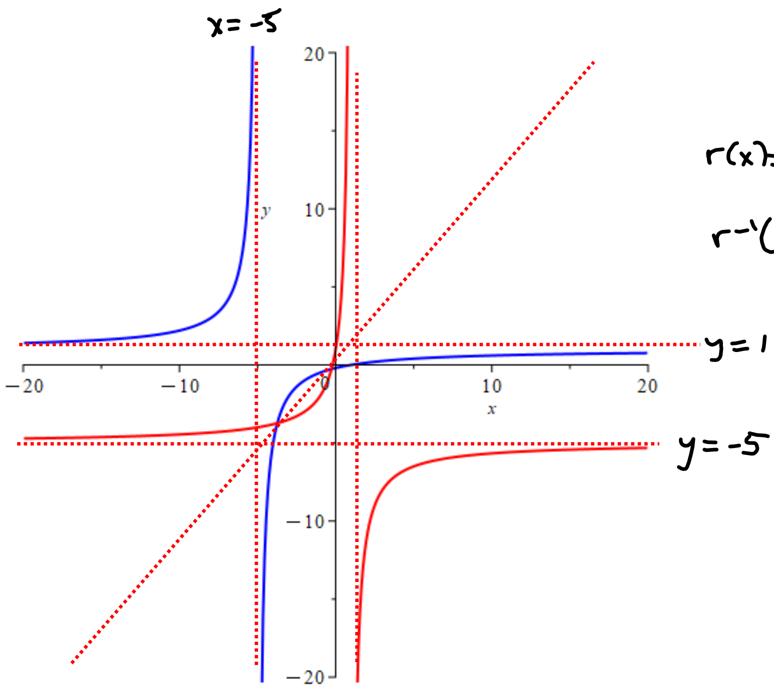
All functions have an inverse relation. Just swap *x* and *y*.

Not all functions have an inverse *function*. For that, you need a 1-to-1 function that passes the horizontal line test.



$r(x) = \frac{x-1}{x+5}$ $\quad x = -5$, $y = 1$

$r^{-1}(x) = -\frac{5x+1}{x-1}$ $\quad x = 1$, $y = -5$

The graph of the inverse function is the reflection of the function about the line *y* = *x*.

We did $\underbrace{\arcsin(x)}_{\text{for writing}} = \underbrace{\sin^{-1}(x)}_{\text{Calculator key (probably)}}$

The "-1" in $\sin^{-1}(x)$ means inverse with respect to function composition, NOT arithmetic.

$\sin^{-1}(x) \neq \frac{1}{\sin(x)} = \csc(x)$ = multiplicative/arithmetic inverse.

$\arcsin(x) = \sin^{-1}(x)$ is the inverse wrt function composition (Function of a function).

$$\left.\begin{aligned} \sin(\arcsin(x)) &= x \\ \arcsin(\sin(x)) &= x \end{aligned}\right\} \text{Almost true 100\%, but not quite.}$$

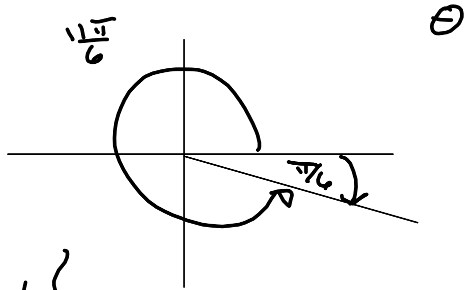
I say "not quite," because we have to restrict *x* to a smaller domain to make sine 1-to-1.

Suppose $x = \frac{11\pi}{6}$

Then $\sin(x) = -\frac{1}{2}$

$\arcsin(\sin(x)) =$

$\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$



But we wanted $\frac{11\pi}{6}$ back!

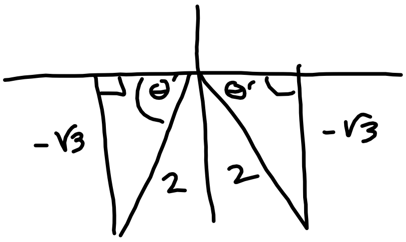
So $\arcsin(\sin(x)) \neq x$!

