J F M A M J J A S O N D

31 28 31 30 31 30 31 31 30 31 30 31

$\underbrace{31 \quad 28 \quad 31 \quad 30 \quad 31}_{151}$ $\quad \underbrace{\qquad}_{334}$

June 21st $- t = 172$

$$\begin{array}{r} 151 \\ +21 \\ \hline 172 \end{array} \qquad \begin{array}{r} 334 \\ +21 \\ \hline 355 \end{array}$$

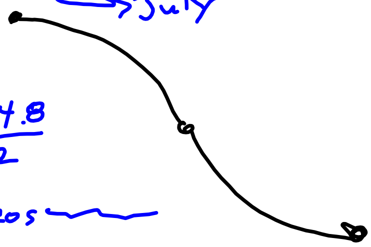
Dec 21st $- 355 = t$

Let's do International Fallls!

$(7, 78.6) \to$ July

$t =$ month

$a(\cos(b(x-c))) + d$



January = 1, 13, 25, ...

$(13, 13.8)$

$a: \dfrac{78.6 - 13.8}{2} = \dfrac{64.8}{2}$

$= 32.4 \qquad 32.4\cos$ ~~~

$d: \dfrac{78.6 + 13.8}{2} = \dfrac{92.4}{2} = 46.2 = y =$ midline (Good Guess for the average temp.

$32.4\cos(b(x-c)) + 46.2$

Period = 12 months

want $bx = 2\pi$ when $x = 12$

$12b = 2\pi$

$b = \dfrac{2\pi}{12} = \boxed{\dfrac{\pi}{6} = b}$

start @ high point: $x = \boxed{7 = c}$

$$\boxed{32.4\cos\left(\frac{\pi}{6}(x-7)\right) + 46.2}$$

This is what I'd be looking for on a test.

A function $f(x)$ is a rule that assigns to each x in some set (called the Domain, D) to <u>exactly one</u> y in another set (called the Range, R).

$$x \longrightarrow f(x) = y$$
$$f : D \longrightarrow R$$

$D = \{x \mid f(x) \text{ is real}\}$ FORMAL

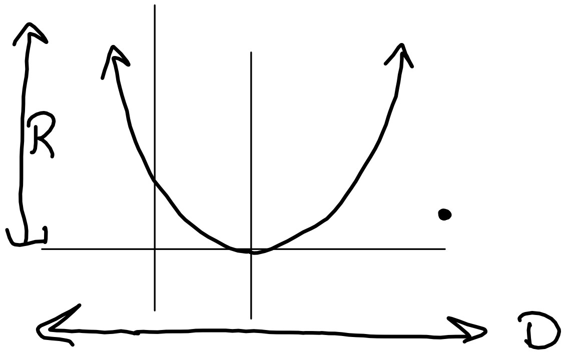
$= \{x \mid f(x) \text{ makes sense}\}$ REAL-WORLD

(No such thing as negative time, e.g.)

$R = \{y \mid y = f(x) \text{ for some } x \in D\}$

$f(x) = x^2 \implies D(f) = (-\infty, \infty) = \mathbb{R}$

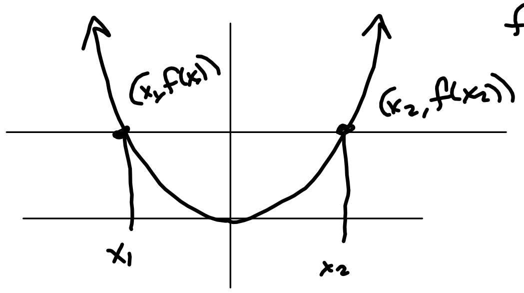
$\& \ R(f) = [0, \infty)$



"exactly one y"
Gives us the
<u>VERTICAL LINE</u> test.

A 1-to-1 function $f(x)$ is a function such that each $y \in R$ corresponds to exactly one $x \in D$.

This gives us the HORIZONTAL LINE Test



$f(x) = x^2$ is NOT 1-to-1.

$f(x_1) = f(x_2)$, but $x_1 \neq x_2$

$\Longrightarrow$

Solve $x^2 = y$ for x!

