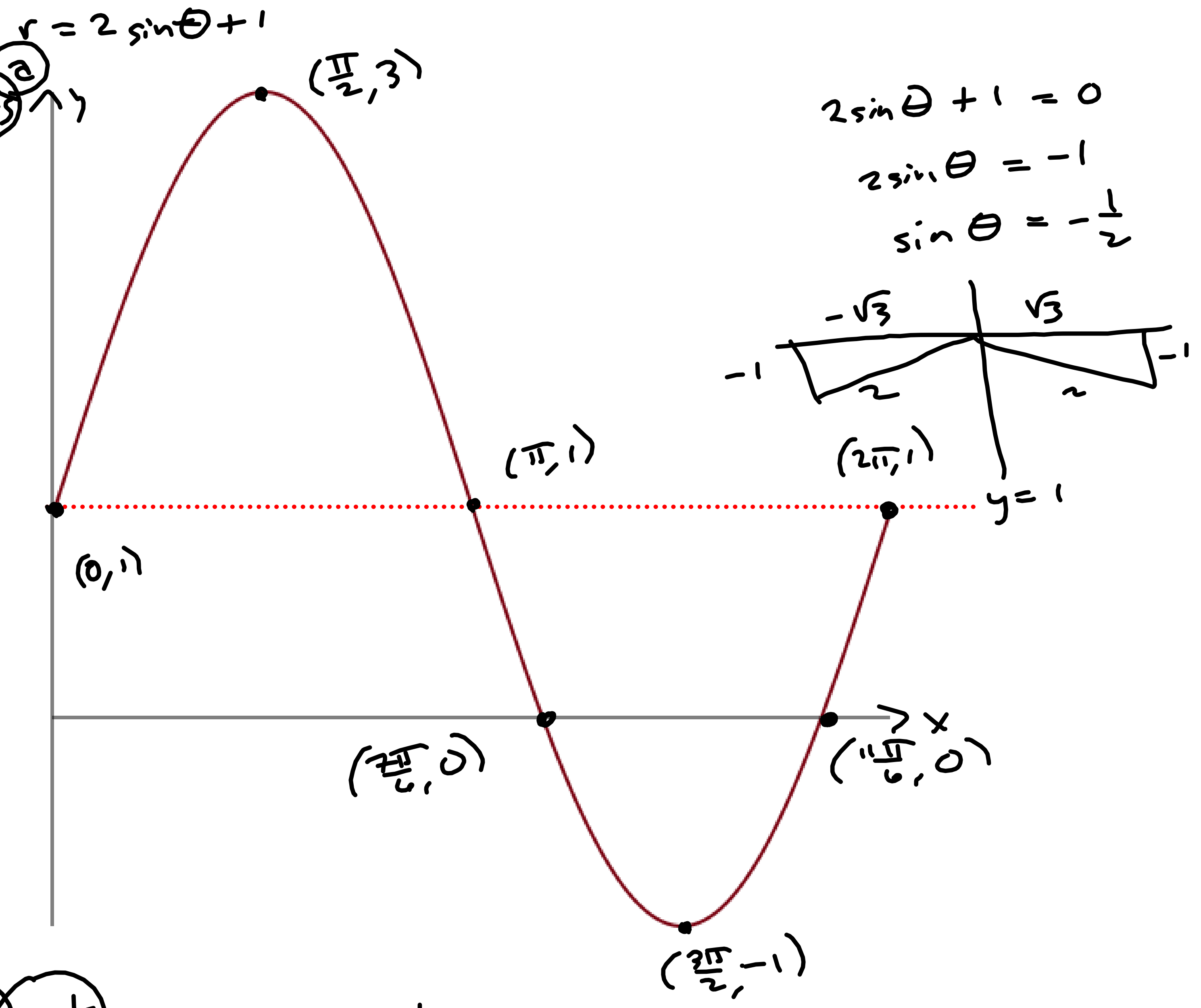
$= \left\langle \dfrac{-3(26)+7}{26}, \dfrac{2(26)+35}{26}\right\rangle = \left\langle \dfrac{-78-7}{26}, \dfrac{52-35}{26}\right\rangle \boxed{= \left\langle \dfrac{-85}{26}, \dfrac{17}{26}\right\rangle = \bar{w}_2}$

$\approx \langle .269, 1.346\rangle$

g 5 pts Bonus GRAM-SCHMIDT ORTHOGONALIZATION.

