§2.1 I

Factor the expression

$a^2 - b^2 = (a-b)(a+b)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

SLEDGE HAMMER:

$f(x)$ a polynomial $\Longrightarrow$

$f(x) = 0$ @ $x = x_1, x_2 \Longrightarrow$

$(x - x_1)$ & $(x - x_2)$ are factors.

When in doubt with a factoring problem, find the zeros with quadratic formula and use the converse of the Factor Theorem.

$$x^2 + 3x + 2 = \overset{(x-(-2))\,(x-(-3))}{(x+2)(x+1)} \Longrightarrow$$

$\Longleftarrow$ $x^2 + 3x + 2 = 0$ when $x = -2, -3$.

Factor

$\sin^2(x) - 5\sin(x) - 14$

Let $u = \sin(x) \Longrightarrow$

$u^2 - 5u - 14$

$a = 1, b = -5, c = -14$

$\Longrightarrow b^2 - 4ac = 5^2 - 4(1)(-14)$

$= 25 + 56 = 81$

$\sqrt{81} = 9$

$$\Longrightarrow u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm 9}{2(1)} = \begin{cases} \frac{14}{2} = 7 \\ \frac{-4}{2} = -2 \end{cases}$$

$\Longrightarrow (u-7)(u+2)$

$= (\sin(x) - 7)(\sin(x) + 2)$

$\sin^4(x) - \cos^4(x)$

Let $u = \sin(x)$
$v = \cos(x)$

Then $u^4 - v^4 = (u^2)^2 - (v^2)^2$

$= (u^2 - v^2)(u^2 + v^2)$

$= (u-v)(u+v)(u^2+v^2)$

$= (\sin(x) - \cos(x))(\sin(x) + \cos(x))\underbrace{(\sin^2 + \cos^2(x))}_{1}$

$= (\sin(x) - \cos(x))(\sin(x) + \cos(x))$

$\boxed{3.1\ \text{II}}$ #17 → $\S 2.2$!

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$$

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}} = \frac{\sqrt{1-\cos^2\theta}}{\sqrt{(1+\cos\theta)^2}} = \frac{\sqrt{\sin^2\theta}}{|1+\cos\theta|}$$

$= \dfrac{|\sin\theta|}{1+\cos\theta}$ Nope. Not what we wanted.

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta}} = \frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{1-\cos^2\theta}}$$

$$= \frac{|1-\cos\theta|}{\sqrt{\sin^2\theta}} = \frac{1-\cos\theta}{|\sin\theta|}$$

$-1 \le \cos\theta \le 1$

$1 \ge -\cos\theta \ge -1$

$2 \ge 1-\cos\theta \ge 0$

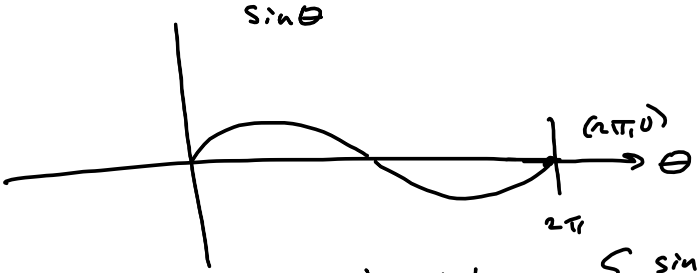
$\Longrightarrow |1-\cos\theta| = \begin{cases} 1-\cos\theta & \text{if } 1-\cos\theta \ge 0 \\ -(1-\cos\theta) & \text{if } 1-\cos\theta < 0 \end{cases}$

$= 1-\cos\theta$!

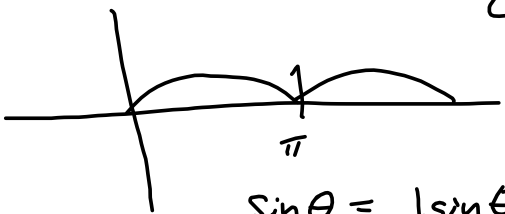
Solve

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{\sin^2\theta} = |\sin\theta|$

& that $= \sin\theta$ when $\sin\theta \geq 0$



$|\sin\theta| = \begin{cases} \sin\theta & \text{if } \sin\theta \geq 0 \\ -\sin\theta & \text{if } \sin\theta < 0 \end{cases}$



$\sin\theta = |\sin\theta|$ when

$\boxed{0 \leq \theta \leq \pi}$ at least

on $[0, 2\pi)$

Solve for μ S'2,1 II #18

$\mu W \cos\theta = W \sin\theta$

$\Longrightarrow \mu = \dfrac{W \sin\theta}{W \cos\theta} = \tan\theta = \mu$

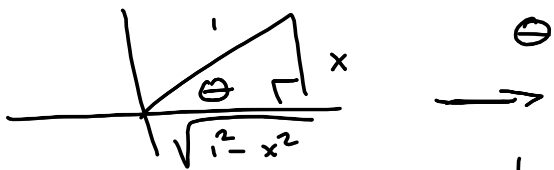
§2.2 Not much fun, 'til the end.