$\sqrt{x^2} = \sqrt{y}$

$|x| = \sqrt{y}$

$x = \pm\sqrt{y}$

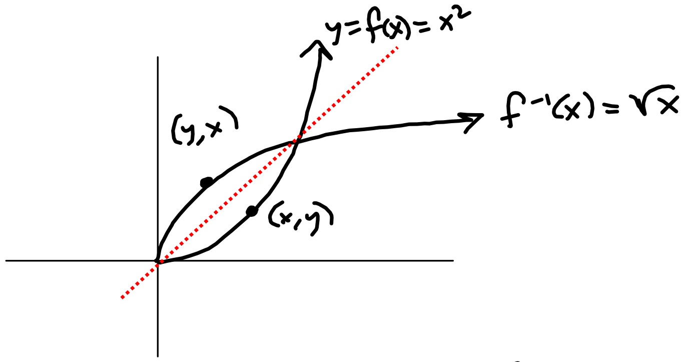
So if I want to have an inverse function, this fails!

$f^{-1}(x) = \pm\sqrt{x}$ Not a function.

Just an inverse relation.

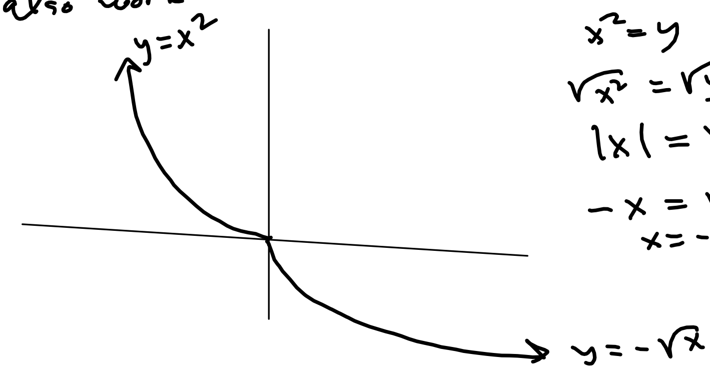
The fix: Restrict the domain!

Restrict to $x \in [0, \infty)$



That's the conventional restriction, but $x \in (-\infty, 0]$ would also work

$x \leq 0$



$x^2 = y$

$\sqrt{x^2} = \sqrt{y}$

$|x| = \sqrt{y}$ (If $x < 0$, then $|x| = -x$!)

$-x = \sqrt{y}$

$x = -\sqrt{y}$ $\Longrightarrow$ $f^{-1}(x) = -\sqrt{y}$.

Show that $\frac{x-1}{x+5}$ is 1-to-1.

Suppose $f(x_1) = f(x_2)$

$$\frac{x_1-1}{x_1+5} = \frac{x_2-1}{x_2+5}$$

$$(x_1-1)(x_2+5) = (x_2-1)(x_1+5)$$

$$x_1x_2 + 5x_1 - x_2 - 5 = x_2x_1 + 5x_2 - x_1 - 5$$

$$+x_1 \quad +x_2 \quad = \quad +x_2 \quad +x_1$$

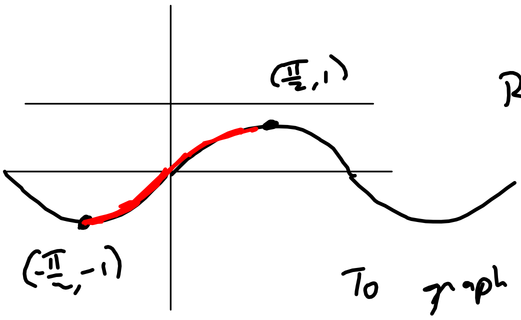
$$6x_1 = 6x_2$$

$x_1 = x_2 \Longrightarrow$ They're the same point & not different, after all

$\Longrightarrow f$ is 1-to-1.

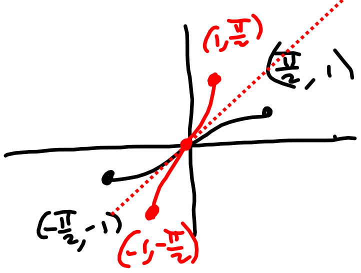
Good Bonus Question!