10 $r = 2\sin\theta + 1$

a) 5pts



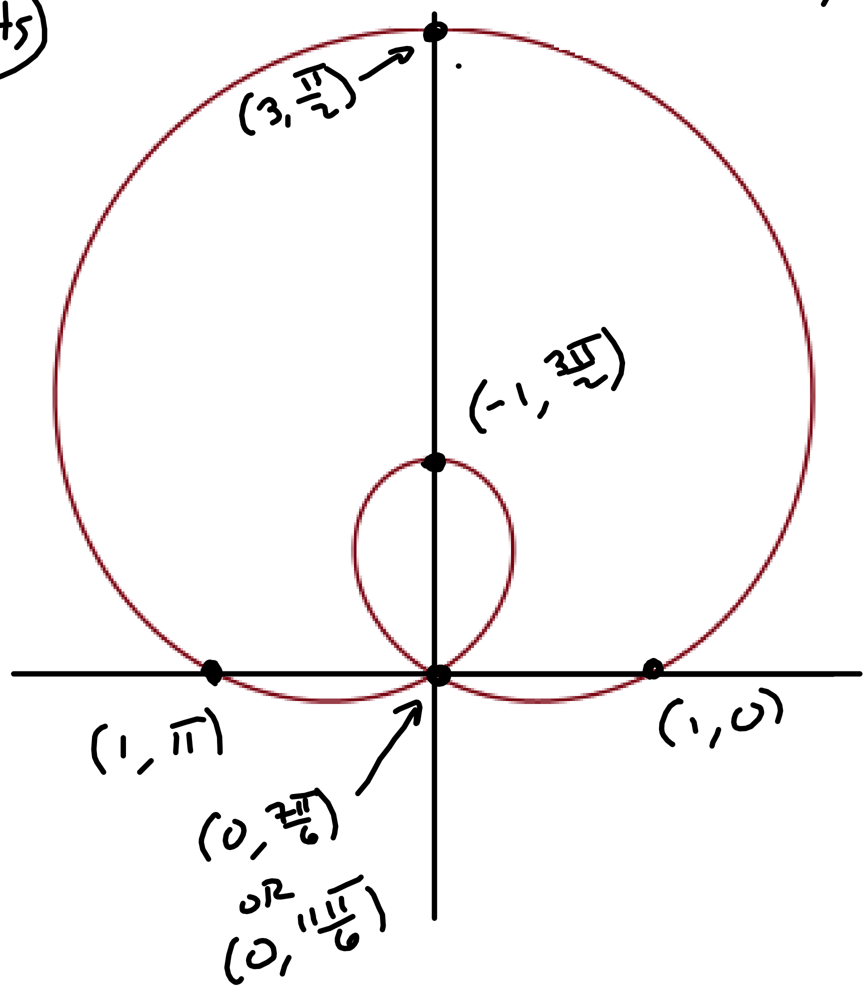
$2\sin\theta + 1 = 0$

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

b) 5pts



(11) (5 pts) $r = \frac{16}{4+2\sin\theta} = \frac{16}{4(1+\frac{1}{2}\sin\theta)} = \frac{4}{1+\frac{1}{2}\sin\theta}$ $e=\frac{1}{2}, ep=4$

$= \frac{ep}{1+e\sin\theta} \Rightarrow \boxed{e=\frac{1}{2}} \Rightarrow ep = \frac{1}{2}p = 4 \Rightarrow \boxed{p=8}$

. Distance from focus to directrix,

$e = \frac{1}{2} < 1 \Rightarrow$ ellipse

$1+e\sin\theta \Rightarrow$ Horizontal Directrix, above the pole

$y = 8 = r\sin\theta \Rightarrow r = \frac{8}{\sin\theta} \Rightarrow$ $\boxed{r = 8\csc\theta \text{ is its polar eq'n}}$

