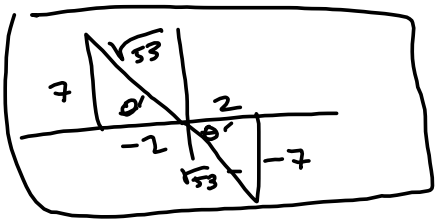
#s 9, 12, 13, 14 on Spring 24 midterm aren't covered on this one.

#s 9, 12 - end on Spring '25 isn't covered.

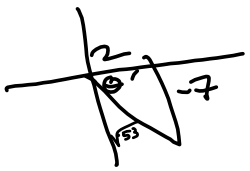
That's all Angle sum, half-angle, law of Sines and Vector stuff.

Draw 2 pictures for $\tan\theta = -\frac{7}{2}$



$2^2+7^2 = 4+49$

Suppose for this, $\cos\theta > 0$. Find all six trig values



$\sin\theta = \frac{-7}{\sqrt{53}}$ $\quad \csc\theta = -\frac{\sqrt{53}}{7}$

$\cos\theta = \frac{2}{\sqrt{53}}$ $\quad \sec\theta = \frac{\sqrt{53}}{2}$

$\tan\theta = -\frac{7}{2}$ $\quad \cot\theta = -\frac{2}{7}$

Find all solns in $[0, 2\pi)$

$\theta' = \arctan\left(\frac{7}{2}\right) \Rightarrow$

$\theta = \pi - \theta', \ 2\pi - \theta'$

Find all solns

$\theta = \pi - \theta' + 2n\pi, \ 2\pi - \theta' + 2n\pi, \ n \in \mathbb{Z}$

short answer: They're π apart

$\theta = \pi - \theta' + n\pi, \ n \in \mathbb{Z}$

$(x-\sqrt{3})(x+\sqrt{3})$

$= x^2 - \frac{3}{4}$

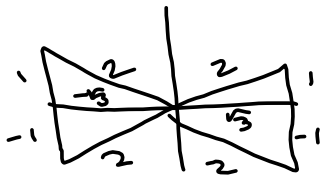
Solve ~~$\cos^2\theta - \frac{3}{4} = 0$~~

$4\cos^2\theta - 3 = 0$ on $[0, 2\pi)$

$4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$



$\theta' = \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$A = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\} = \theta \rightarrow \theta \in A$

Find ALL SOLUTIONS.

$A = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$\theta \in \{ y + 2n\pi \mid y \in A, n \in \mathbb{Z} \}$

$= \{ y + n\pi \mid y = \frac{\pi}{6}, \frac{5\pi}{6} \}$

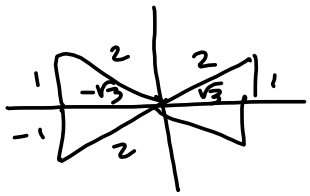
$\cos^2(2\theta) = \frac{3}{4}$

$4\cos^2(2\theta) - 3 = 0$

$\cos(2\theta) = \pm \frac{\sqrt{3}}{2}$

All $\theta \in [0, 2\pi)$

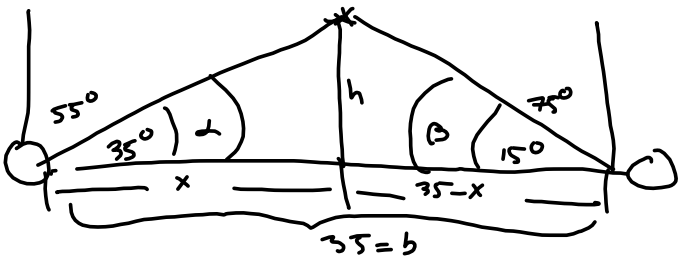
$2\theta \in [0, 4\pi)$



$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{6}+\frac{12\pi}{6}, \frac{5\pi}{6}+\frac{12\pi}{6}, \frac{7\pi}{6}+\frac{12\pi}{6}, \frac{11\pi}{6}+\frac{12\pi}{6}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$\longrightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$



Formula for height of triangle

$$h = \frac{35\sin(35^\circ)\sin(15^\circ)}{\sin(35^\circ)+\sin(15^\circ)} = \frac{b\sin\alpha\sin\beta}{\sin\alpha+\sin\beta}$$

b = base length

$$\frac{h}{x} = \tan 35^\circ$$

$$\frac{h}{35-x} = \tan(15^\circ)$$

$$h = x\tan 35^\circ$$

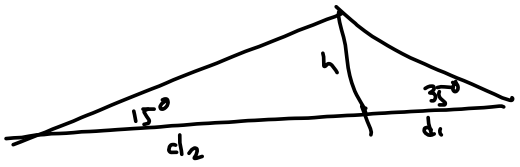
$$h = (35-x)\tan(15^\circ)$$

$$h = h$$

$$x\tan 35^\circ = 35\tan 15^\circ - x\tan 15^\circ$$

$$x(\tan(35^\circ)+\tan(15^\circ)) = 35\tan 15^\circ$$

$$\Rightarrow x = \frac{35\tan 15^\circ}{\tan 35^\circ+\tan 15^\circ}$$



$$\frac{d_2}{h} = \cot 15^\circ \qquad \frac{d_1}{h} = \cot 35^\circ$$

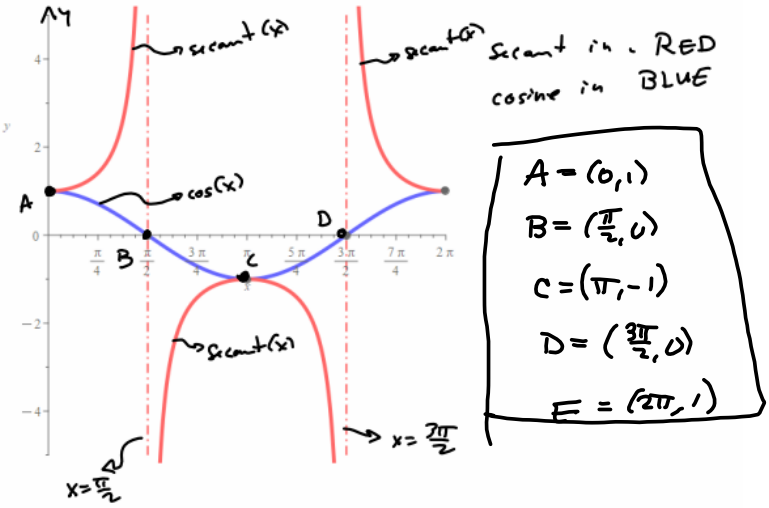
$$d_2 = h\cot 15^\circ \qquad d_1 = h\cot 35^\circ$$

$$d_1 + d_2 = 35 \Rightarrow h\cot 15^\circ + h\cot 35^\circ = 35$$

$$\Rightarrow h(\cot 15^\circ + \cot 35^\circ) = 35$$

$$\Rightarrow h = \frac{35}{\cot 15^\circ + \cot 35^\circ} \approx 6.78268440025 \text{ mi.}$$

3. (5 pts) Graph one period of cosine and secant on the same coordinate axes.



Graph $\cos(x)$ & $\arccos(x) = \cos^{-1}(x)$ on same graph.