Modular arithmetic.

7 = 2 modulo 5, because 2 is the remainder when you divide 7 by 5.

26 = 1 mod 5. 26 = 5*5 + 1

$\mathbb{Z}/5\mathbb{Z} = \bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}$

$\bar{1} = \{1, 6, 11, 16, 21, 26, \ldots\}$

Pythagorean Identities:

on the unit circle $x^2+y^2=1$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

$\tan^2\theta + 1 =$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{1}\cdot\frac{\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$ $\sec^2\theta - 1 = \tan^2\theta$

$\boxed{\cot^2\theta + 1 = \csc^2\theta}$

Cofunction Identities

$\sin(\pi - \theta) = \cos\theta$

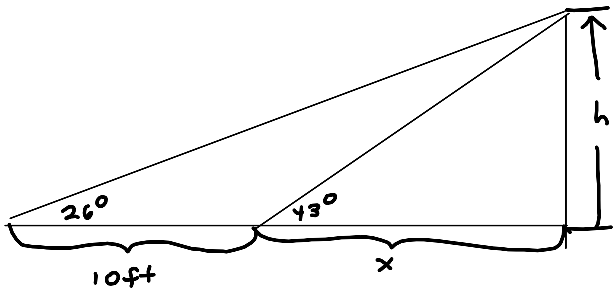
$\cos(\pi - \theta) = \sin\theta$

$\tan(\pi - \theta) = \cot\theta$

$\csc(\pi - \theta) = \sec\theta$

$\cot(\pi - \theta) = \tan\theta$

$\sec(\pi - \theta) = \csc\theta$



How tall is the light pole?

Let $\boxed{\begin{array}{l} h = \text{height of the pole (in feet)} \\ x = \text{distance from } 2^{nd} \text{ elevation reading to the pole (ft)} \end{array}}$

$\frac{h}{x} = \tan 43^\circ \implies h = x\tan(43^\circ) = ax$, where $a = \tan 43^\circ$

$\frac{h}{x+10} = \tan 26^\circ \implies h = (x+10)\tan(26^\circ) = b(x+10)$, where $b = \tan(26^\circ)$

$\implies ax = b(x+10)$, since $h = h$

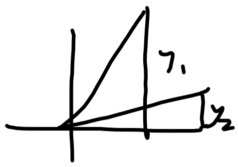
Solve for x!

$ax = bx + 10b \longrightarrow$

$ax - bx = 10b \longrightarrow$

$x(a-b) = 10b \longrightarrow$

$x = \frac{10b}{a-b} = \cancel{\frac{10\tan 43^\circ}{\tan 26^\circ - \tan 43^\circ}}$ Nope!



$= \frac{10\tan 26^\circ}{\tan 43^\circ - \tan 26^\circ} \approx$ 10.9656425608 ft

$h = x\tan 26^\circ$

$\approx \boxed{5.34830123147 \text{ ft} \approx h}$

$\cos^3\theta = (\cos\theta)^3$

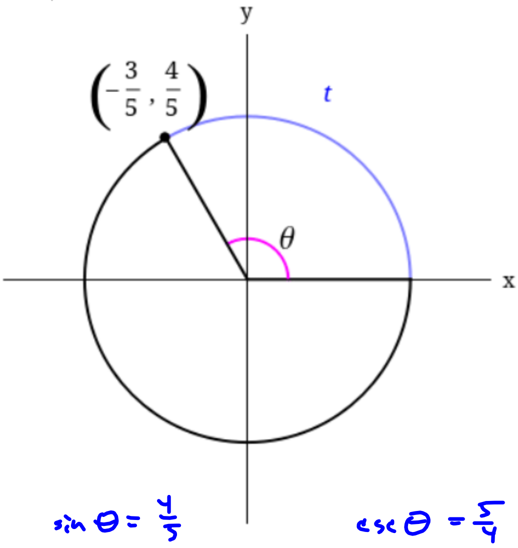
$\cos^{17}\theta = (\cos\theta)^{17}$

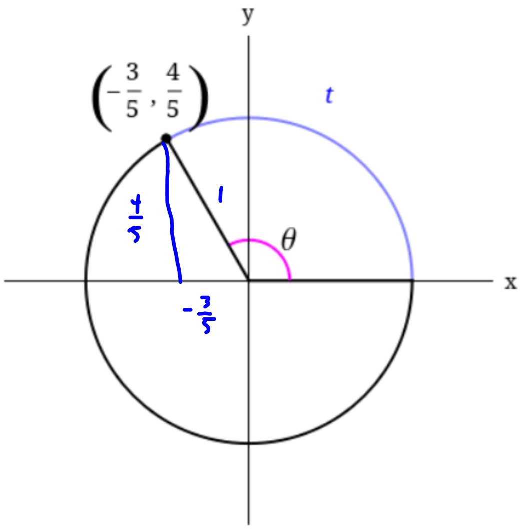
$\cos^{-1}\theta$ is forbidden?!