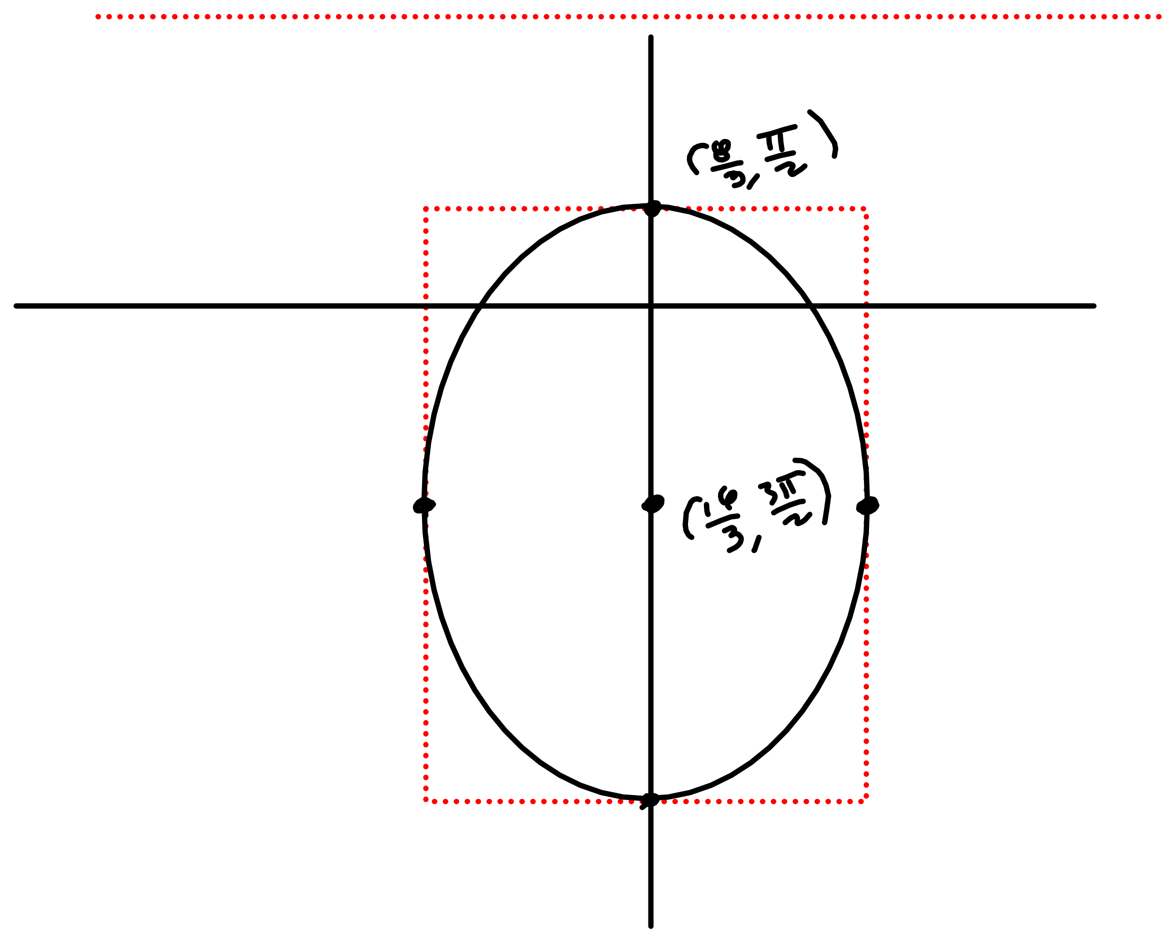
$r(\frac{\pi}{2}) = \frac{16}{4+2} = \frac{16}{6} = \frac{8}{3} \rightsquigarrow (\frac{8}{3}, \frac{\pi}{2}) \rightsquigarrow (0, \frac{8}{3})$ Rectangular

$r(\frac{3\pi}{2}) = \frac{16}{4-2} = 8 \rightsquigarrow (8, \frac{3\pi}{2}) \rightsquigarrow (0,-8)$ Rect.

$\Rightarrow 2a = \frac{8}{3} + 8\cdot\frac{3}{3} = \frac{8+24}{3} = \frac{32}{3} \Rightarrow \boxed{a = \frac{16}{3}}$

center = midpoint of $(0,\frac{8}{3})$ & $(0,-8) \Rightarrow (0, \frac{\frac{8-24}{3}}{2}) = (0, -\frac{16}{3})$ = center

