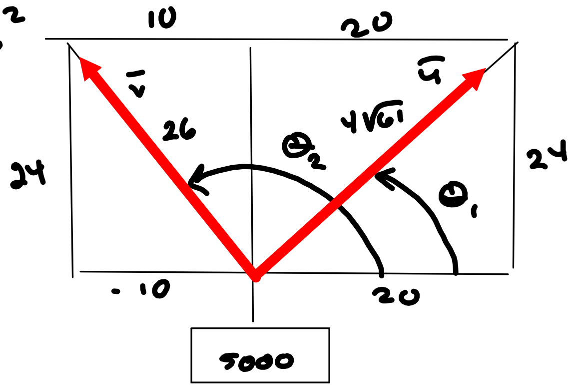
$10^2+24^2=676=26^2$

$20^2+24^2=976$

$$\begin{array}{r|l} 2 & 976 \\ 2 & 488 \\ 2 & 244 \\ 2 & 122 \\ & 61 \end{array}$$

Let $x=\|\bar{u}\|$

& $y=\|\bar{v}\|$



$\bar{u}=\|\bar{u}\|\langle\cos\theta_1,\sin\theta_1\rangle$

$\bar{v}=\|\bar{v}\|\langle\cos\theta_2,\sin\theta_2\rangle$

$\bar{w}=\bar{u}+\bar{v}=\langle 0,5000\rangle$

$\bar{u}+\bar{v}=\bar{w}$

$\bar{u}=x\langle\cos\theta_1,\sin\theta_1\rangle=x\langle\cos(\arctan(\frac{24}{20})),\sin(\arctan(\frac{24}{20}))\rangle$

$=x\langle\frac{20}{4\sqrt{61}},\frac{24}{4\sqrt{61}}\rangle=\langle\frac{5}{\sqrt{61}}x,\frac{6}{\sqrt{61}}x\rangle$

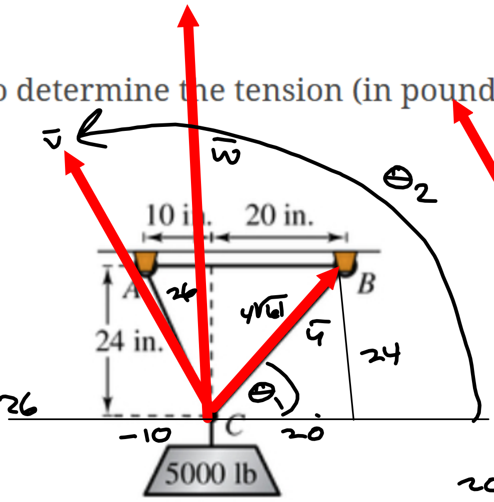
$\bar{v}=y\langle\cos\theta_2,\sin\theta_2\rangle=y\langle-\frac{10}{26},\frac{24}{26}\rangle=\langle-\frac{5}{13}y,\frac{6}{13}y\rangle$

$\bar{u}+\bar{v}=\langle\frac{5}{\sqrt{61}}x-\frac{5}{13}y,\frac{6}{\sqrt{61}}x+\frac{6}{13}y\rangle=\langle 0,5000$

$\left(\frac{5}{\sqrt{61}}x-\frac{5}{13}y=0\right)(6)\qquad\frac{30}{\sqrt{61}}x-\frac{30}{13}y=0$

86. Physics Use the figure to determine the tension (in pounds) in each cable supporting the load.

Section 3.3 II - Applications.



$10^2 + 24^2 = 100 + 576 = 676$

2 | 676
2 | 338
13 | 169
13

$\sqrt{676} = 2 \cdot 13 = 26$

$20^2 + 24^2$
$= 400 + 576$
$= 976 \Rightarrow$
$\sqrt{976} = 4\sqrt{61}$

24×24: 96, 48, 576

2 | 976
2 | 488
2 | 244
2 | 122
61

$\bar{w} = \bar{u} + \bar{v} = \langle 0, 5000 \text{ lbs} \rangle$

$\bar{u} = \|\bar{u}\| \langle \cos\theta_1, \sin\theta_1 \rangle = \|\bar{u}\| \langle \frac{20}{4\sqrt{61}}, \frac{24}{4\sqrt{61}} \rangle =$

$= \boxed{\|\bar{u}\| \langle \frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}} \rangle}$

Let $x = \|\bar{u}\|$ = magnitude of $\bar{u}$ = Tension in the right cable (lbs)

