Final is May 11th. Let me know if you need to take it somewhere other than Horizon Hall in Greeley.

Send me an e-mail on D2L and I'll send your test to the testing center of your choice, and we'll hook you up by this Friday.

Section 6.9 is totally optional. (Conic sections in polar coordinates)

Today, Finish off Section 4.5 - DeMoivre's Theorem.

Want to kick off Section 6.6 - Parametric Equations.

DeMoivre's Theorem

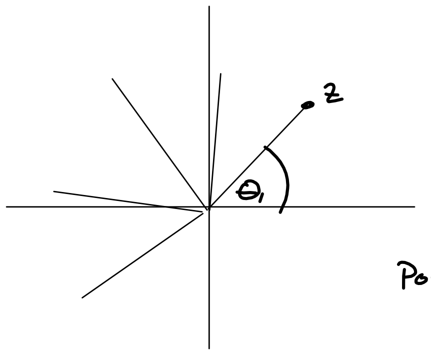
$z_1 = r_1(\cos(\theta_1) + i\sin(\theta_1))$ $\longrightarrow$

$z_2 = r_2(\cos(\theta_2) + i\sin(\theta_2))$

$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$

$z_2 = \cos(45^\circ) + i\sin(45^\circ)$

$|z_2| = \sqrt{\cos^2(45^\circ) + \sin^2(45^\circ)} = 1$



Assume $r = 1$

$z = r(\cos\theta + i\sin\theta)$, then

$z^2 = r^2(\cos(2\theta) + i\sin(2\theta))$

$= \cos(2\theta) + i\sin(2\theta)$

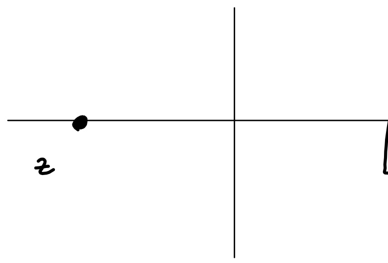
Powers

Roots:

Recall: Principle square root for real #s is the 1^{st} or positive square root.

Notice if $\theta = 180°$.

Then $\frac{\theta}{2} = 90°$



$\sqrt{z} = \sqrt{r}\left(\cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)\right)$

$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\theta/n\right)\right)$

$(\sqrt{z})^2 = \left(\sqrt{r}\left(\cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)\right)\right)^2$

$= (\sqrt{r})^2(\cos(\theta) + i\sin(\theta)) = z$

$\frac{\theta}{n}$ = increment = argument of the Principle n^{th} root.

There are another $n-1$ n^{th} roots.

Find 3^{rd} roots of $z = 8(\cos(90°) + i\sin(90°))$

$K=0$ $\sqrt[3]{z} = \sqrt[3]{8}\left(\cos\left(\frac{90°}{3}\right) + i\sin(30°)\right)$ is 1^{st}

$K=1$ $\sqrt[3]{8}(\cos(30° + 30°) + i\sin(60°))$

$K=2$ $\sqrt[3]{8}(\cos($

$\frac{\theta}{n} + \frac{2K\pi}{n}$

Increment is $\frac{2\pi}{n}$, in this case $\frac{2\pi}{3}$

$30° \rightarrow \frac{\pi}{6}$

$K=0$ $\frac{\pi}{6}$

$K=1$ $\frac{\pi}{6} + \frac{2\pi}{3} = \frac{\pi}{6} + \frac{4\pi}{6} = \frac{5\pi}{6}$

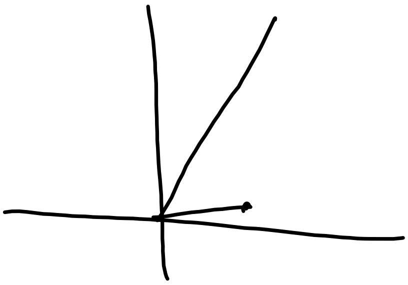
$K=2$ $\frac{5\pi}{6} + \frac{4\pi}{6} = \frac{9\pi}{6}$

$K=3$ $\frac{9\pi}{6} + \frac{4\pi}{6} = \frac{13\pi}{6} = \frac{12\pi}{6} + \frac{\pi}{6} = 2\pi + \frac{\pi}{6}$

Fifth Roots

5th Roots

$$z = 32\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$



$\frac{3\pi}{5}$ = inc.