$e = \frac{c}{a} \Rightarrow ae = c \Rightarrow \frac{16}{3}\cdot\frac{1}{2} = \frac{8}{3} = c$ = focal length



$$r = \frac{16}{4+2\left(\frac{y}{r}\right)} = \frac{16}{\frac{4r+2y}{r}} = \frac{16r}{4r+2y}$$

$$\Longrightarrow \frac{16}{4r+2y} = 1 \longrightarrow 4r+2y = 16$$

$$\Longrightarrow 2y + 4\sqrt{x^2+y^2} = 16$$

$$\Longrightarrow y + 2\sqrt{x^2+y^2} = 8$$

$$\Longrightarrow 2\sqrt{x^2+y^2} = 8-y$$

$$\Longrightarrow 4(x^2+y^2) = y^2 - 16y + 64$$

$$\begin{array}{r} 64 \\ \underline{3} \\ 192 \end{array}$$

$$4x^2 + 4y^2 - y^2 + 16y = 64$$

$$4x^2 + 3y^2 + 16y = 64$$

$$4x^2 + 3\left(y^2 + \frac{16}{3}y + \left(\frac{8}{3}\right)^2\right) = 64 + 3\left(\frac{64}{9}\right) = \frac{192+64}{3} = \frac{256}{3}$$

$$4x^2 + 3\left(y+\frac{8}{3}\right)^2 = \frac{256}{3}$$

$$\frac{3}{256}\left(\qquad \right)$$

$$\frac{12}{256}x^2 + \frac{9}{256}\left(y+\frac{8}{3}\right)^2 = 1$$

$$\begin{array}{c} 3 \\ \cancel{6} \\ \cancel{12} \\ \cancel{256} \\ \cancel{128} \\ 64 \end{array}$$

$$\frac{3}{64}x^2 + \frac{9}{256}\left(y+\frac{8}{3}\right)^2 = 1$$

$$\frac{x^2}{\frac{64}{3}} + \frac{\left(y+\frac{8}{3}\right)^2}{\frac{256}{9}} = 1$$

$$\frac{x^2}{\left(\frac{8}{\sqrt{3}}\right)^2} + \frac{\left(y+\frac{8}{3}\right)^2}{\left(\frac{16}{3}\right)^2} = 1$$

$$5.\overline{3} = \boxed{a = \frac{16}{3},} \; \boxed{b = \frac{8}{\sqrt{3}}} = \frac{8\sqrt{3}}{3} \approx 4.618802155$$

$$c^2 = a^2 - b^2 = \frac{256}{9} - \frac{64}{3}\cdot\frac{3}{3} = \frac{256-192}{9} = \frac{64}{9}$$

$$\longrightarrow c = \sqrt{\frac{64}{9}} = \frac{8}{3} = 2.\overline{6}$$

11 5pts $r = \frac{16}{4+2\sin\theta} = \frac{16}{4(1+\frac{1}{2}\sin\theta)} = \frac{4}{1+\frac{1}{2}\sin\theta} = \frac{ep}{1+e\sin\theta}$

Above Horizontal Directrix

$\Rightarrow$ e = eccentricity = $\boxed{\frac{1}{2} = e}$

$ep = \frac{1}{2}p = 4 \Rightarrow \boxed{p = 8}$

Focus @ pole & $p = 8 \rightarrow$ Directrix is $y = 8$ OR $\boxed{r = 8\csc\theta \text{ Directrix}}$

$e = \frac{1}{2} < 1 \Rightarrow$ Ellipse

$r(\frac{\pi}{2}) = \frac{4}{1+\frac{1}{2}(1)} = \frac{4}{\frac{3}{2}} = \frac{8}{3} \rightarrow (\frac{8}{3}, \frac{\pi}{2})$

$r(\frac{3\pi}{2}) = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8 \rightarrow (8, \frac{3\pi}{2})$