$\bar{v} = \|\bar{v}\| \langle \cos\theta_2, \sin\theta_2 \rangle = y \langle -\frac{10}{26}, \frac{24}{26} \rangle = y \langle -\frac{5}{13}, \frac{12}{13} \rangle$,

where $\boxed{y = \|\bar{v}\| \text{, in lbs}}$

$\bar{u} + \bar{v} = \bar{w} \Longrightarrow$

$x \langle \frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}} \rangle + y \langle -\frac{5}{13}, \frac{12}{13} \rangle = \langle 0, 5000 \rangle$

$$\frac{5}{\sqrt{61}}x - \frac{5}{13}y = 0 \longrightarrow \frac{5}{\sqrt{61}}x = \frac{5}{13}y \longrightarrow$$

$$\frac{6}{\sqrt{61}}x + \frac{12}{13}y = 5000 \qquad x = \frac{\sqrt{61}}{5}\cdot\frac{5}{13}y = \boxed{\frac{\sqrt{61}}{13}y = x}$$

$$\longrightarrow \frac{6}{\sqrt{61}}x + \frac{12}{13}y = \left(\frac{6}{\sqrt{61}}\right)\left(\frac{\sqrt{61}}{13}y\right) + \frac{12}{13}y =$$

$$= \frac{6}{13}y + \frac{12}{13}y = \frac{18}{13}y = 5000 \longrightarrow$$

$$y = (5000)\left(\frac{13}{18}\right) = \frac{(2500)(13)}{9}$$

$$\approx \boxed{3611.11111111 \text{ lbs} \approx y} \longrightarrow$$

$$x = \left(\frac{\sqrt{61}}{13}\right)(y) = \left(\frac{\sqrt{61}}{\cancel{13}}\right)\left(\frac{2500\cancel{(13)}}{9}\right) = \frac{(\sqrt{61})(2500)}{9}$$

$$\approx \boxed{2169.51379886 \text{ lbs} \approx x}$$

Check:

$$\bar{u} + \bar{v} = \langle 0, 5000 \rangle \, ?$$

$$\bar{u} \approx 2169.51379886 \left\langle \frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle \approx \langle 1388.88888889, 1666.66666666 \rangle$$

$$\bar{v} \approx 3611.11111111 \left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle \approx \langle -1388.88888889, 3333.33333333 \rangle$$

$$\approx \langle 0, 5000 \rangle \checkmark$$

Section 3.4 - Dot Product

$\bar{u} = \langle 1, 3 \rangle$

$\bar{v} = \langle -5, 2 \rangle$

In general:

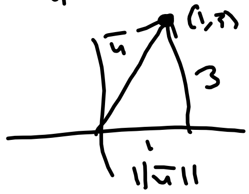
$\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$ $\rightarrow$ $\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2$

& $\bar{u} \cdot \bar{u} = u_1 u_1 + v_1 u_1$

→ The dot product of $\bar{u}$ & $\bar{v}$ is

$\bar{u} \cdot \bar{v} = (1)(-5) + (3)(2) = -5 + 6 = 1$

$\bar{u} \cdot \bar{u} = 1^2 + 3^2 = 10 = \|\bar{u}\|^2$, since $\|\bar{u}\| = \sqrt{1^2 + 3^2}$, by Pythagorus.



$1^2 + 3^2 = 10$

$\sqrt{10} = \|\bar{u}\|$

$(3\bar{u}) \cdot (5\bar{v}) = 15\,(\bar{u} \cdot \bar{v})$

$3\langle u_1, u_2 \rangle = \langle 3u_1, 3u_2 \rangle$

$\bar{u} \cdot \bar{v} = \bar{v} \cdot \bar{u}$ so Dot product is commutative.*

Dot product is a <u>number</u>, not a vector.

* by inheritance from commutativity of multiplication of real #s.

$$\bar{u} \cdot \bar{v} = \underbrace{u_1 v_1 + u_2 v_2 = v_1 u_1 + v_2 u_2}_{\text{commutativity of multiplication of real \#s}} = \bar{v} \cdot \bar{u}$$

It also obeys a distributive law

$\bar{u} \cdot (\bar{v} + \bar{w}) = \bar{u} \cdot \bar{v} + \bar{u} \cdot \bar{w}$

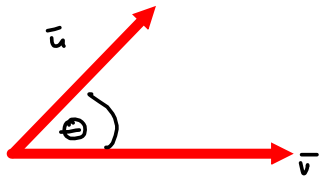
$\langle u_1, u_2 \rangle \cdot (\langle v_1 + w_1, v_2 + w_2 \rangle$

$= u_1 (v_1 + w_1) + u_2 (v_2 + w_2)$

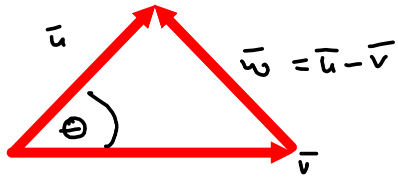
$= \underline{u_1 v_1} + \underline{u_1 w_1} + \underline{u_2 v_2} + \underline{u_2 w_2}$

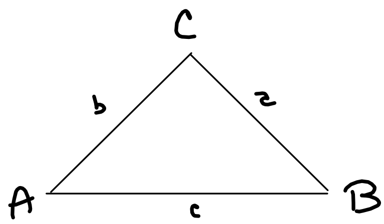
$= \underbrace{u_1 v_1 + u_2 v_2} + \underbrace{u_1 w_1 + u_2 w_2}$

