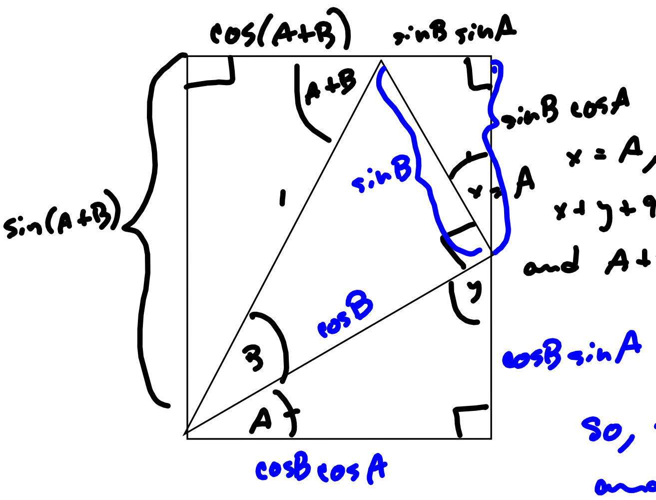
Link to Nice Video Proof

$\sin(u+v) = \sin u \cos v + \sin v \cos u$



$x = A$, b/c

$x + y + 90 = 180^\circ$

and $A + y + 90^\circ = 180^\circ$

$\cos A = \frac{ADJ}{\sin B} \rightarrow$

$ADJ = \cos A \sin B$

So, $\sin(A+B) = \sin A \cos B + \sin B \cos A$

and

$\cos(A+B) + \sin B \sin A$

$= \cos B \cos A \rightarrow$

$\cos(A+B) = \cos B \cos A - \sin B \sin A$

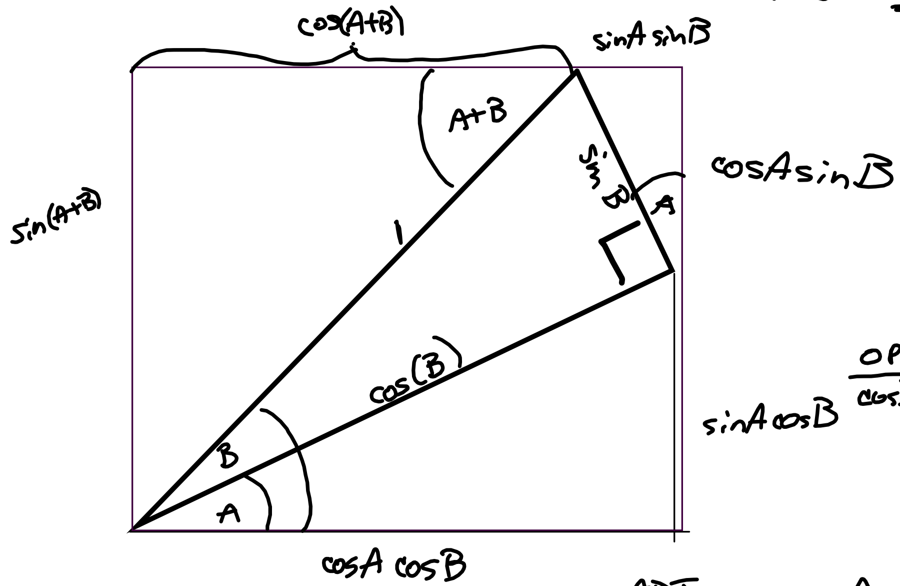
$= \cos A \cos B - \sin A \sin B$

Angle Sum Identities:

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\frac{opp}{\sin B} = \sin(A) \longrightarrow opp = \sin A\sin B$



$\frac{opp}{\cos B} = \sin A$

$\frac{ADJ}{\cos B} = \cos A$

$ADJ = \cos A\cos B$

$$\boxed{\sin(A+B) = \sin A\cos B + \sin B\cos A}$$

$\cos(A+B) + \sin A\sin B = \cos A\cos B \longrightarrow$

$$\boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$$

$$\boxed{\sin(A-B) = \sin(A+(-B)) = \sin A\cos(-B) + \sin(-B)\cos A = \sin A\cos B - \sin B\cos A}$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \left(\frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B}\right)\left(\frac{\frac{1}{\cos A\cos B}}{\frac{1}{\cos A\cos B}}\right)$$

$$= \frac{\frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B}}{\frac{\cos A\cos B - \sin A\sin B}{\cos A\cos B}} = \frac{\frac{\sin A\cancel{\cos B}}{\cos A\cancel{\cos B}} + \frac{\sin B\cancel{\cos A}}{\cancel{\cos A}\cos B}}{\frac{\cancel{\cos A\cos B}}{\cancel{\cos A\cos B}} - \frac{\sin A\sin B}{\cos A\cos B}}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}}$$