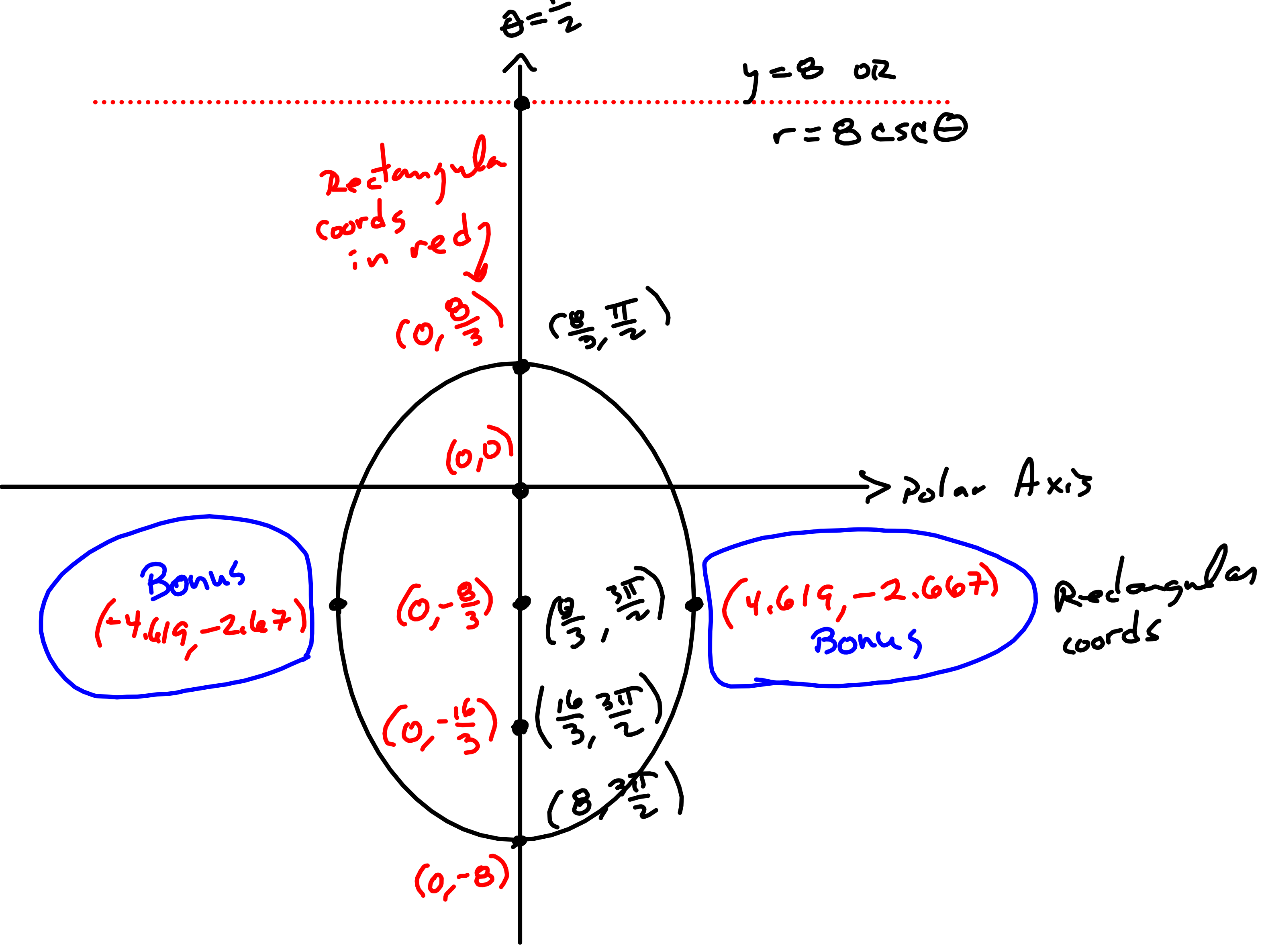
Rect:

$(0, \frac{8}{3})$

$(0, -8)$

$2a = 8 + \frac{8}{3} = \frac{24+8}{3} = \frac{32}{3} \Rightarrow \boxed{a = \frac{16}{3}}$

$e = \frac{c}{a} \Rightarrow c$ = focal length $= ea = \frac{1}{2}(\frac{16}{3}) = \boxed{\frac{8}{3} = c}$



