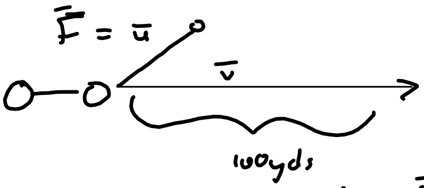
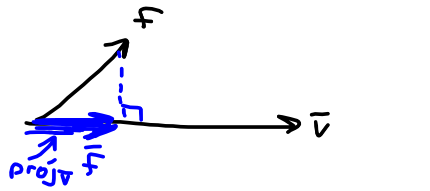




$\bar{u} = 30\langle \cos 45°, \sin 45° \rangle = \bar{F} = \langle \frac{30\sqrt{2}}{2}, \frac{30\sqrt{2}}{2} \rangle = \langle 15\sqrt{2}, 15\sqrt{2} \rangle$

$\bar{v} = (100\text{yds})\left(\frac{3\text{ft}}{1\text{yd}}\right)\langle 1, 0 \rangle$

$= \langle 300, 0 \rangle$

Work $= \bar{F} \cdot \bar{v} = \langle 15\sqrt{2}, 15\sqrt{2} \rangle \cdot \langle 300, 0 \rangle$

$= (45\sqrt{2})(100) = 4500\sqrt{2}$ ft-lbs.

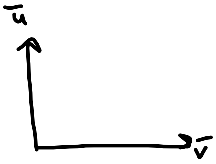
Component of $\bar{F}$ in the direction of $\bar{v}$ times $\|\bar{v}\|$

$\bar{F} = \langle 15\sqrt{2}, 15\sqrt{2} \rangle$

$\text{proj}_{\bar{v}}\bar{F} = \langle 15\sqrt{2}, 0 \rangle$

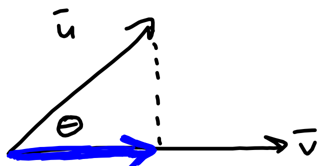
Work $= (\text{proj}_{\bar{v}}\bar{F}) \cdot \bar{v}$

= "Force times distance."

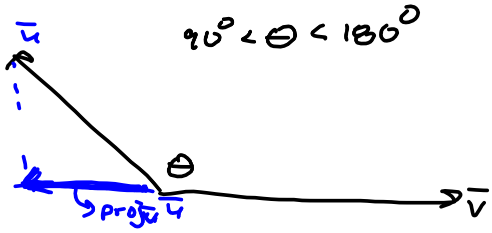


$\text{proj}_{\bar{v}}\bar{u} = \bar{0}$

$0 < \theta < 90^\circ$



$90^\circ < \theta < 180^\circ$



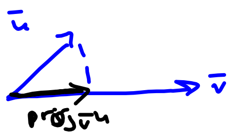
$\text{proj}_{\bar{v}}\bar{u}$ = Projection of $\bar{u}$ onto $\bar{v}$

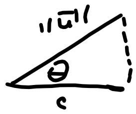
= Component of $\bar{u}$ in the direction of $\bar{v}$.

= Shadow $\bar{u}$ casts on $\bar{v}$ times a unit vector in the direction of $\bar{v}$

$\text{proj}_{\bar{v}}\bar{u} = -\bar{u}$

$$\text{proj}_{\bar{v}}\bar{u} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v}$$





$$\frac{c}{\|\bar{u}\|} = \cos\theta \longrightarrow c = \|\bar{u}\|\cos\theta$$

$$\text{proj}_{\bar{v}}\bar{u} = \underbrace{\left(\|\bar{u}\|\cos\theta\right)}_{\text{size}}\underbrace{\frac{\bar{v}}{\|\bar{v}\|}}_{\text{direction}}$$

$$= \left(\cancel{\|\bar{u}\|}\frac{\bar{u}\cdot\bar{v}}{\cancel{\|\bar{u}\|}\|\bar{v}\|}\right)\frac{\bar{v}}{\|\bar{v}\|}$$

$$\frac{1}{\|\bar{v}\|}\bar{v}$$

$$= \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v}$$

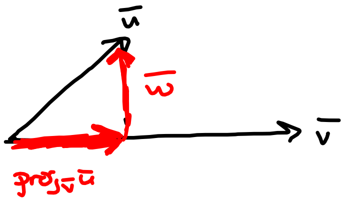
is fine for $0 < \theta < 90^\circ$

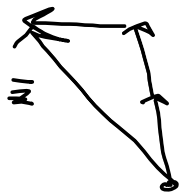
For $90^\circ < \theta < 180^\circ$, $\text{proj}_{\bar{v}}\bar{u}$ is in the opposite direction from $\bar{v}$.

$\theta = 0^\circ \implies \text{proj}_{\bar{v}}\bar{u} = \bar{u}$

$\theta = 90^\circ \implies \text{proj}_{\bar{v}}\bar{u} = \bar{0} = \langle 0, 0 \rangle$

$\theta = 180^\circ \longrightarrow \text{proj}_{\bar{v}}\bar{u} = -\bar{u}$





$\bar{z} = 2\bar{w} - 1\,\text{proj}_{\bar{v}}\bar{u}$

NOTE: $\bar{u} = \text{proj}_{\bar{v}}\bar{u} + \bar{w}$

$\bar{w} = ? = \bar{u} - \text{proj}_{\bar{v}}\bar{u}$ = Decomposition of $\bar{u}$ into the sum of a vector parallel to $\bar{v}$ and a vector perpendicular (orthogonal) to $\bar{v}$.

