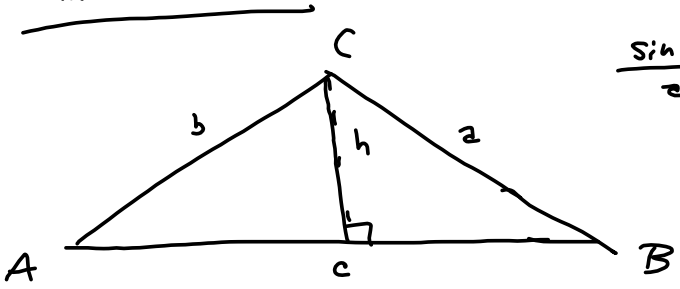
We haven't covered $\sin A \sin B$, $\sin A \cos B$, $\cos A \cos B$ } Product-to-Sum Formulas.

§3.1 Law of Sines



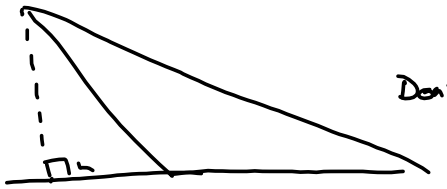
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

SSS Nope
SAS
ASA
ASS = DonKey

$\sin A = \frac{h}{b}$ $\sin B = \frac{h}{a}$

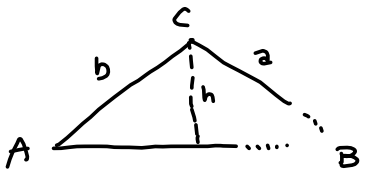
$h = b\sin A$ = $h = a\sin B$

$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$

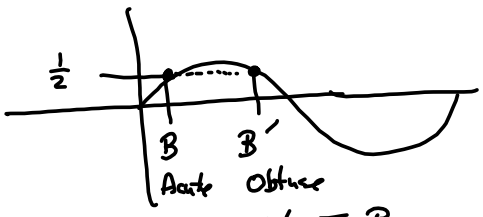


Don't want to do.

Special: ASS could be one unique solution
that's when $a > b$



two solutions:
$h < a < b$
when $h < a < b$, we get 2 cases:
B is acute - what your calculator says when you hit it, with arcsine = SIN⁻

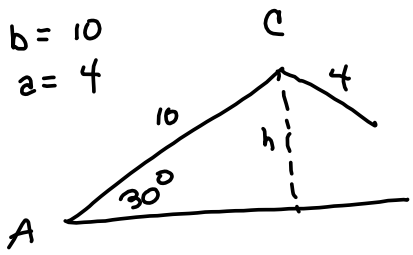


B is obtuse - Then the answer will be

$B' = \pi - B$
$= 180^\circ - B$

No Solution
$a < h$

$A = 30^\circ$
$b = 10$
$a = 4$



$\frac{h}{10} = \sin 30^\circ \Rightarrow$
$h = 10 \sin 30^\circ = 10\left(\frac{1}{2}\right) = 5 > 4$, so
a can't reach.
NO SOLUTION!

Unique Solution

Round to 3 places:

$A = 30^\circ$
$b = 10$
$a = 11$

$a > b \rightarrow$ unique solution:

$\frac{\sin B}{b} = \frac{\sin A}{a} \rightarrow$

$\sin B = \frac{b \sin A}{a} = \frac{10 \sin 30^\circ}{11} \approx 0.454545454545 \rightarrow$

$B \approx \sin^{-1}(0.454545454545) \approx 27.0356917894^\circ \approx B \rightarrow$

$C = 180^\circ - A - B \approx 180^\circ - 30^\circ - 27.0356917894^\circ$

$\approx 122.964308211^\circ \approx C$

$A = 30^\circ$
$B \approx 27.036^\circ$
$C \approx 122.964^\circ$

Finally

$a = 11$
$b = 10$

$\frac{\sin C}{c}$

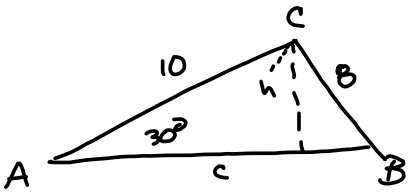
$\frac{c}{\sin C} = \frac{a}{\sin A}$ $(122.964\ldots^\circ)$

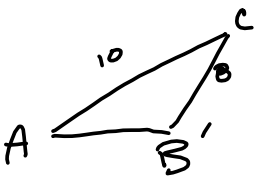
$\rightarrow c = \frac{a \sin C}{\sin A} \approx \frac{11 \sin(122.964308211^\circ)}{\sin(30^\circ)}$

$a = 11$
$b = 10$
$c \approx 18.458$

≈ 18.4582130089

Two Solutions in ASS situation.





$A = 30^\circ$

$b = 10$

$a = 5$

$\frac{h}{10} = \sin 30^\circ$

$h = 10 \sin 30^\circ = 5$

$a > h$ & $a < b$ ⟹

2 solns

B is acute. – Calculator Answer.

B′ is obtuse.

Solve for B is acute

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{b \sin A}{a} = \frac{10 \sin 30^\circ}{8} = \frac{5}{8}$$

$$B = \arcsin\left(\frac{5}{8}\right) \approx 38.6821874535^\circ$$

$$C = 180^\circ - A - B \approx 150^\circ - 38.6821874535^\circ \approx 111.317812546^\circ \approx C$$

$A = 30^\circ$
$B \approx 38.682^\circ$
$C \approx 111.318^\circ$

$a = 11$
$b = 10$
$c \approx 20.495$

No, dummy!
B is Acute situation
$a = 8$, $b = 10$

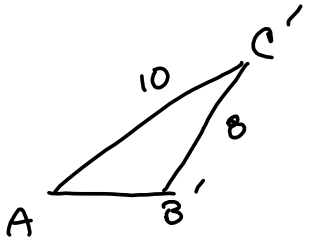
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} \approx \frac{11 \sin(111.317812546^\circ)}{\sin(30^\circ)} \approx 20.4947215499$$

11 → B

Nope

$$B' = 180^\circ - B \approx 180^\circ - 38.6821874535^\circ \approx 141.317812546^\circ$$



$A = 30^\circ$
$B' \approx 141.318^\circ$
$C' \approx 8.682^\circ$

$B' \approx 141.318^\circ$

$C' = 180^\circ - A - B' \approx$

$180^\circ - 30^\circ - 141.317812546^\circ$

$\approx 8.682187454^\circ \approx C'$

Sorry I got the numbers wrong!

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1$$

$$= \frac{\cos\theta\cot\theta - (1-\sin\theta)}{1-\sin\theta}$$

$$= \left(\frac{\cos\theta\left(\frac{\cos\theta}{\sin\theta}\right) - \left(\frac{1-\sin\theta}{1}\right)}{1-\sin\theta} \right) \frac{\sin\theta}{\sin\theta}$$

$$\frac{\cos^2\theta - (\sin\theta - \sin^2\theta)}{\sin\theta - \sin^2\theta} = \frac{\cos^2\theta - \sin\theta + \sin^2\theta}{\sin\theta(1-\sin\theta)}$$

$$= \frac{1-\sin\theta}{\sin\theta(1-\sin\theta)} = \frac{1}{\sin\theta} = \csc\theta$$

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1 = \frac{\cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)}{1-\sin\theta}$$