12 Bonus 5pts $c = \frac{8}{3}, a = \frac{16}{3}$

$\rightarrow b^2 = a^2 - c^2 = \frac{256-64}{9} = \frac{192}{9} = b^2$

$\rightarrow b = \sqrt{\frac{192}{9}} = \frac{8\sqrt{3}}{3} \approx 4.618802155$

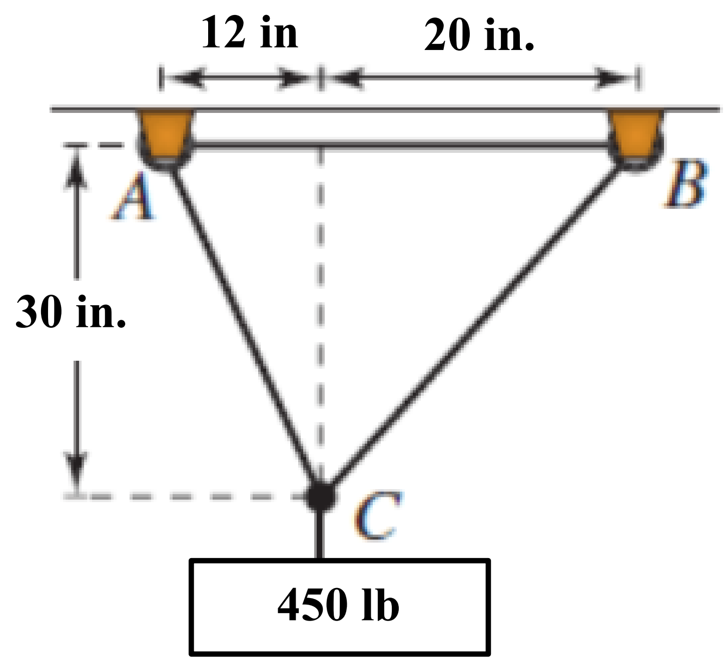
2 | 192
2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3

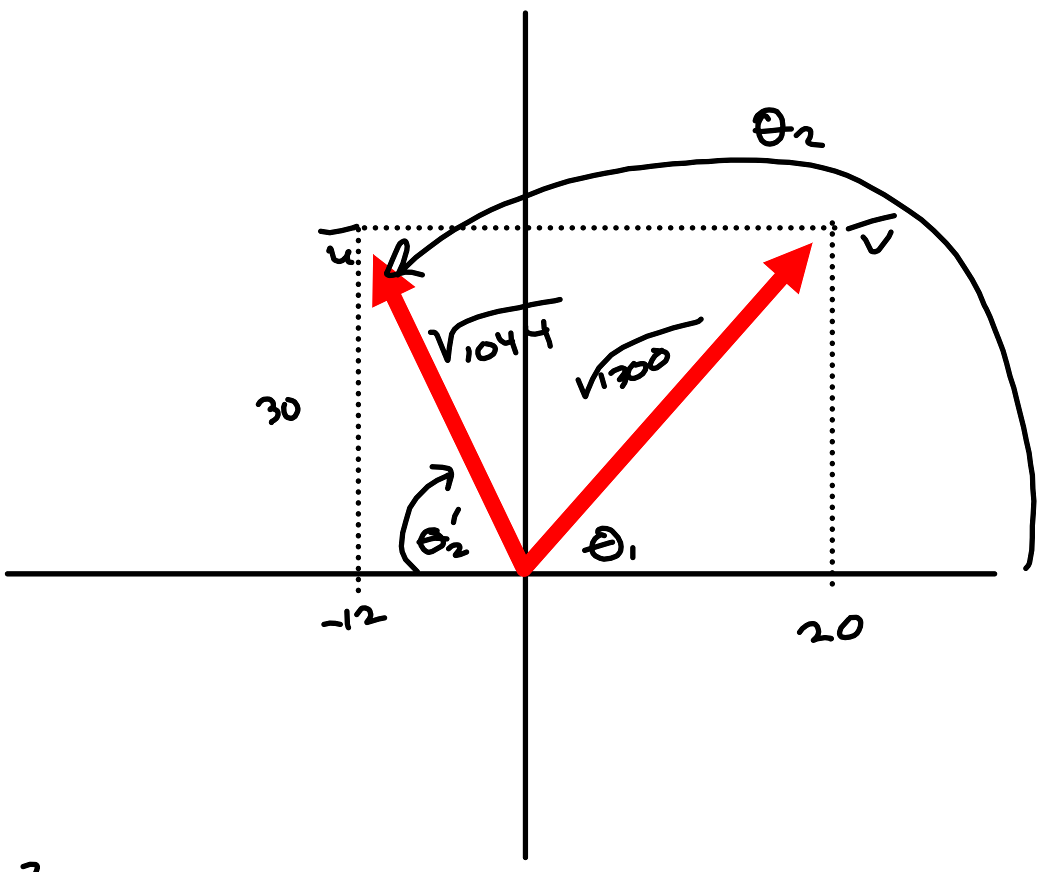
Endpoints of minor axis are level with the center @ $(0, -\frac{8}{3})$

So, $(-\frac{8\sqrt{3}}{3}, -\frac{8}{3}) \approx (-4.619, -2.667)$

and $(\frac{8\sqrt{3}}{3}, -\frac{8}{3}) \approx (4.619, -2.667)$ are the endpoints of the minor axis.

