**Please check e-mail on D2L for Week 6 Homework and Grade Report, post Midterm.**

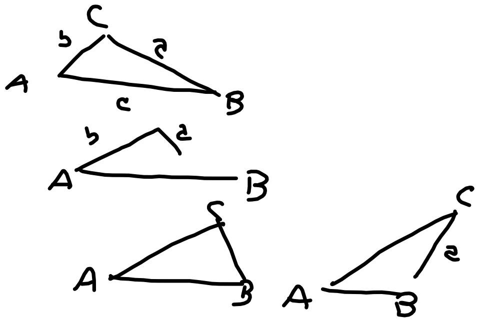
Recall: Law of Sines

"ASS" situation:

1 unique soln $h < b < a$

0 soln $a < h$

2 solns $h < a < b$




I think I messed-up the last example Law of Sines, 2-solution case

$A = 30^\circ$
$a = 8$
$b = 10$

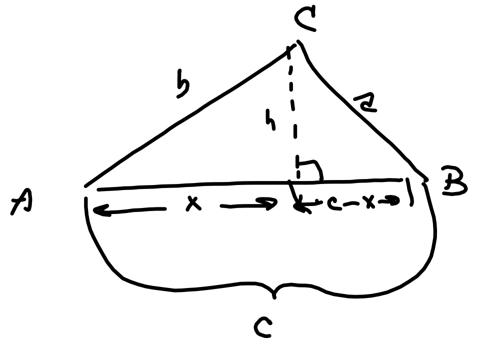
I mixed this up with

$A = 30^\circ$
$b = 10$
$a = 11$

from previous example

But the rest of the theory was pretty sound.

# Law of Cosines




Law of Cosines Says

$a^2 = b^2 + c^2 - 2bc\cos A$

$\frac{h}{a} = \sin B$ $\qquad$ $\frac{h}{c-x} = \cos B$

$h = a\sin B$

$(c-x)^2 = (c-x)(c-x)$

$= c^2 - xc - xc + x^2$

$= c^2 - 2cx + x^2$

$a^2 = h^2 + (c-x)^2$

$= (a\sin B)^2 + c^2 - 2cx + x^2$

$= a^2\sin^2 B + c^2 - 2c(b\cos A) + b^2\cos^2 A$

$= \underline{b^2\sin^2 A} + c^2 - 2bc\cos A + \underline{b^2\cos^2 A}$

$= b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc\cos A$

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ only version I remember

$\frac{x}{b} = \cos A$

$x = b\cos A$

$\frac{h}{b} = \sin A$

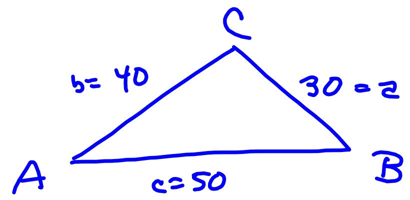
$h = b\sin A$

Likewise:

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Only way to solve SSS situation.




For THIS, we need to solve for cos A

$a^2 = b^2 + c^2 - 2bc\cos A$ ✓

$\Rightarrow a^2 - b^2 - c^2 = -2bc\cos A$

$\Rightarrow \frac{a^2 - b^2 - c^2}{-2bc} = \cos A$

$\Rightarrow \boxed{\frac{b^2 + c^2 - a^2}{2bc} = \cos A}$ Some put THIS on their Cheat Sheet.

$$\cos A = \frac{40^2+50^2-30^2}{2(40)(50)} = 0.8$$

$$\Rightarrow A = \cos^{-1}(0.8) = \arccos(0.8)$$

$$\Rightarrow \boxed{A \approx 36.8698976458^\circ}$$

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \sin B = \frac{b\sin A}{a} = \frac{40\sin A}{30} \approx 0.799999999999$$

$$\Rightarrow B \approx \arcsin(\sin B) \approx 53.1301023541^\circ \approx B$$

$$C = 180^\circ - A - B$$

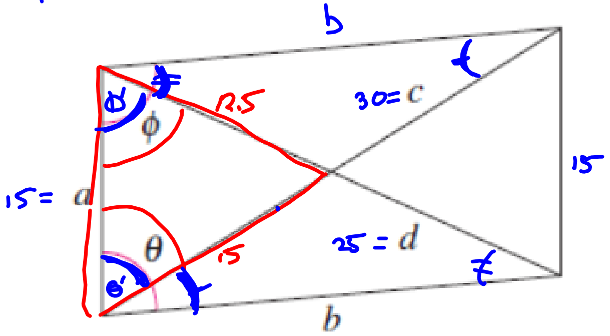
$$\approx 180^\circ - 36.8698976458^\circ - 53.1301023541^\circ$$

$$\approx 90.0000000001^\circ$$

To 3 decimal places:

$$\boxed{\begin{aligned} A &\approx 36.870^\circ \\ B &\approx 53.130^\circ \\ C &\approx 90.000^\circ \end{aligned}}$$