For inverse cosine, it were better to use

arccos(some #)

23. Find the exact values of the six trigonometric functions of the real number t.

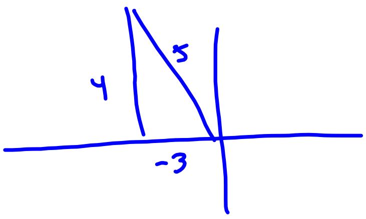




$\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\sec\theta = -\frac{5}{3}$

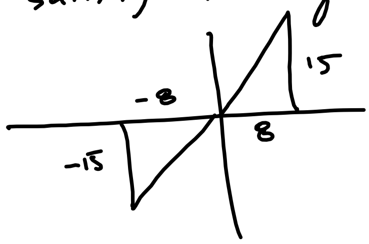
$\tan\theta = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$



Don't let 1.3 trick you into only seeing Quadrant I.

Find the 6 trig values for the angle satisfying $\tan\theta = \frac{15}{8}$

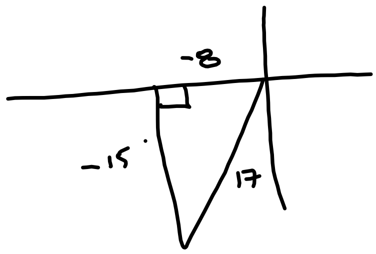
This is poorly posed! There are $\underline{2}$ angles in $[0, 2\pi)$ that satisfy this equation



One in QI.
.. .. QII!

The book adds the condition $\sin\theta < 0$.

This means: Quadrant III ($\sin\theta > 0$ QI)



$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

$\Rightarrow r = 17$

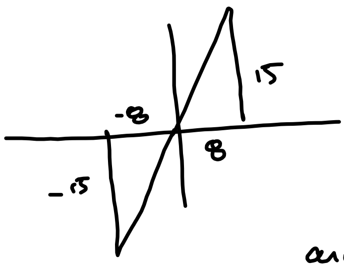
$\sin\theta = -\frac{15}{17}$ $\qquad \csc\theta = -\frac{17}{15}$

$\cos\theta = -\frac{8}{17}$ $\qquad \sec\theta = -\frac{17}{8}$

$\tan\theta = \frac{15}{8}$ $\qquad \cot\theta = \frac{8}{15}$

Find the EXACT Solutions in $[0, 2\pi)$ that solve

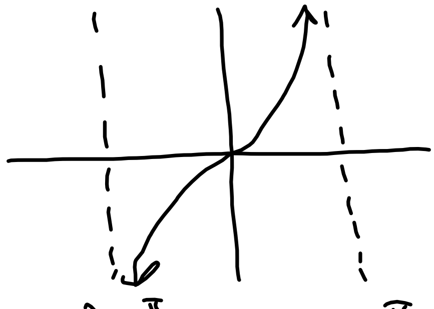
$\tan\theta = \frac{15}{8}$

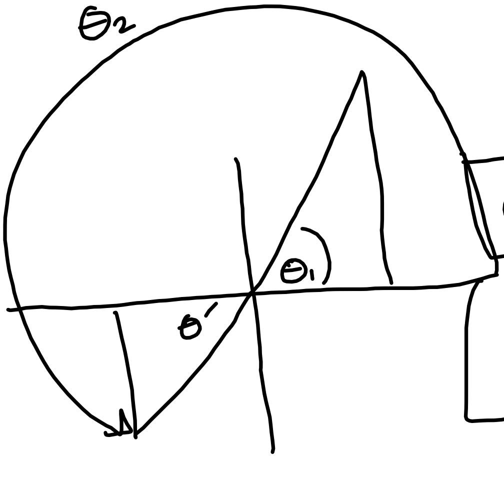


$\boxed{\arctan\left(\frac{15}{8}\right)}$ = the exact soln in $[0, \frac{\pi}{2})$