(13) Bonus 10 pts





$12^2+30^2 = 144+900$
$= 1044$

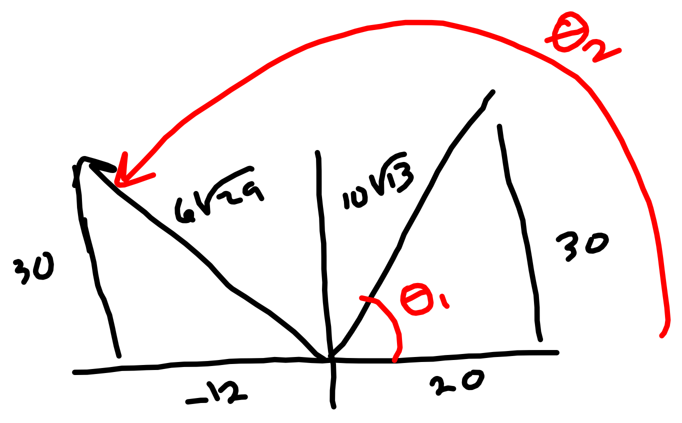
$$\begin{array}{r|l} 2 & 1044 \\ 2 & 522 \\ 3 & 261 \\ 3 & 87 \\ & 29 \end{array}$$

$\sqrt{1044} = 6\sqrt{29}$

$20^2+30^2 = 400+900 = 1300$

$$\begin{array}{r|l} 10 & 1300 \\ 10 & 130 \\ & 13 \end{array}$$

$\sqrt{1300} = 10\sqrt{13}$



$\bar{u} = \|\bar{u}\| \langle \cos\theta_1, \sin\theta_1 \rangle$

$\bar{v} = \|\bar{v}\| \langle \cos\theta_2, \sin\theta_2 \rangle$

$\bar{u} = \|\bar{u}\| \left\langle -\frac{12}{6\sqrt{29}}, \frac{30}{6\sqrt{29}} \right\rangle = \boxed{x \left\langle -\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle = \bar{u}}$

where $x = \|\bar{u}\|$

$\bar{v} = \|\bar{v}\| \left\langle \frac{20}{10\sqrt{13}}, \frac{30}{10\sqrt{13}} \right\rangle = \boxed{y \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle = \bar{v}}$

where $y = \|\bar{v}\|$

$\bar{u} + \bar{v} = \langle 0, 450 \rangle \longrightarrow$

$\left\langle -\frac{2}{\sqrt{29}} x, \frac{5}{\sqrt{29}} x \right\rangle + \left\langle \frac{2}{\sqrt{13}} y, \frac{3}{\sqrt{13}} y \right\rangle = \langle 0, 450 \rangle \longrightarrow$

$-\frac{2}{\sqrt{29}} x + \frac{2}{\sqrt{13}} y = 0 \longrightarrow x = \frac{\sqrt{29}}{2} \cdot \frac{2}{\sqrt{13}} y = \frac{\sqrt{29}}{\sqrt{13}} y$

$\frac{5}{\sqrt{29}} x \; \frac{3}{\sqrt{13}} y = 450$

$\longrightarrow \frac{5}{\sqrt{29}} \left(\frac{\sqrt{29}}{\sqrt{13}} \right) y + \frac{3}{\sqrt{13}} y = 450$

$\longrightarrow \frac{8}{\sqrt{13}} y = 450 \longrightarrow y = \frac{450\sqrt{13}}{8} = \boxed{\frac{225\sqrt{13}}{4} \text{ lbs} = \|\bar{v}\| \approx 203 \text{ lbs}}$

202.8122592

$x = \sqrt{\frac{29}{13}} \left(\frac{225\sqrt{13}}{4} \right) = \boxed{\frac{225\sqrt{29}}{4} \approx 303 \text{ lbs} \approx \|\bar{u}\|}$