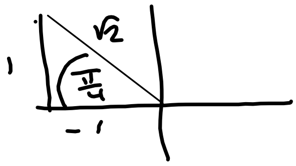
Also: $\tan(2A) = \frac{\tan A + \tan A}{1-\tan A\tan A} = \boxed{\frac{2\tan A}{1-\tan^2 A} = \tan(2A)}$

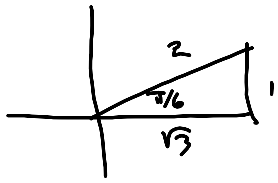
Find exact value of

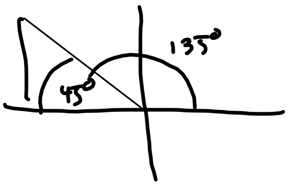
$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$$

$11 = 10+1 = 9+2$

$$= \sin\frac{3\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{3\pi}{4} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\left(-\frac{1}{\sqrt{2}}\right)$$







$$= \boxed{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}}$$

$$= \frac{\sqrt{3}\sqrt{2}}{2\sqrt{2}\sqrt{2}} - \frac{1\sqrt{2}}{2\sqrt{2}\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$ is simplified radical form.

$\sin(2v) = \sin(v+v) = \sin v \cos v + \sin v \cos v = 2\sin v \cos v$

$\boxed{\sin(2v) = 2\sin v \cos v}$ Double-Angle

$\cos(2v) = \cos v \cos v - \sin v \sin v = \cos^2 v - \sin^2(v)$

$= \cos^2 v - (1-\cos^2 v)$

$= \cos^2 v - 1 + \cos^2 v$

$= 2\cos^2 v - 1$

$1-\sin^2 v - \sin^2 v$
$= 1 - 2\sin^2 v$

$$\boxed{\begin{aligned}\cos(2v) &= \cos^2 v - \sin^2 v \\ &= 2\cos^2 v - 1 \\ &= 1 - 2\sin^2 v\end{aligned}}$$

$\frac{1}{2}$-Angle:

$\cos(2v) = 2\cos^2(v) - 1$

$\longrightarrow 2\cos^2(v) = \cos(2v) + 1$

$\longrightarrow \cos^2(v) = \frac{1+\cos(2v)}{2}$

$\longrightarrow \boxed{\cos(v) = \pm\sqrt{\frac{1+\cos(2v)}{2}}}$

Actually, letting $u = 2v$, $\frac{u}{2} = v$

$\boxed{\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}}$ use this.

$\cos(2v) = 1 - 2\sin^2(v)$

$2\sin^2(v) = 1 - \cos(2v)$

$\sin^2(v) = \frac{1-\cos(2v)}{2}$

$\sin(v) = \pm\sqrt{\frac{1-\cos(2v)}{2}}$

$\boxed{\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}}$

$\boxed{\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}}$

Find $\sin\left(\frac{11\pi}{12}\right)$ using a half-angle formula.

$$\frac{u}{2} = \frac{11\pi}{12} = \frac{\frac{11\pi}{6}}{2} = \frac{u}{2}$$

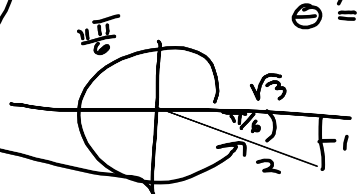
$$\sin\left(\frac{11\pi}{12}\right) = \pm\sqrt{\frac{1-\cos\left(\frac{11\pi}{6}\right)}{2}} = +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

is it "+" or "−"

Method 1:
Just stick it in calculator
$\sin\left(\frac{11\pi}{12}\right) \approx + \ldots$

$\theta \doteq \frac{\pi}{6}$

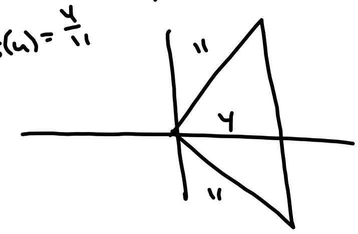


So, $\sin\left(\frac{11\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$ 1/2 − Angle

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ Angle − sum

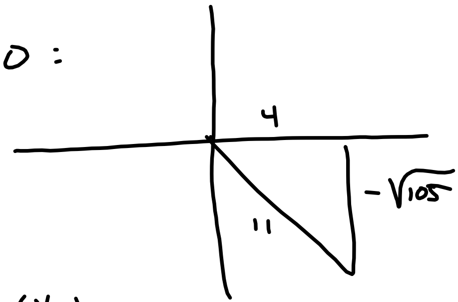
Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given $\cos(u) = \frac{4}{11}$ and $\sin(u) < 0$

$\cos(u) = \frac{4}{11}$



and $\sin(u) < 0$:

$11^2 - 4^2$
$= 121 - 16$
$= 105$



$$\sin(2u) = 2\sin u \cos u = 2\left(-\frac{\sqrt{105}}{11}\right)\left(\frac{4}{11}\right)$$
$$= \boxed{\frac{-8\sqrt{105}}{121} = \sin(2u)}$$

$$\cos(2u) = 2\cos^2 u - 1$$
$$= 2\left(\frac{4}{11}\right)^2 - 1 = \frac{2(16)}{121} - \frac{121}{121} = \frac{32-121}{121} = \boxed{\frac{-89}{121} = \cos(2u)}$$

$$\tan(2u) = \frac{\frac{-8\sqrt{105}}{121}}{\frac{-89}{121}} = \frac{-8\sqrt{105}}{-89} = \boxed{\frac{8\sqrt{105}}{89} = \tan(2u)}$$

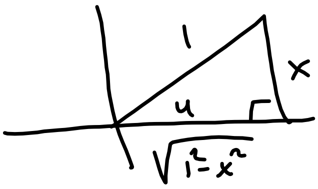
$$= \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{\sqrt{105}}{4}\right)}{1 - \left(\frac{105}{16}\right)} = \frac{-\frac{\sqrt{105}}{2}}{\frac{16-105}{16}}$$

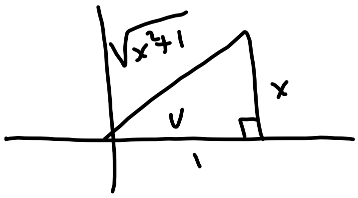
$$= \frac{-\frac{\sqrt{105}}{2}}{\frac{-89}{16}} = \left(-\frac{\sqrt{105}}{2}\right)\left(-\frac{16}{89}\right)$$
$$= \boxed{\frac{-8\sqrt{105}}{89} = \tan(2u)}$$

$\cos(\arcsin(x) + \arctan(x)) = \cos(u+v)$, where

$u = \arcsin(x)$, $v = \arctan(x)$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v = \sqrt{1-x^2}\left(\frac{1}{\sqrt{x^2+1}}\right) - x\left(\frac{x}{\sqrt{x^2+1}}\right)$$





$$= \boxed{\frac{\sqrt{1-x^2} - x^2}{\sqrt{x^2+1}}}$$

And finally, the power-reduction formulas

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$$

$$\sin^2\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{2}$$

$$\boxed{\sin^2(v) = \frac{1-\cos(2v)}{2}}$$

Likewise

$$\boxed{\cos^2(v) = \frac{1+\cos(2v)}{2}}$$

$$\tan^2(v) = \frac{1-\cos(2v)}{1+\cos(2v)}$$

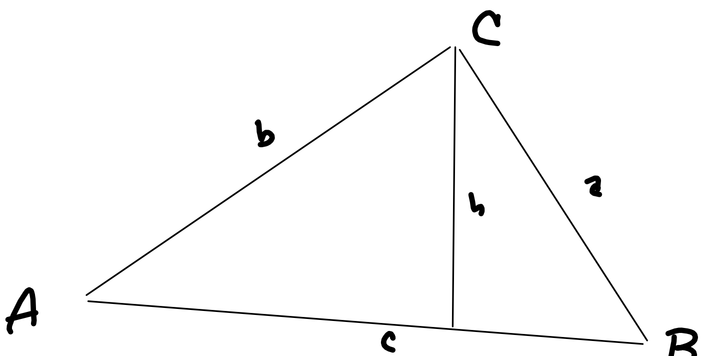
write $\tan^4(x)$ in terms of cosines to the 1^{st} power.

$$\tan^4(x) = \left(\tan^2(x)\right)^2 = \left(\frac{\sin^2(x)}{\cos^2(x)}\right)^2 = \left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)^2$$

$$= \frac{1-2\cos(2x)+\cos^2(2x)}{1+2\cos(2x)+\cos^2(2x)}$$

$$= \left(\frac{1-2\cos(2x)+\frac{1+\cos(4x)}{2}}{1+2\cos(2x)+\frac{1+\cos(4x)}{2}}\right)\left(\frac{2}{2}\right)$$

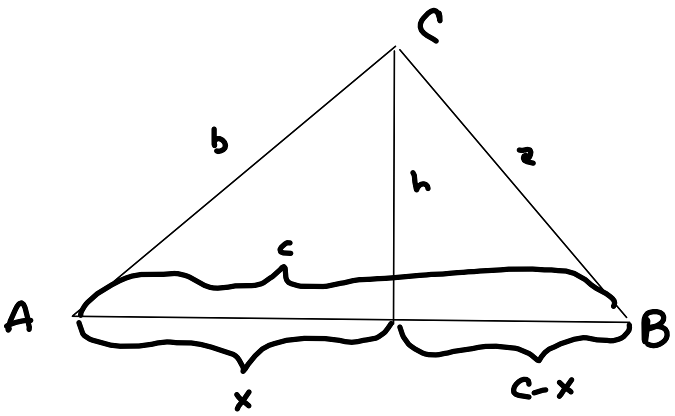
$$= \frac{2-4\cos(2x)+1+\cos(4x)}{2+4\cos(2x)+1+\cos(4x)} = \boxed{\frac{\cos(4x)-4\cos(2x)+3}{\cos(4x)+4\cos(2x)+3}}$$



