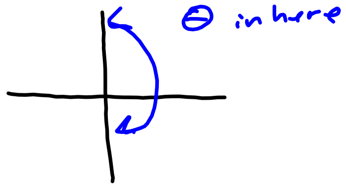
§ 2.1 II #5

$4\sqrt{3} = \sqrt{64-x^2}$. Make the substitution $x = 8\cos\theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



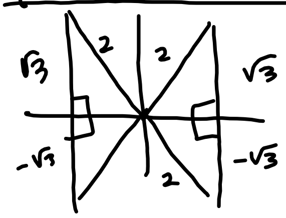
$$\sqrt{64-(8\cos\theta)^2} = \sqrt{64-64\cos^2\theta} = \sqrt{64(1-\cos^2\theta)}$$

$$= 8\sqrt{1-\cos^2\theta} = 8\sqrt{\sin^2\theta} = \boxed{8|\sin\theta|} = 4\sqrt{3}$$

WebAssign told you that $8\sin\theta$ was OK.

$$\Rightarrow \quad |\sin\theta| = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$$

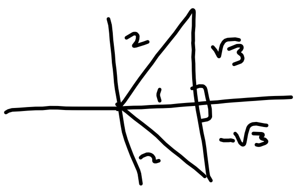
$$\Rightarrow \quad \boxed{\sin\theta = \pm\frac{\sqrt{3}}{2}}$$



$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

But problem says $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

That means QI or QIV, so



$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$

($-\frac{\pi}{3}$ is coterminal with $\frac{5\pi}{3}$)

Then it asked what $\cos\theta$ is.

$$\boxed{\cos\theta = \frac{1}{2}}$$

Verify the identity algebraically. Use a graphing utility to check your result graphically. (Simplify at each step.)

$$\frac{6\sec(\theta) - 6}{1 - \cos(\theta)} = 6\sec(\theta)$$

$$\frac{6\sec\theta - 6}{1-\cos\theta} = \frac{6(\sec\theta - 1)}{1-\cos\theta} = \left(\frac{6\left(\frac{1}{\cos\theta} - 1\right)}{1-\cos\theta}\right)\left(\frac{\cos\theta}{\cos\theta}\right)$$

$$= \frac{6\cos\theta\left(\frac{1}{\cos\theta} - 1\right)}{\cos\theta(1-\cos\theta)} = \frac{6\cancel{(1-\cos\theta)}}{\cos\theta\cancel{(1-\cos\theta)}} = \frac{6}{\cos\theta} = 6\sec\theta.$$

I like working with cosines more than secants.

WebAssign / Book way

$$\frac{6(\sec\theta - 1)}{1-\cos\theta} = \left(\frac{6(\sec\theta - 1)}{1 - \frac{1}{\sec\theta}}\right)\left(\frac{\sec\theta}{\sec\theta}\right)$$

$$= \frac{6\sec\theta\cancel{(\sec\theta - 1)}}{\cancel{\sec\theta - 1}} = 6\sec\theta.$$

Factor the expression.

S 2.1 Part II #2

$15\cos^2(x) + 14\cos(x) - 8 = f(x) \to f(\cos(x))$

Let $u = \cos\theta \longrightarrow$

$f(x) \to f(u)$

$f(x) = 15u^2 + 14u - 8$

$= 15u^2 + 20u - 6u - 8$

$= 5u(3u+4) - 2(3u+4)$

$= (3u+4)(5u-2)$

$= (3\cos(x)+4)(5\cos(x)-2)$

$-\ (3)(5)(2)(2)(2)$

$20 - 6$

SLEDGEHAMMER

$a = 15,\ b = 14,\ c = -8 \longrightarrow$

$b^2 - 4ac = 14^2 - 4(15)(-8)$

$= 196 + (32)(15)$

$= 196 + 480 = 676$

$\sqrt{676} = 26$

When it factors "over the rationals," b^2-4ac will always be a perfect square.

$$\begin{array}{r|l} 2 & 676 \\ 2 & 338 \\ 13 & 169 \\ & 13 \end{array}$$

$$\begin{array}{r} 32 \\ \times\ 15 \\ \hline 160 \\ 320 \\ \hline 480 \\ 196 \\ \hline 676 \end{array}$$