302.9155204

$$\tan(u+v) = \frac{\sin u\cos v + \sin v\cos u}{\cos u\cos v - \sin u\sin v} =$$

$$= \frac{\dfrac{\sin u\cos v + \sin v\cos u}{\cos u\cos v}}{\dfrac{\cos u\cos v - \sin u\sin v}{\cos u\cos v}} = \frac{\tan u + \tan v}{1 - \tan u\tan v}$$

$$\tan(2u) = \frac{2\tan u}{1-\tan^2 u} = \frac{2\dfrac{\sin u}{\cos u}}{1-\dfrac{\sin^2 u}{\cos^2 u}} = \frac{\dfrac{2\sin u}{\cos u}}{\dfrac{\cos^2 u-\sin^2 u}{\cos^2 u}}$$

$$= \frac{2\sin u}{\cos^2 u-\sin^2 u}\cdot\frac{\cos^2 u}{\cos u}$$

$$= \frac{2\sin u\cos u}{\cos^2 u-\sin^2 u}$$

$$\frac{2\tan\left(\frac{u}{2}\right)}{1-\tan^2\left(\frac{u}{2}\right)} = \tan(u)$$

$$1-\tan^2\left(\tfrac{u}{2}\right)\tan u = 2\tan\left(\tfrac{u}{2}\right)$$

$$-\tan^2\left(\tfrac{u}{2}\right)\tan(u) - 2\tan\left(\tfrac{u}{2}\right) + 1 = 0$$

$$\tan(u)\tan^2\left(\tfrac{u}{2}\right) - 2\tan\left(\tfrac{u}{2}\right) - 1 = 0$$

$$a = \tan(u),\ b = -2,\ c = -1$$

$$b^2 - 4ac = 4 - 4(\tan(u))(-1) = 4 + 4\tan u$$

$$\tan\left(\tfrac{u}{2}\right) = \frac{2 \pm \sqrt{4+4\tan u}}{2\tan u} = 1 \pm \tan u + 1$$

$$\frac{2\tan\frac{u}{2}}{1-\tan^2\frac{u}{2}} = \tan(u)$$

$$2\tan\left(\frac{u}{2}\right) = \tan(u) - \tan(u)\tan^2\left(\frac{u}{2}\right)$$

$$\tan(u)\tan^2\left(\frac{u}{2}\right) + 2\tan\left(\frac{u}{2}\right) - \tan(u) = 0$$

$$a = \tan(u),\ b = 2,\ c = -\tan(u)$$

$$\longrightarrow b^2 - 4ac = 4 - 4(\tan(u))(-\tan(u))$$

$$= 4 + 4\tan^2 u$$

$$\tan\left(\frac{u}{2}\right) = \frac{-2 \pm 2|\tan u|}{\tan u} = \frac{-2 \pm 2\sqrt{1+\tan^2 u}}{\tan(u)}$$

$$\frac{2\tan\frac{u}{2}}{1-\tan^2\frac{u}{2}} = \tan(u)$$

$$\frac{2\frac{\sin\frac{u}{2}}{\cos\frac{u}{2}}}{\frac{\cos^2\frac{u}{2}-\sin^2\frac{u}{2}}{\cos^2\frac{u}{2}}} = \frac{\cancel{2}\frac{\sin\frac{u}{2}}{\cos\frac{u}{2}}}{\frac{\cos(u)}{\left(\frac{1+\cos(u)}{\cancel{2}}\right)}} = \frac{\tan\left(\frac{u}{2}\right)}{\frac{\cos(u)}{1+\cos(u)}} = \tan(u)$$

$$\tan\left(\frac{u}{2}\right) = \frac{\tan(u)\,(1+\cos(u))}{\cos(u)} = \frac{\sin u\,(1+\cos(u))}{\cos^2 u}$$

$$\frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \pm\sqrt{\frac{1-\cos u}{1+\cos u}} = \pm\sqrt{\frac{(1-\cos u)(1-\cos u)}{\sin^2 u}}$$

$$= \frac{1-\cos u}{|\sin u|}$$

$u \in QI$ $\tan\left(\frac{u}{2}\right) = +$ $\sin(u) > 0$

$\sin\left(\frac{u}{2}\right) > 0$

$u \in QII$ $\tan\left(\frac{u}{2}\right) > 0$ $\sin(u) > 0$

$\sin(u/2) > 0$

$$\frac{1-\cos u}{\sin(u)} = \frac{+}{+}$$

$\cos(u) < 0$

$\cos\left(\frac{u}{2}\right) > 0$

$\pi < \theta < \frac{3\pi}{2}$

$u \in QIII$ $\frac{1-\cos u}{\sin u} = \frac{+}{-}$ $\tan\left(\frac{u}{2}\right) < 0$

$\frac{u}{2} \in QII$

$\sin\left(\frac{u}{2}\right) > 0$

$u \in QIV$ $\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

$\frac{u}{2} \in QII$