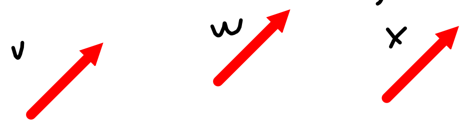
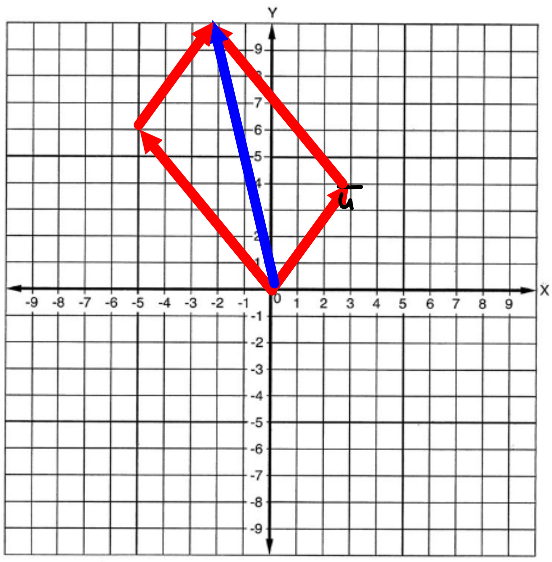
§3.3 Vectors

A vector is a directed line segment of a specified length



Standard position:
Butt at the origin.
That will represent ALL vectors of that length and direction.



$\bar{u}$ = the equivalence class of all vectors with that length and direction.

$\bar{u} = \langle 3, 4 \rangle$

Vector Addition
(Parallelogram Law)

Let $\bar{v} = \langle -5, 6 \rangle$

$\bar{u} + \bar{v} = \langle 3 + (-5), 4 + 6 \rangle$
$= \langle -2, 10 \rangle$
= "resultant" = the vector formed by placing the butt of $\bar{v}$ to the tip of $\bar{u}$.

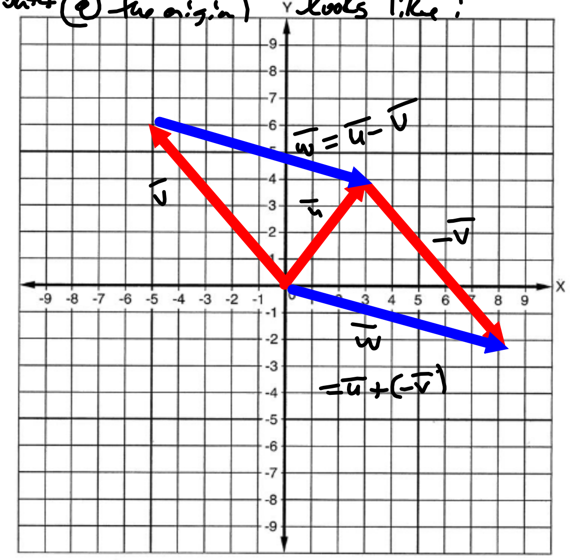
Note: $\bar{u} + \bar{v} = \bar{v} + \bar{u}$

subtraction of vectors

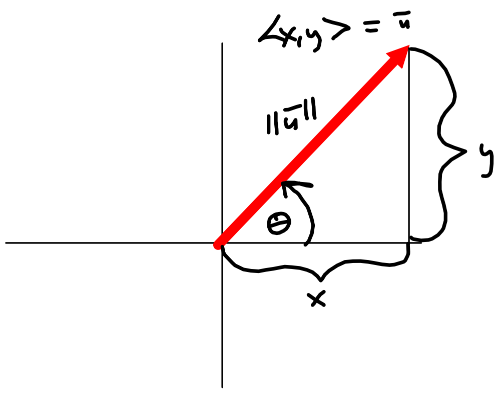
$\bar{u} - \bar{v} = \bar{u} + (-\bar{v}) = \bar{w}$

= vector from tip of $\bar{v}$ to the tip of $\bar{u}$
(and then put its butt @ the origin)

Remember what $\bar{u} - \bar{v}$ looks like!



Direction Angle of $\bar{u}$ is the angle it makes with the positive x-axis, measured counter-clockwise.



Direction Angle θ of $\bar{u}$ is the angle it makes with the positive x-axis, measured counter-clockwise.

Magnitude of $\|\bar{u}\|$ is its length

Notice $\frac{y}{\|\bar{u}\|} = \sin\theta \longrightarrow$ $\boxed{y = \|\bar{u}\| \sin\theta}$

$\frac{x}{\|\bar{u}\|} = \cos\theta \longrightarrow$ $\boxed{x = \|\bar{u}\| \cos\theta}$

$\bar{u} = \langle x, y\rangle = \langle \|\bar{u}\|\cos\theta, \|\bar{v}\|\sin\theta\rangle$

$= \|\bar{u}\| \langle \cos\theta, \sin\theta\rangle$

$\|\bar{u}\| = \sqrt{(3-0)^2 + (4-0)}$

$= \sqrt{3^2+4^2}$

Properties of Vectors

Scalar multiples

$3\bar{u} = 3\langle 3, 4\rangle$

$= \langle 3\cdot 3, 3\cdot 4\rangle$

$= \langle 9, 12\rangle$

$= 3\langle 3, 4\rangle$

$\|\bar{u}\| = \|\langle x, y\rangle\|$

$$\boxed{\|\bar{u}\| = \sqrt{x^2+y^2}}$$

Let $\bar{v} = \langle \cos\theta, \sin\theta\rangle$

$\longrightarrow \|\bar{v}\| = \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$!