This gives us an orthogonal basis for $\mathbb{R}^2$ (the Cartesian Plane)

Canonical Basis for $\mathbb{R}^2$ is

$\{\langle 1,0\rangle, \langle 0,1\rangle\} = \{e_1, e_2\}$

Basis "spans" $\mathbb{R}^2$:

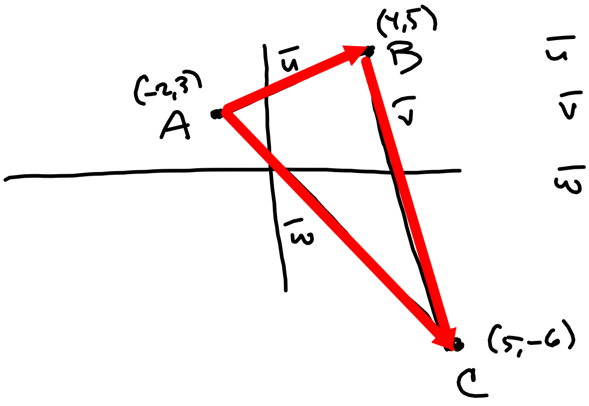
$\bar{u} = \langle u_1, u_2\rangle = u_1\langle 1,0\rangle + u_2\langle 0,1\rangle = u_1e_1 + u_2e_2$

Linear Combo on e_1, e_2

Basis ALSO means it's a minimal spanning set.

$\mathbb{R}^3$: $\langle 1,0,0\rangle, \langle 0,1,0\rangle, \langle 0,0,1\rangle = \bar{i}, \bar{j}, \bar{k}$, respectively

$\langle 3,5,6\rangle = 3\bar{i} + 5\bar{j} + 6\bar{k}$



$\bar{u} = \langle 4-(-2), 5-3 \rangle = \langle 6, 2 \rangle = \bar{u}$

$\bar{v} = \langle 5-4, -6-5 \rangle = \langle 1, -11 \rangle = \bar{v}$

$\bar{w} = \langle 5-(-2), -6-3 \rangle = \langle 7, -9 \rangle$

$\|\bar{u}\| = \sqrt{6^2+2^2} = \sqrt{40} = 2\sqrt{10}$

$\|\bar{v}\| = \sqrt{1^2+11^2} = \sqrt{122}$

$\|\bar{w}\| = \sqrt{7^2+9^2} = \sqrt{49+81} = \sqrt{130}$

$2\,|\,130$
$5\,|\,65$
13

$$\cos A = \frac{\bar{u}\cdot\bar{w}}{\|\bar{u}\|\|\bar{w}\|} = \frac{\langle 6,2 \rangle \cdot \langle 7,-9 \rangle}{(2\sqrt{10})(\sqrt{130})}$$

$$= \frac{42-18}{2\sqrt{1300}} = \frac{24}{20\sqrt{13}} = \frac{6}{5\sqrt{13}}$$

$$\cos B = \frac{(-\bar{u})\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} \rightarrow \frac{\langle -6,-2 \rangle \cdot \langle 1,-11 \rangle}{(2\sqrt{10})(\sqrt{122})} \Rightarrow \frac{-6+22}{2\sqrt{1220}} \Rightarrow \frac{16}{2\sqrt{1220}}$$

$2\,|\,1220$
$2\,|\,610$
$5\,|\,305$
61

$$\Rightarrow \frac{8}{\sqrt{1220}} \Rightarrow \frac{8}{2\sqrt{305}} \Rightarrow \frac{4}{\sqrt{305}} \Rightarrow \cos B \longrightarrow \boxed{B \approx 76.7594800848^\circ}$$

$$\& \cos A \Rightarrow \frac{6}{5\sqrt{13}} \longrightarrow \boxed{A \approx 70.5599651718^\circ}$$

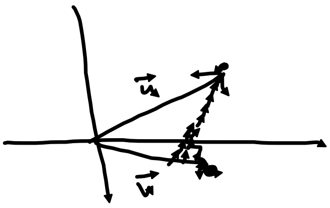
$$\longrightarrow C = 180^\circ - A - B \approx \boxed{32.6805547434^\circ \approx C}$$

109.10721471

To 3 places:

$$\boxed{\begin{aligned} A &\approx 70.560^\circ \\ B &\approx 76.759^\circ \\ C &\approx 32.681^\circ \end{aligned}}$$

Let $\vec{u} = \langle 8, 5 \rangle$, $\vec{v} = \langle 7, -1 \rangle$



$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v}$$

$$= \frac{56-5}{\sqrt{8^2+5^2}\sqrt{7^2+1^2}}\vec{v} = \frac{51}{\sqrt{89}\sqrt{50}}\vec{v} = \frac{51}{5\sqrt{89}\sqrt{2}}$$

$$= \frac{51}{5\sqrt{178}}\vec{v} = \frac{51\sqrt{178}}{5(178)}\vec{v} = \boxed{\frac{51\sqrt{178}}{890}\langle 7, -1 \rangle}$$

$$\frac{\cancel{51}^{17}}{\cancel{890}_{330}} \qquad = \boxed{\frac{17\sqrt{178}}{330}\langle 7, -1 \rangle} \qquad \frac{1780}{2} = 890 \qquad 2\,|\,50,\ 5\,|\,25,\ 5$$