New! #19 §2.2

Trigonometric Substitution

Draw the cotton-pickin' triangle!

Use $\arcsin(x)$ for $\sin^{-1}(x)$



$\theta = \arcsin(x)$

Always draw in QI ~~⇒ in stan~~ standard position

$\Rightarrow \cos\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

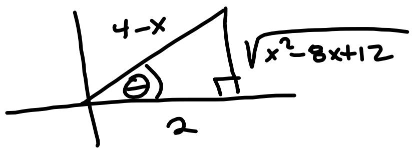
Problem Statement

Verify that

$\cos(\sin^{-1}x) = \sqrt{1-x^2}$

$\tan(\cos^{-1}(\frac{2}{4-x}))$ WebAssign

$\tan(\arccos(\frac{2}{4-x})) = \tan\theta = \frac{\sqrt{x^2-8x+12}}{2}$



$\sqrt{(4-x)^2-2^2}$

$= 16-8x+x^2-4$

$= 12-8x+x^2$

$= x^2-8x+12$

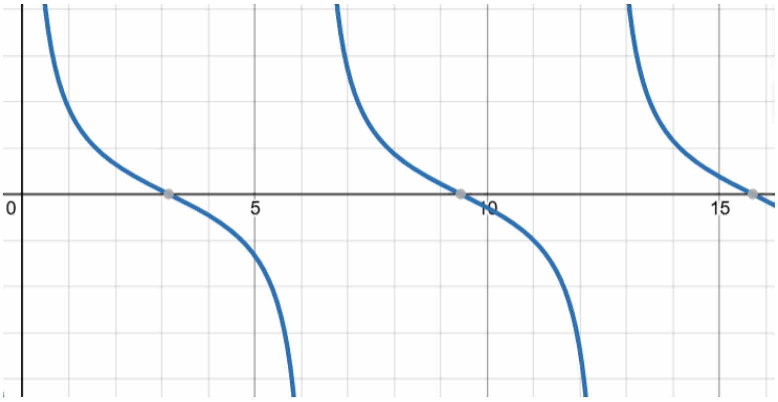
WebAssign says "Use the table feature ..."

Just enter the function into Desmos and evaluate it at the indicated values.

Consider the following equation.

$$\frac{3 + 3\cos x}{3\sin x} = \frac{3\sin x}{3 - 3\cos x}$$

(a) Use a graphing utility to graph each side of the equation.



$$f(x) = \frac{(1+\cos(x))}{\sin(x)}$$

$$g(x) = \frac{\sin x}{1-\cos x}$$

Don't have a table feature, but you can crank these out fairly quickly.

$f(0)$ undefined

$f(1) = 1.83048772171$

$f(2) = 0.642092615934$

$f(3) = 0.0709148443027$

It goes fairly quickly

$f(-1) = -1.83048772171$

$f(-2) = -0.642092615934$

$f(-3) = -0.0709148443027$

$g(0)$ undefined

$g(-1) = -1.83048772171$

$g(-2) = -0.642092615934$

$g(-3) = -0.0709148443027$

S2.2 #23

Verify the identity. (Simplify at each step.)

$$\tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$$

$\tan^4 x = (\tan^2 x)\left(\boxed{\tan^2(x)}\right)$ ✗

$= (\tan^2 x)\left(\boxed{\sec^2(x) - 1}\right)$ ✗

$= \tan^2 x \sec^2 x - \tan^2 x$

$\left.\begin{array}{l} \frac{\pi}{2} - x \\ 90^\circ - x \end{array}\right\}$ For cofunction identities.

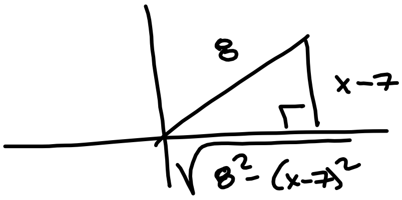
$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$ $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$

$\sec\left(\frac{\pi}{2} - x\right) = \csc(x)$

$\tan\left(\frac{\pi}{2} - x\right) = \cot(x)$

$\sin^{-1}\left(\frac{x-7}{8}\right)$ or $\arcsin\left(\frac{x-7}{8}\right)$

WebAssign Mills



$$\sin\left(\arctan\left(\frac{x+5}{2}\right)\right) = \frac{x+5}{\sqrt{(x+5)^2+4}}$$