$\sqrt{2^2 \cdot 13^2} = 2 \cdot 13 = 26$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm 26}{2(15)}$$

$$= \frac{-14 \pm 26}{30} \to \frac{-12}{30} = -\frac{2}{5}, \quad \frac{40}{30} = \frac{4}{3}$$

$$\Longrightarrow \quad 15\left(u + \frac{2}{5}\right)\left(u - \frac{4}{3}\right)$$

$$= \cancel{5 \cdot 3 \left(u + \frac{2}{5}\right)}$$

$$= 5\left(u + \frac{2}{5}\right)(3)\left(u - \frac{4}{3}\right)$$

$$= (5u+2)(3u-4)$$

$$= (5\cos(x)+2)(3\cos(x)-4)$$

Other Factoring Models

Difference / sum of cubes

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

Difference of Squares

$$x^2 - y^2 = (x-y)(x+y)$$

$15u^2 + 14u - 8$

$= 15\left(u^2 + \frac{14}{15}u + \left(\frac{7}{15}\right)^2\right) - 8 - 15\left(\frac{49}{225}\right)$

$= 15\left(u + \frac{7}{15}\right)^2 - \frac{169}{15}$ SET $= 0$

$(-8)\left(\frac{15}{15}\right) - \frac{49}{15}$

$= \frac{-120 - 49}{15} = -\frac{169}{15}$

$\Longrightarrow 15\left(u + \frac{7}{15}\right)^2 = \frac{169}{15}$

$\Longrightarrow \left(u + \frac{7}{15}\right)^2 = \frac{169}{225}$

$\Longrightarrow u + \frac{7}{15} = \pm\sqrt{\frac{169}{225}} = \pm\frac{13}{15}$

$\Longrightarrow u = \frac{-7 \pm 13}{15}$

$\frac{6}{15} = \frac{2}{5}$

$\frac{-20}{15} = \frac{-4}{3}$

$15\left(u - \frac{2}{5}\right)\left(u + \frac{4}{3}\right) = 3 \cdot 5 \left(u - \frac{2}{5}\right)\left(u + \frac{4}{3}\right) = (5u - 2)(3u + 4)$

Use the cofunction identities to evaluate the expression without using a calculator.

$$\tan^2 81° + \cot^2 45° - \sec^2 45° - \csc^2 9°$$

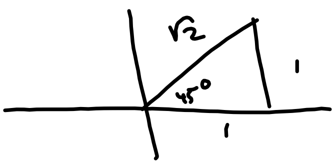
$81^\circ + 9^\circ = 90^\circ$ (complementary) $\to -(\tan^2 45^\circ + 1)$

$\cot^2(90^\circ - 81^\circ) + \cot^2(45^\circ) \boxed{-(\cot^2 45^\circ + 1)} - \csc^2(9^\circ)$

$\cot^2(9^\circ) + \cot^2(45^\circ) - \tan^2(45^\circ) - 1 - \csc^2(9^\circ)$

$\cancel{\csc^2(9^\circ)} - 1 + \cot^2(45^\circ) - \tan^2(45^\circ) - \cancel{\csc^2(9^\circ)}$

$\cot^2(45^\circ) - \tan^2(45^\circ) - 1 = 1^2 - 1^2 - 1 = \boxed{-1}$



I think I lost a "1"

Answer is -2

Try again:

$\tan^2(81^\circ) + \cot^2(45^\circ) - \sec^2(45^\circ) - \csc^2(9^\circ)$

$\cot^2(9^\circ) + 1^2 - \left(\frac{\sqrt{2}}{1}\right)^2 - \csc^2(9^\circ)$

$= \csc^2(9^\circ) - 1 + 1 - 2 - \csc^2(9^\circ)$

$= \boxed{-2}$

Extraneous Solutions: To solve an equation, we cast a wide net. Sometimes that means throwing back a few little fishies.

$A = B \longrightarrow$

$A^2 = B^2$, but

$A^2 = B^2$ DOES NOT IMPLY $A = B$

It implies $A = \pm B$, i.e.

$A^2 = B^2$

$\sqrt{A^2} = \sqrt{B^2}$

$|A| = |B|$

$\longrightarrow A = \pm B$

$\sqrt{x+1} = \sqrt{2x+3} = \sqrt{2(x+\frac{3}{2})}$

$(\quad)^2 = (\quad)^2$

$x+1 = 2x+3$

$-x = 2$

$x = -2$

But $\sqrt{2+1} = \sqrt{3}$

$\sqrt{2(-2)+3} = \sqrt{-1}$ oops! $x=-2$ is not in the domain!

$\underbrace{x=-2 \text{ is an extraneous solution.}}$