$= \bar{u} \cdot \bar{v} + \bar{u} \cdot \bar{w}$



$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$ Proof by Law of Cosines





$a^2 = b^2 + c^2 - 2bc\cos A$

$\|\bar{u}-\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$

$(\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v}) = \bar{u}\cdot\bar{u} \underbrace{-\bar{u}\cdot\bar{v} - \bar{v}\cdot\bar{u}} - \bar{v}\cdot(-\bar{v})$

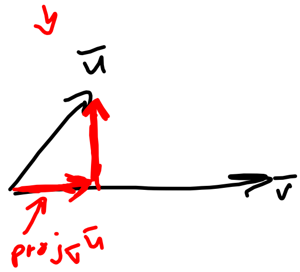
$= \|\bar{u}\|^2 \cancel{-2\bar{u}\cdot\bar{v}} + \|\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 \cancel{-2}\|\bar{u}\|\|\bar{v}\|\cos\theta$

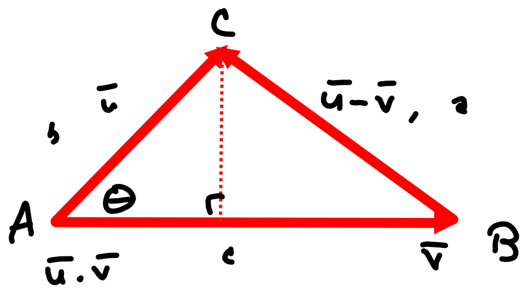
$\rightarrow \quad \bar{u}\cdot\bar{v} = \|\bar{u}\|\|\bar{v}\|\cos\theta \quad \rightarrow$

$$\boxed{\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}}$$

$\boxed{\text{Next up: } \operatorname{proj}_{\bar{v}}\bar{u}}$

Decomposing into Orthogonal Components





$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\|\bar{u}-\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

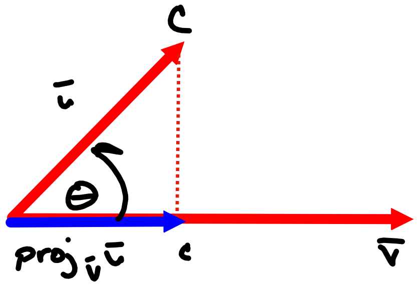
$$(\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v}) = \bar{u}\cdot\bar{u} - \bar{u}\cdot\bar{v} - \bar{u}\cdot\bar{v} + \bar{v}\cdot\bar{v}$$

$$= \|\bar{u}\|^2 - 2\bar{u}\cdot\bar{v} + \|\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\Rightarrow -2\bar{u}\cdot\bar{v} = -2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

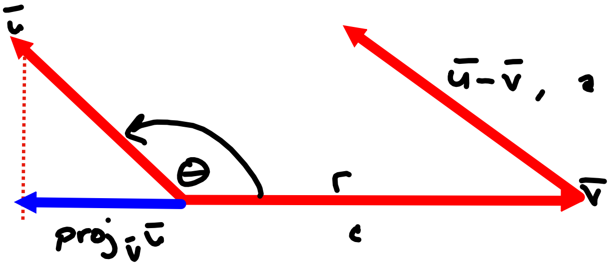
$$\Rightarrow \cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|v\|}$$

$0 < \theta < 90^\circ$



$\cos\theta > 0$

$90 < \theta < 180^\circ$



$\cos\theta < 0$

$\text{proj}_{\bar{v}}\bar{u} = \|\bar{u}\| \cos\theta \cdot$ (unit vector in direction of $\bar{v}$)

$$= \|\bar{u}\| \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} \cdot \frac{\bar{v}}{\|\bar{v}\|} = \boxed{\frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}\,\bar{v}} = \text{proj}_{\bar{v}}\bar{u}$$

Written Q7

$2\sin(4x) + 4\sin(2x)$

$= 4\sin(2x)\cos(2x) + 4\sin(2x)$

$= (8\sin(x)\cos x)(1 - 2\sin^2(x)) + 8\sin(x)\cos(x)$

$= \cos(x)\left[8\sin(x) - 16\sin^3(x) + 8\sin(x)\right]$

$= \sin(x)\cos(x)\left[8 - 16\sin^2(x) + 8\right] = \cos(x)\left[16 - 16\sin^2(x)\right]$

(a)

$= \sin(x)\cos(x)(16 - 16\sin^2(x)$

$= 16\sin(x)\cos(x)\cos^2(x) = 0$

$\rightarrow \sin(x) = 0$ or $\cos^3(x) = 0$

$x = 0, \pi$ $\quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{x \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}}$

(b) $\boxed{x \in \left\{\frac{n\pi}{2} \mid n \in \mathbb{Z}\right\}}$

Degrees Answers:

$\boxed{x = 0, 90°, 180°, 270°}$

Display sol'n set.

$\boxed{\left\{90°n \mid n \in \mathbb{Z}\right\}}$