It's RELATED to x, but is in a different quadrant.

If $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\arcsin(\sin(x)) = x$.!

Find 2 angles in [0, 2pi)

that solve $\sin(x) = -\frac{\sqrt{3}}{2}$



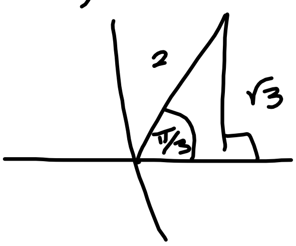
$\arcsin(\sin(x)) = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\pi}{3} = -60^\circ$

$-\frac{\pi}{3} \notin [0, 2\pi)$

We have to do some figuring

θ' = positive angle in QI that gives the same triangle



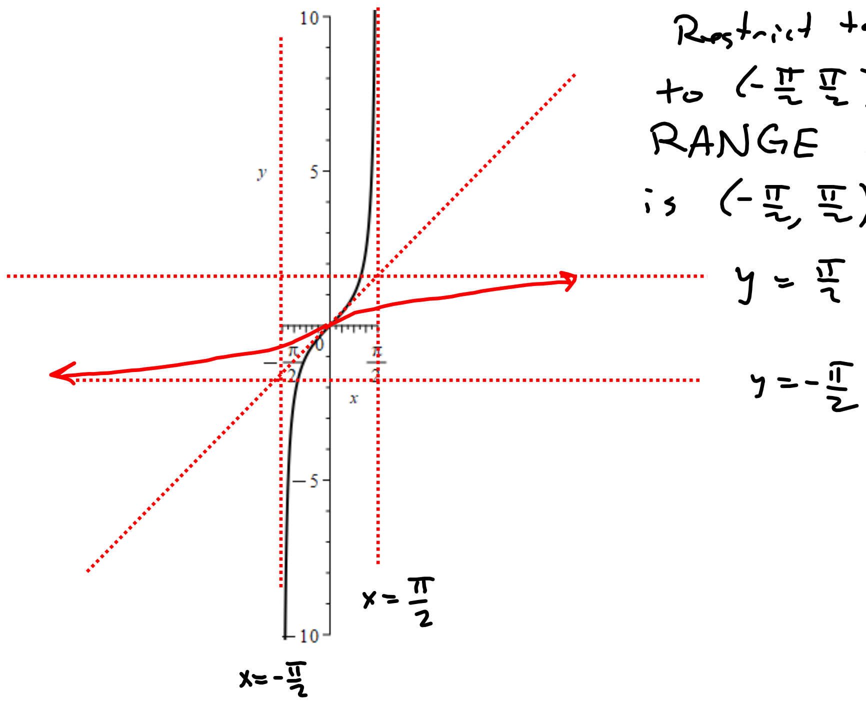
This is $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \theta'$

The two solutions in $[0, 2\pi)$ are

$\pi + \theta' = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$ in QIII

and $2\pi - \theta' = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$ The 2 sol'ns in $[0, 2\pi)$

Tangent inverse is much the same, as far as domains go,
we restrict tangent to $(-\frac{\pi}{2}, \frac{\pi}{2})$

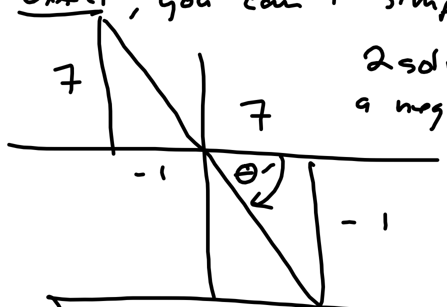


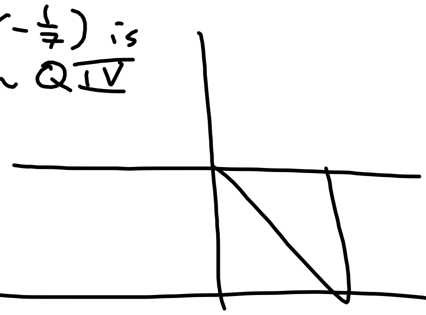
Restrict tangent's domain to $(-\frac{\pi}{2}, \frac{\pi}{2})$, so the RANGE of arctangent is $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan\theta = -\frac{1}{7}$

$\arctan(-\frac{1}{7}) = ?$ can be approximated, but to be EXACT, you can't simplify any more.

