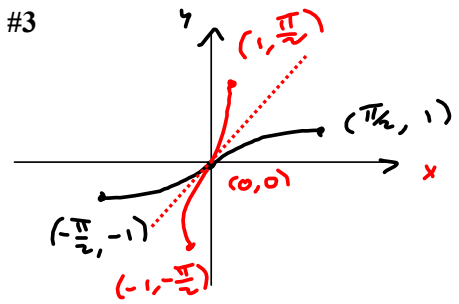
Midterm Test Spring, '24

#3



arcsin(x)

sin(x)

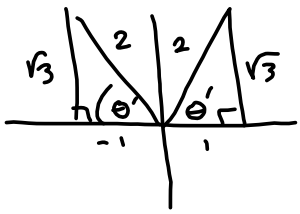
Solve the equation

$\sin(x) - \frac{\sqrt{3}}{2} = 0$

$2\sin(x) - \sqrt{3} = 0$

$2\sin(x) = \sqrt{3}$

$\sin(x) = \frac{\sqrt{3}}{2}$



Degrees: $\theta' = 60^\circ$ or $\frac{\pi}{3}$

$\theta = \theta'$ or $180^\circ - \theta' = 180^\circ - 60^\circ = 120^\circ$

Radians: $\theta = \theta'$ or $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$2\sin^2\theta + 3\sin\theta + 1 = 0$ Bad question fixed.

(a) Find all solns in $[0, 2\pi)$

Let $u = \sin\theta$. Then

$2u^2 + 3u + 1 = 0$

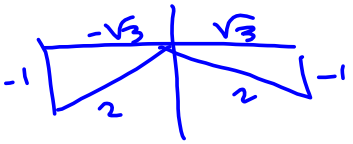
$(2u+1)(u+1) = 0$

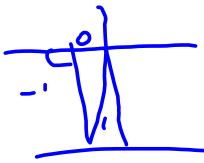
$2u + 1 = 0$ or $u + 1 = 0$

$2u = -1$ $u = -1$

$u = -\frac{1}{2}$ $\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$





$\theta = \frac{3\pi}{2}$

Let $\theta' = \frac{\pi}{6}$, Then

$\theta = 2\pi - \pi/6, \ \pi + \frac{\pi}{6}$

$\therefore \frac{11\pi}{6}, \frac{7\pi}{6} = \theta$

$\theta = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

(b) Find all solns

Funky way:

$A = \{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

$\rightarrow$ Soln Set is $\{x + 2n\pi \mid x \in A \ n \in \mathbb{Z}\}$

$x \in \{x + 2n\pi \mid x \in \{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\} \ n \in \mathbb{Z}\}$

$x = \frac{7\pi}{6} + 2n\pi, \ \frac{3\pi}{2} + 2n\pi, \ \frac{11\pi}{6} + 2n\pi, \ n \in \mathbb{Z}$

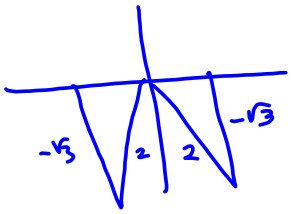
Find all solns x in $[0, 2\pi)$

$2\sin(5x) + \sqrt{3} = 0$

$5x \in [0, 10\pi)$

$2\sin(5x) = -\sqrt{3}$

$\sin(5x) = -\frac{\sqrt{3}}{2}$



$5x = \frac{4\pi}{3}$ or $5x = \frac{5\pi}{3}$

$5x = \frac{4\pi}{3}, \frac{10\pi}{3}, \frac{4\pi}{3}, \frac{22\pi}{3}, \frac{28\pi}{3}, \frac{34\pi}{3} > 10\pi$

$2\pi = \frac{6\pi}{3}$

$x = \frac{4\pi}{15}, \frac{10\pi}{15}, \frac{16\pi}{15}, \frac{22\pi}{15}, \frac{28\pi}{15}$, ...

$\frac{2\pi}{3}$ Lowest terms

$x = \frac{5\pi}{15}, \frac{11\pi}{15}$, ...

Not that smart

$5x = \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}, \frac{23\pi}{3}, \frac{29\pi}{3}, \frac{35\pi}{3}$ ~~10π~~

That's all of them in $[0, 2\pi)$

$x = \frac{5\pi}{15}, \frac{11\pi}{15}, \frac{17\pi}{15}, \frac{23\pi}{15}, \frac{29\pi}{15}$, ...