$\sin A = \frac{h}{b}$, $\sin B = \frac{h}{a}$

$b \sin A = h = a \sin B \Longrightarrow$

$\frac{\sin A}{a} = \frac{\sin B}{b}$. The rest can be done in similar way.



$$a^2 = h^2 + (c-x)^2$$

$$= b^2 \sin^2 A + c^2 - 2cx + x^2$$

$$= b^2 \sin^2 A + c^2 - 2c(b\cos A) + b^2\cos^2 A$$

$$= b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc\cos A$$

$$= b^2 + c^2 - 2bc\cos A$$

$$\frac{h}{b} = \sin A$$

$$h = b\sin A$$

$$\frac{x}{b} = \cos A$$

$$x = b\cos A$$

Double-Angle

$\cos(2v) = \cos(v+v) = \cos v \cos v - \sin v \sin v = \cos^2 v - \sin^2 v$

$\sin(2v) = \sin(v+v) = \sin v \cos v + \sin v \cos v = 2\sin v \cos v$

HALF-ANGLE

Recall $\cos(2v) = \cos^2 v - \sin^2 v$

$= \cos^2(v) - (1 - \cos^2 v) = 2\cos^2 v - 1$

$\longrightarrow \cos(2v) = 2\cos^2 v - 1 = \cos(2v)$

$\Rightarrow 2\cos^2 v = \cos(2v) + 1$

$\longrightarrow \cos^2(v) = \frac{1+\cos(2v)}{2} \longrightarrow$

$\cos(v) = \pm\sqrt{\frac{1+\cos(2v)}{2}}$

Let $u = 2v$. Then $v = \frac{u}{2}$ and the above becomes

$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$

Recall

$\cos(2v) = \cos^2(v) - \sin^2(v) = 1 - \sin^2(v) - \sin^2(v)$

$= 1 - 2\sin^2(v) \Longrightarrow$

$2\sin^2(v) = 1 - \cos(2v)$

$\Longrightarrow \sin^2(v) = \dfrac{1-\cos(2v)}{2}$ Also used, separately as a Power-Reducing formula.

$\Longrightarrow \sin(v) = \pm\sqrt{\dfrac{1-\cos(2v)}{2}} \Longrightarrow$

$\boxed{\sin\left(\frac{u}{2}\right) = \pm\sqrt{\dfrac{1-\cos(u)}{2}}}$ via $u = 2v$, $\frac{u}{2} = v$ sub.

$$\boxed{\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)}} = \left(\frac{\sin u \cos v + \sin v \cos u}{\cos u \cos v - \sin u \sin v}\right)\left(\frac{\cos u \cos v}{\cos u \cos v}\right)$$

$$= \frac{\frac{\sin u \cos v}{\cos u \cos v} + \frac{\sin v \cos u}{\cos u \cos v}}{\frac{\cos v \cos u}{\cos u \cos v} - \frac{\sin u \sin v}{\cos u \cos v}} = \boxed{\frac{\tan u + \tan v}{1 - \tan u \tan v}}$$

Sometimes 1 version is easier than the other, especially considering you usually already have $\sin(u+v)$ & $\cos(u+v)$.

$\tan^4(x) = \left(\tan^2(x)\right)^2$ Re-write as first powers of cosines of multiple angles.

$$= \left(\frac{\sin^2(x)}{\cos^2(x)}\right)^2 = \left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)^2 = \frac{\cos^2(2x) - 2\cos(2x)+1}{\cos^2(2x)+2\cos(2x)+1}$$

$$= \frac{\frac{1+\cos(4x)}{2} - 2\cos(2x)+1}{\frac{1+\cos(4x)}{2} + 2\cos(2x)+1} = \frac{1+\cos(4x) - 4\cos(2x)+2}{1+\cos(4x)+4\cos(4x)+2}$$

$$= \boxed{\frac{\cos(4x) - 4\cos(2x)+3}{\cos(4x)+4\cos(2x)+3}}$$

Product-to-sum

Via angle-sum formulas

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$+\quad \sin(A-B) = \sin A\cos B - \sin B\cos A$$

$$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$$

$$\boxed{\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]}$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$$

$$\boxed{\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$