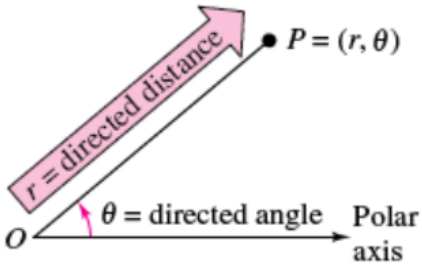
Section 6.7 - Polar Coordinates

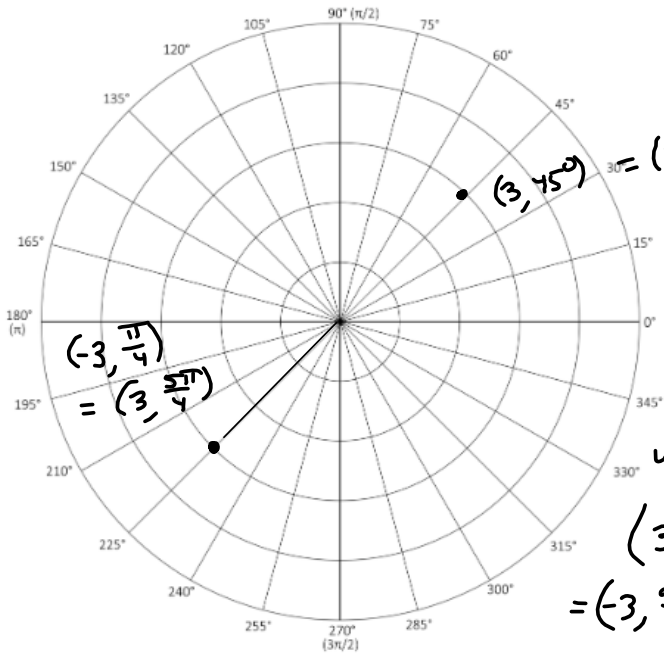
In these sections, 6.7 and 6.8, we allow $r < 0$ and interpret it as a negative direction

1. $r = directed\ distance$ from O to P
2. $\theta = directed\ angle$, counterclockwise from the polar axis to segment $\overline{OP}$



x-axis is Polar Axis

y-axis is $\theta = \frac{\pi}{2}$



$= (3, \frac{\pi}{4})$

Prefer π radians where possible.

r can be negative.

Representation is not unique!

$(3, \frac{5\pi}{4}) = (-3, \frac{\pi}{4}) = (3, \frac{13\pi}{4}) = (3, \frac{21\pi}{4})$

$= (-3, \frac{9\pi}{4}) = (-3, \frac{17\pi}{4}) = (-3, -\frac{7\pi}{4}) = ($

$= (3, -\frac{3\pi}{4}) =$ same point!

Converting rectangular to polar and vice-versa

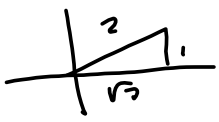
$r^2 = \sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$

$(x,y) = (-5\sqrt{3}, -5)$

$r = \sqrt{(5\sqrt{3})^2 + 5^2} = \sqrt{25\cdot 3 + 25} = \sqrt{100} = 10$

$\tan\theta = \frac{y}{x} = \frac{-5}{-5\sqrt{3}} = \frac{1}{\sqrt{3}}$



Temptation is to just write $\theta = \arctan\left(\frac{1}{\sqrt{3}}\right)$, but that's the wrong quadrant.

$\theta = \pi + \arctan\left(\frac{1}{\sqrt{3}}\right)$

$= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

So $(r,\theta) = \left(10, \frac{7\pi}{6}\right)$

$= \left(-10, \frac{\pi}{6}\right)$

From Polar to Rectangular

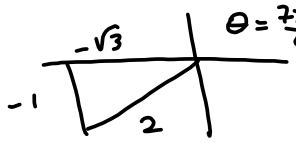
$x = r\cos\theta$

$y = r\sin\theta$

Convert $\left(10, \frac{7\pi}{6}\right)$ to rectangular coords

$(x,y) = (r\cos\theta, r\sin\theta) = \left(10\cdot\cos\left(\frac{7\pi}{6}\right), 10\sin\left(\frac{7\pi}{6}\right)\right)$

$= \left(10\left(-\frac{\sqrt{3}}{2}\right), 10\left(-\frac{1}{2}\right)\right) = (-5\sqrt{3}, -5)$



Convert to polar coords

$xy = 16$

$r\cos\theta \cdot r\sin\theta = 16$

$r^2 \sin\theta\cos\theta = 16$

$r^2\left(\frac{\sin(2\theta)}{2}\right) = 16$

$r^2 = 16 \cdot 2 \cdot \frac{1}{\sin(2\theta)} = 32\csc(2\theta)$

$r^2 = 32\csc(2\theta)$

$x^2 + y^2 - 10x = 0$

$r^2 - 10(r\cos\theta) = 0$

$\boxed{r^2 = 10\cos\theta}$