2 solns i $\arctan(-\frac{1}{7})$ is a negative angle in QIV





$\boxed{\theta' = \arctan(+\frac{1}{7})}$

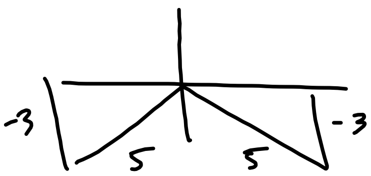
$\arctan(\frac{1}{7})$

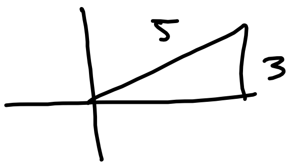
$\boxed{\theta = \pi - \theta',\ 2\pi - \theta' \text{ is exact}}$

7. Sketch the equation situation and find 2 solutions to each of the following. Give final answers in both radians and degrees, to 4 decimal places.

→ in $[0, 2\pi)$

a. (5 pts) $\sin(\theta) = -\frac{3}{5}$





$\boxed{\theta' = \arcsin\left(\frac{3}{5}\right)}$

$\boxed{\theta = \pi + \theta',\ 2\pi - \theta'}$

EXACT ANSWERS!

$2\pi - \arcsin\left(\frac{3}{5}\right)$

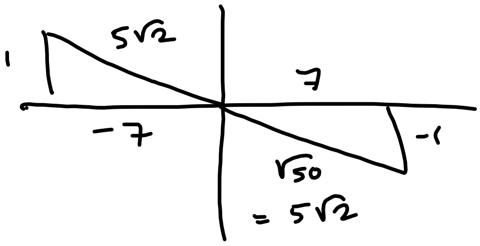
$= 5.63968419839$

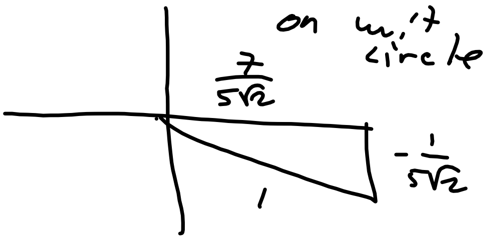
$\pi + \arcsin\left(\frac{3}{5}\right)$

$= 3.78509376238$

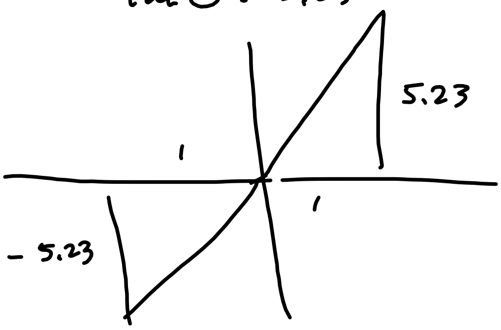
$\boxed{\theta \approx 5.6397,\ 3.7851}$

$\pi - \arcsin\left(-\frac{3}{5}\right)$, $2\pi + \arcsin\left(-\frac{3}{5}\right)$ is also good, but I prefer to think of $\theta' \in$ QI.