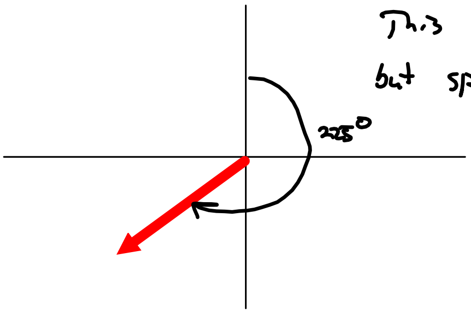
A vector of magnitude $r = 1$ is called a UNIT VECTOR

ANY vector $\bar{u}$ may be represented as the scalar product of $\|\bar{u}\|$ and the unit vector $\langle \cos\theta, \sin\theta\rangle$ where θ = direction angle of $\bar{u}$

just a number

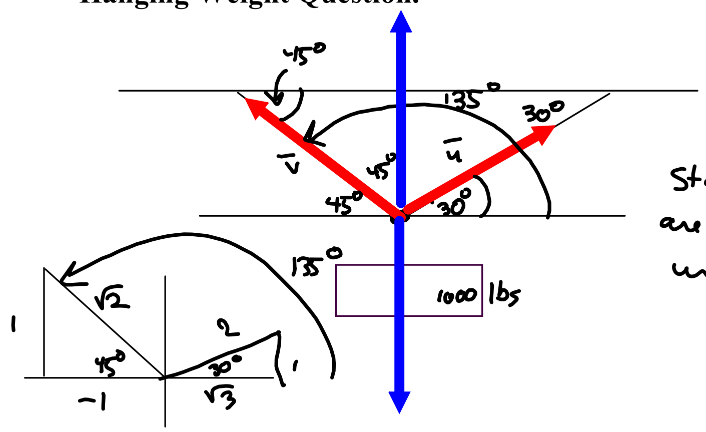
$\bar{u} = \|\bar{u}\| \langle \cos\theta, \sin\theta\rangle$

NAVIGATION
Bearings : " A heading of 225° means
225° East of North
(measured clockwise from Due North



This is NOT Direction angle,
but specific to navigation.

Hanging Weight Question.



Need Direction ANGLES!

Stationary weight, so forces are perfectly balanced and unchanging

Forces Balanced, so $\bar{u} + \bar{v} = 1000\langle 0, 1\rangle$, because the weight is 1000 lbs straight down

Find the tension in both of the cables, $\|\bar{u}\|, \|\bar{v}\|$

Let $x = \|\bar{u}\|$
$y = \|\bar{v}\|$

Then $\bar{u} = \|\bar{u}\| \langle \cos 30^\circ, \sin 30^\circ \rangle = \langle \frac{\sqrt{3}}{2}x, \frac{1}{2}x \rangle = x\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$\bar{v} = \|\bar{v}\| \langle \cos 135^\circ, \sin 135^\circ \rangle = \langle -\frac{1}{\sqrt{2}}y, \frac{1}{\sqrt{2}}y \rangle = y\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

FACT: $\bar{u} + \bar{v} = \langle 0, 1000 \rangle$ = resultant.

$x\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle + y\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \langle 0, 1000 \rangle$

$\Rightarrow \langle \frac{\sqrt{3}}{2}x, \frac{1}{2}x \rangle + \langle -\frac{1}{\sqrt{2}}y, \frac{1}{\sqrt{2}}y \rangle = \langle 0, 1000 \rangle$

$\Rightarrow \frac{\sqrt{3}}{2}x - \frac{1}{\sqrt{2}}y = 0$
$\frac{1}{2}x + \frac{1}{\sqrt{2}}y = 1000$

Dummy! the y's are all ready to be cancelled in step!!

$$\begin{aligned} &\tfrac{\sqrt{3}}{2}x - \tfrac{1}{\sqrt{2}}y = 0 \\ +\ &\tfrac{1}{2}x + \tfrac{1}{\sqrt{2}}y = 1000 \\ \hline &\left(\tfrac{\sqrt{3}}{2} + \tfrac{1}{2}\right)x = 1000 \end{aligned}$$

$x = \dfrac{1000}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \dfrac{2000}{\sqrt{3}+1}$ ✓

$\boxed{\|\bar{u}\| = \dfrac{2000}{\sqrt{3}+1}}$

$x = \dfrac{2000}{1+\sqrt{3}}$ and $y = \dfrac{1000\sqrt{6}}{1+\sqrt{3}}$, i.e.,

$x \approx 732.05$ and $y \approx 896.58$

Command on WolframAlpha.com

Solve(sqrt(3)/2*x-1/sqrt(2)*y=0,1/2*x+1/sqrt(2)*y=1000)

TI-84 or TI-83 can handle this, digitally, with Matrix commands, in particular rref(A), where A is the coefficient matrix.

By hand:

$$E1 \left(\frac{\sqrt{3}}{2}x - \frac{1}{\sqrt{2}}y = 0 \right)\frac{2}{\sqrt{3}} \Longrightarrow x - \frac{2}{\sqrt{6}}y = 0 \quad E1$$

$$E2 \left(\frac{1}{2}x + \frac{1}{\sqrt{2}}y = 1000 \right)\frac{1}{2} \Longrightarrow -\left(x + \frac{1}{2\sqrt{2}}y = 500\right) \quad E2$$

$$E1 - E2: \ -\frac{2}{\sqrt{6}}y - \frac{1}{2\sqrt{2}}y = -500$$

Messed-up Somewhere

$$\Longrightarrow \left(-\frac{2}{\sqrt{6}} - \frac{1}{2\sqrt{2}}\right)y = -500$$

$$\Longrightarrow \quad y = \frac{-500}{-\frac{2}{\sqrt{6}} - \frac{1}{2\sqrt{2}}} = \frac{500}{\frac{2}{\sqrt{6}} + \frac{1}{2\sqrt{2}}}$$