S3.2 #8




$\phi = ?$
$\theta = ?$
$b = ?$

$8^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \frac{b^2 + c^2 - 8^2}{2bc}$

$\cos\theta' = \frac{15^2 + 15^2 - 12.5^2}{2(15)^2}$

$\approx 0.652777777778$

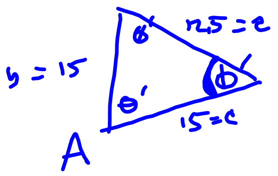
$A = \theta', \approx 49.2486367043°$

$$\frac{(15^2 + 15^2 - 12.5^2)}{2(15^2)} = 0.652777777778$$

$$\arccos(0.652777777778) = 49.2486367043$$

$a = 15, d = 25, b = ?, \theta = ?$

$\cos\theta' = 1$




Since △ is isosceles,
The other 2 angles are the same!

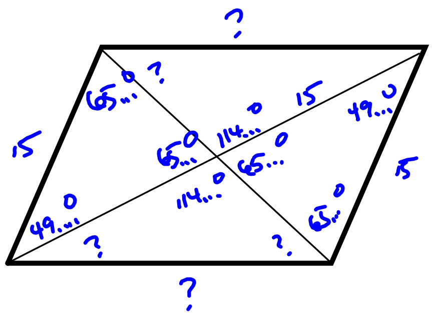
$180° - \theta' - 2\phi' = 0$

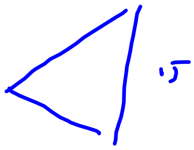
$180° - \theta' = 2\phi' \approx 130.751363296°$

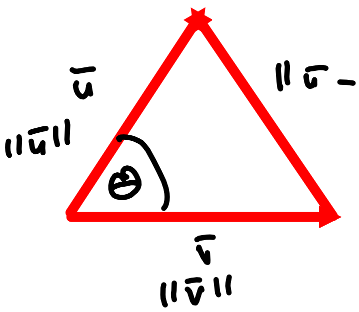
$\Rightarrow \boxed{\phi' \approx 65.375681648°}$

$180° - 65..°$

$\approx 114.624318352°$






$$\|\bar{u}-\bar{v}\| = \sqrt{(\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v})}$$

$$\|\bar{u}-\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

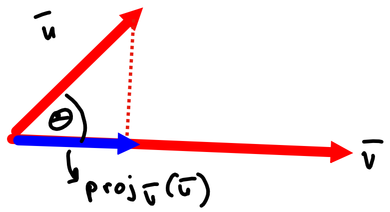
$$(\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v}) = \bar{u}\cdot\bar{u} - 2\bar{u}\cdot\bar{v} + \bar{v}\cdot\bar{v}$$

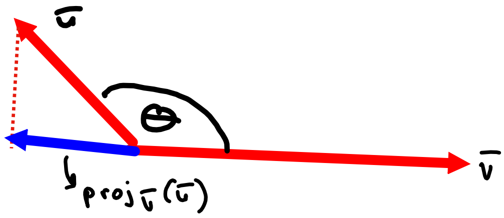
$$= \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\Longrightarrow \|\bar{u}\|^2 - 2\bar{u}\cdot\bar{v} + \|\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\Longrightarrow \quad \bar{u}\cdot\bar{v} = \|\bar{u}\|\|\bar{v}\|\cos\theta \quad \Longrightarrow$$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$




$0 \le \theta < 90^\circ$

$$\text{proj}_{\bar v}(\bar u) = \left(\|\bar u\| \cos\theta\right) \frac{\bar v}{\|\bar v\|}$$

$$= \|\bar u\| \frac{\bar u \cdot \bar v}{\|\bar u\|\|\bar v\|}\left(\frac{\bar v}{\|\bar v\|}\right) = \frac{\bar u \cdot \bar v}{\|\bar v\|^2}\, \bar v$$

$90^\circ < \theta \le 180^\circ$

$$\text{proj}_{\bar v}\bar u = \|\bar u\| \cos\theta \frac{\bar v}{\|\bar v\|}$$

$$= \dots = \frac{\bar u \cdot \bar v}{\|\bar v\|^2}$$

$$\theta = 90^\circ \Rightarrow \text{proj}_{\bar v}\bar u = \bar 0$$

$$\theta = 180^\circ \Rightarrow \text{proj}_{\bar u}\bar v = -\bar u$$