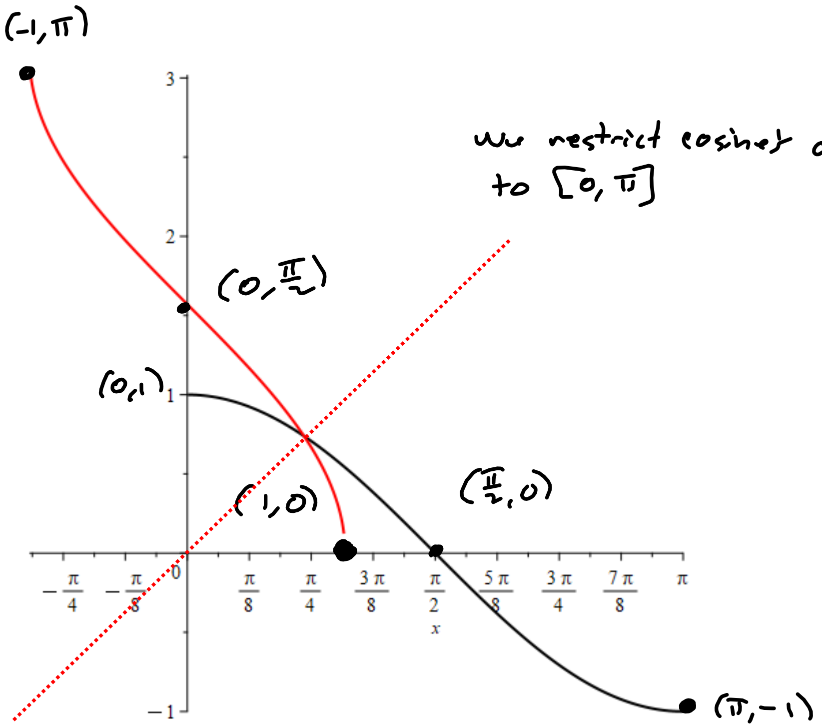




$\tan\theta = 5.23$

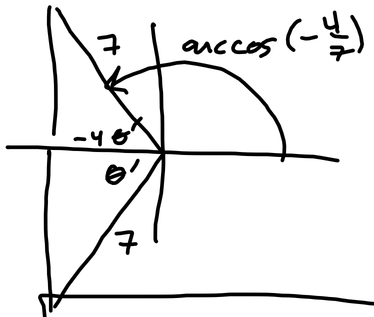


Now, cosine & arccosine.



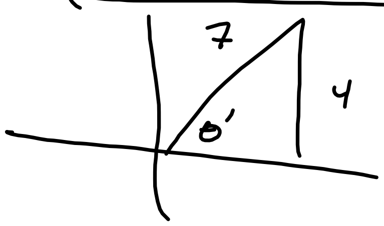
we restrict cosine's domain to $[0, \pi]$

b. (5 pts) $\sec(\theta) = -\dfrac{7}{4}$ $\Longleftrightarrow$ $\cos(\theta) = -\frac{4}{7}$



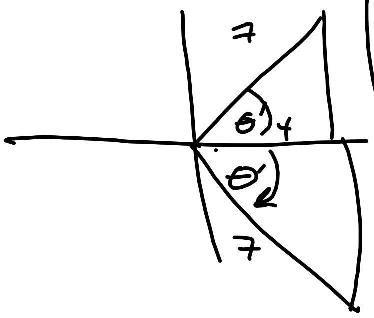
$\arccos(-\frac{4}{7}) \approx$

Look @ $\boxed{\arccos(\frac{4}{7}) = \theta'}$



$\boxed{\text{So } \theta = \pi - \theta', \pi + \theta'}$

a. $\cos\theta = \frac{4}{7}$



$\arccos\left(\frac{4}{7}\right) = \theta'$

EXACT

$\theta = \arccos\left(\frac{4}{7}\right),\ 2\pi - \arccos\left(\frac{4}{7}\right)$

$\theta = \theta',\ 2\pi - \theta'$

To 4 places: $\theta \approx .9626,\ 3.206$ → 5.3206. Brain fart

In degrees mode, see below:

$\arccos\left(\frac{4}{7}\right)$

$= (0.962550747885)\left(\frac{180°}{\pi}\right)$ To do degrees,...

$2\pi - \arccos\left(\frac{4}{7}\right)$

$= 5.32063455929$

$$\arccos\left(\frac{4}{7}\right) = 55.150095421$$

360 !

$$2\pi - \arccos\left(\frac{4}{7}\right) = -48.8669101138$$

oops!
Went to degrees mode!

$$\arccos\left(\frac{4}{7}\right) = 55.150095421$$

Much better!

$$360 - \arccos\left(\frac{4}{7}\right) = 304.849904579$$