$\pi/3$ Lowest terms

Find All of them.

Let $A = \left\{ \frac{4\pi}{15}, \frac{2\pi}{3}, \frac{16\pi}{15}, \frac{22\pi}{15}, \frac{28\pi}{15}, \frac{\pi}{3}, \frac{11\pi}{15}, \frac{17\pi}{15}, \frac{23\pi}{15}, \frac{25\pi}{15} \right\}$

$$x \in \{ y + 2n\pi \mid y \in A \}$$

WebAssign Answers

MORE EFFICIENT

$5x = \frac{4\pi}{3} + 2n\pi \longrightarrow$ C

$x = \frac{4\pi}{15} + \frac{2n\pi}{5}, \quad n \in \mathbb{Z}$

OR

$5x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{\pi}{3} + \frac{2n\pi}{5}, \quad n \in \mathbb{Z}$

WebAssign FORMAT :

$$\frac{4\pi}{15} + \frac{2n\pi}{15}, \ \frac{\pi}{3} + \frac{2n\pi}{5} \qquad n \in \mathbb{Z}$$

2. Answer the questions about the equation $\sin(\theta)=\frac{2}{3}$.

   a. (5 points) Sketch two triangles that satisfy $\sin(\theta)=\frac{2}{3}$.

   b. (5 pts) Assume the terminal side of the angle θ lies in the 2nd quadrant. Find the other five trigonometric functions of θ.

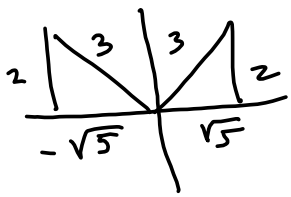
   c. (5 pts) Again, assuming θ's terminal side lies in Q II, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

   Find the *exact* solution to part c.

   d. (5 pts) Give *all* solutions to the equation $\sin(\theta)=\frac{2}{3}$, in degrees *and* radians, rounded to three (3) decimal places.

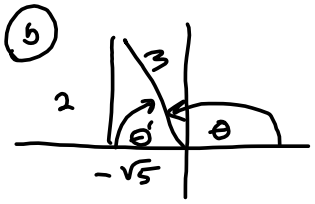
   Find the *exact* solution to part d.

(a) $\sin\theta = \frac{2}{3}$



$\sqrt{3^2-2^2} = \sqrt{9-4} = \sqrt{5}$

(b)



$\sin\theta = \frac{2}{3}$ $\quad \csc\theta = \frac{3}{2}$

$\cos\theta = -\frac{\sqrt{5}}{3}$ $\quad \sec\theta = -\frac{3}{\sqrt{5}}$

$\tan\theta = -\frac{\sqrt{5}}{2}$ $\quad \cot\theta = -\frac{2}{\sqrt{5}}$

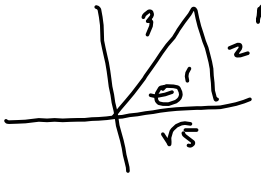
(c) Scientific Calculator
Find θ to 3 decimal places.

$\sin\theta = \frac{2}{3}$

$\arcsin(\sin\theta) = \boxed{\arcsin\left(\frac{2}{3}\right) = \theta' \approx}$

$\approx$ 0.729727656227 $\approx$ 41.8103148958°

Reference angle picture.



$\theta = \pi - \theta' \approx$ 2.41186499736

$\boxed{\theta \approx 2.412}$

$\theta = 180° - \theta' \approx$ 138.189685104°

$\boxed{\theta \approx 138.190°}$

EXACT:

$\theta = \pi - \theta'$
OR
$\theta = 180° - \theta'$

(d) All solns to 3 places

$\pi - \theta' + 2n\pi = (2n+1)\pi - \theta' \approx \boxed{.730 + 2n\pi, n \in \mathbb{Z}}$

$180° - \theta' + 360°n =$

$= \boxed{180° + 360°n - 41.810°}$ $n \in \mathbb{Z}$

I prefer the EXACT answers for these.

Spring, '24 Midterm. Don't worry about

#s 9, 12, 13, 14.

They're all from Section 2.4 and beyond (basically 2.5).

We're only testing through Section 2.3, this semester.

Alex B points out some scheduling stuff.

Week 6 Written Assignment delves into Section 2.4 (#s 4, 5). But those other problems are of a type that could appear on the Midterm.

We'll push the due date back, but don't go to sleep on Week 6.