Can't make it simplify any farther and still be exact.

arctan(z) spits out values between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$



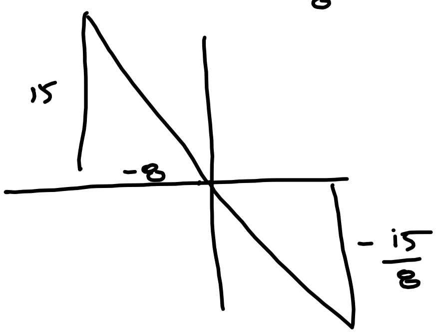


$\boxed{\theta_1 = \arctan\left(\frac{15}{8}\right)}$

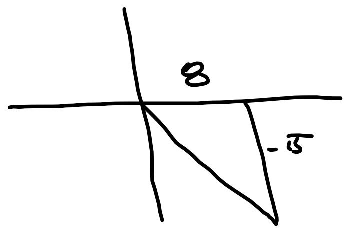
$$\boxed{\begin{aligned}\theta' &= \arctan\left(\tfrac{15}{8}\right)\\ \theta_2 &= \pi + \theta'\end{aligned}}$$

Find all sol'ns $\theta \in [0, 2\pi)$ such that

$$\tan\theta = -\frac{15}{8}$$

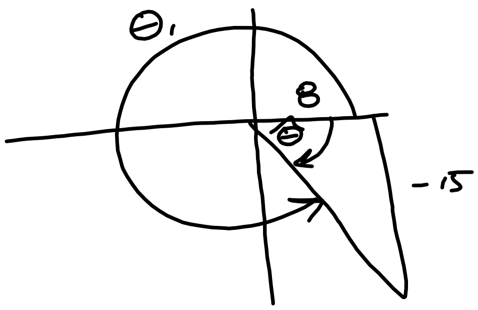


$\arctan\left(-\frac{15}{8}\right)$ calculator:



$\arctan\left(-\frac{15}{8}\right) \approx -61.9275130641^\circ$ oops! Degrees!

$\arctan\left(-\frac{15}{8}\right) \approx -1.08083900054$



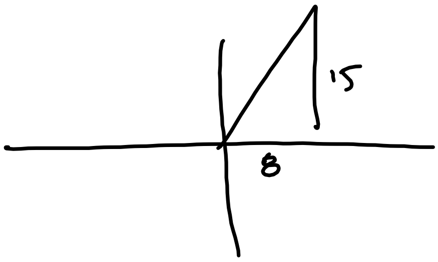
$\hat{\theta} = -1.08\ldots$

Want an angle between 0 & 2π. This isn't!

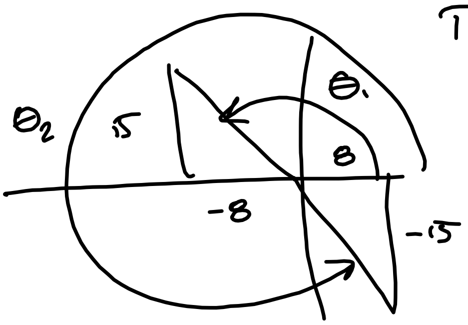
$$\theta_1 = 2\pi + \hat{\theta}$$
$$= 2\pi + \arctan\left(-\frac{15}{8}\right)$$

Another approach:

$\arctan\left(-\frac{15}{8}\right) = -\arctan\left(\frac{15}{8}\right)$



$\arctan\left(\frac{15}{8}\right) \longrightarrow$

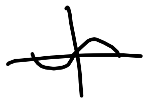


Then $\theta_1 = \pi - \arctan\left(\frac{15}{8}\right)$

$\theta_2 = 2\pi - \arctan\left(\frac{15}{8}\right)$

FACTS:

$\sin\theta$ odd

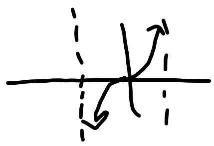


$\sin(-\theta) = -\sin\theta$

$\cos\theta$ even

$\cos(-\theta) = \cos\theta$

$\tan\theta$ odd



$\tan(-\theta) = \frac{\sin\theta}{\cos\theta} = \frac{-}{+} = -$

$= -\tan\theta$

Show that

$$\tan\theta \cot\theta = 1$$

<u>Proof</u>

$$\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = 1$$