Trig functions are NOT 1-to1!!!!!

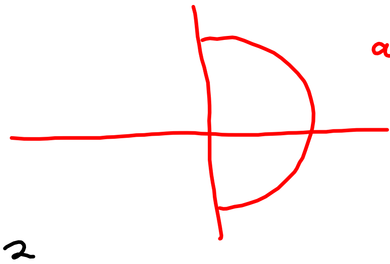


Restriction to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ in red.

To graph the inverse, just swap y's & x's!
" " " " " reflect about the line $y = x$



$\sin^{-1}(x) = \arcsin(x)$ in red!

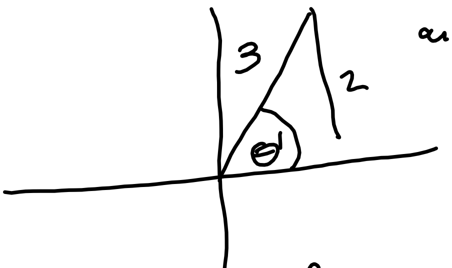


arcsine only reports angles between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$, inclusive.

solve $\sin(x) = \frac{2}{3}$

$\arcsin(\sin(x)) = \arcsin(\frac{2}{3})$

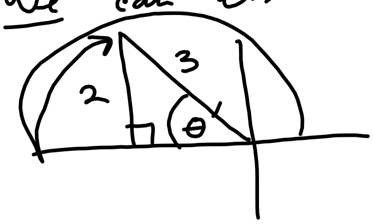
$x = \arcsin(\frac{2}{3})$? Almost, but not quite.



arcsine sees this

θ' = reference angle

We can also see



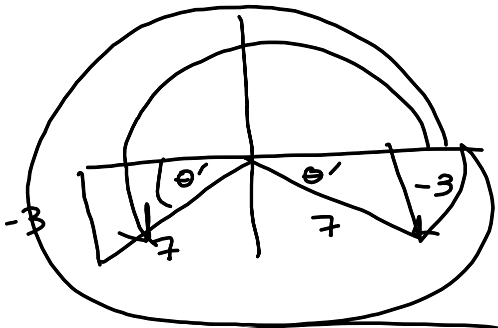
$\sin\theta = \frac{2}{3}$ also!

$\theta = \pi - \theta'$

$= \pi - \arcsin(\frac{2}{3})$

2 solutions in $[0, 2\pi)$: $\arcsin(\frac{2}{3})$, $\pi - \arcsin(\frac{2}{3})$

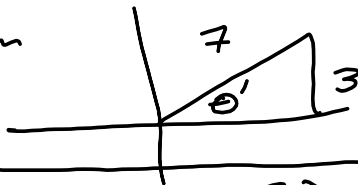
$\sin(x) = -\frac{3}{7}$ Find all sol'ns in $[0, 2\pi)$



$\arcsin\left(-\frac{3}{7}\right) \approx$ -0.442911044074

Want to think of θ' as a positive angle.

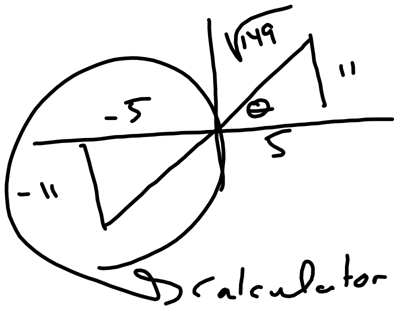
Then



So $\boxed{\theta = \pi + \theta',\ 2\pi - \theta'}$

$$\boxed{\theta' = \arcsin\left(\frac{3}{7}\right)}$$

$$\sin\left(\arctan\left(\frac{11}{5}\right)\right) = \frac{11}{\sqrt{149}}$$



$\theta = \arctan\left(\frac{11}{5}\right)$

$121 + 25 = 149$

calculator (arctangent) doesn't see it)

Next time, we do the same thing for cosine and tangent.