$$\frac{\frac{\pi}{3}}{5} = \frac{\pi}{15}$$

$k=0$ $\quad \sqrt[5]{32}\left(\cos\left(\frac{\pi}{15}\right) + i\sin\left(\frac{\pi}{15}\right)\right)$

$\frac{\pi}{15} + \frac{\pi}{5} = \frac{\pi}{15} + \frac{3\pi}{15}$

4

7

10

13

16

$\frac{16\pi}{15}$ =

Add $\frac{2\pi}{5}$ not $\frac{\pi}{5}$

Try again

$\frac{\pi}{15}$ $\quad k=0$

$\frac{\pi}{15} + \frac{2\pi}{5}\cdot\frac{3}{3} = \frac{\pi}{15} + \frac{6\pi}{15} = \frac{7\pi}{15}$ $\quad 1$

$\frac{13\pi}{15}$ $\quad 2$

$\frac{19\pi}{15}$ $\quad 3$

$\frac{25\pi}{15}$ $\quad 4$

$\frac{31\pi}{15}$ check.

$$2\left(\cos\left(\frac{\pi}{15}\right) + i\sin\left(\frac{\pi}{15}\right)\right)$$

$$2\left(\cos\left(\frac{7\pi}{15}\right) + i\sin\left(\frac{7\pi}{15}\right)\right)$$

$$2\left(\cos\left(\frac{13\pi}{15}\right) + i\sin\left(\frac{13\pi}{15}\right)\right)$$

$$2\left(\cos\left(\frac{19\pi}{15}\right) + i\sin\left(\frac{19\pi}{15}\right)\right)$$

$$2\left(\cos\left(\frac{25\pi}{15}\right) + i\sin\left(\frac{25\pi}{15}\right)\right) = \boxed{2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)}$$

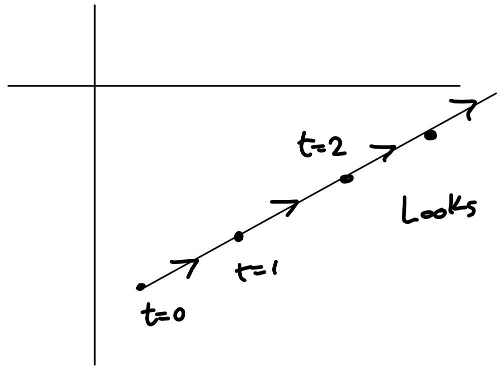
$2\left(\cos\left(\frac{31\pi}{15}\right) + i\sin\left(\frac{31\pi}{15}\right)\right)$ = Full circle check.

Parametric Equations are a way to make everything depend on one variable, like time.

$x = 2t + 1$

$y = t - 5$

t	x	y
0	1	-5
1	3	-4
2	5	-3
3	7	-2



Looks Like a line.

Eliminate parameter by solving one of the eq'ns for 't' and substitute into the other eq'n.

$y + 5 = t$

$x = 2(y+5) + 1 = 2y + 10 + 1 = 2y + 11$

$\Rightarrow 2y = x - 11$

$y = \frac{x-11}{2} = \frac{1}{2}x - \frac{11}{2}$

ORIENTATION – Draw an arrow in the direction of increasing t.

$x = \cos\theta$

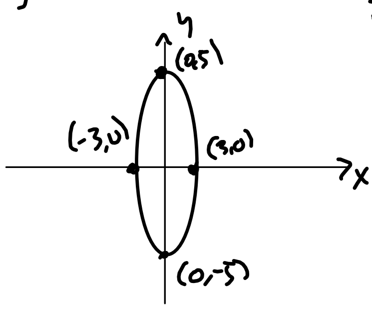
$y = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$x^2 + y^2 = 1$

$x = 3\cos\theta \longrightarrow \frac{x}{3} = \cos\theta \longrightarrow \left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

$y = 5\sin\theta \qquad \frac{y}{5} = \sin\theta$



$c^2 = a^2 - b^2 = $ (Focal Length)

$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1 \qquad a = 5, \; b = 3$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$x = t + 2$ $\Longrightarrow$ $x - 2 = t$ $\Longrightarrow$ $y = (x-2)^2 - 2(x-2)$

$y = t^2 - 2t$

$= x^2 - 4x + 4 - 2x + 4$

$= x^2 - 6x + 8$

$= x^2 - 6x + 3^2 - 9 + 8$

$= (x-3)^2 - 1$