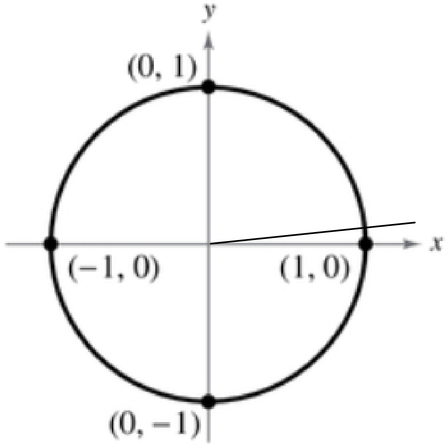
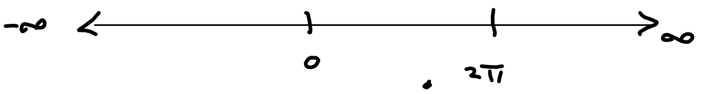
S 1.2 - The Unit Circle.

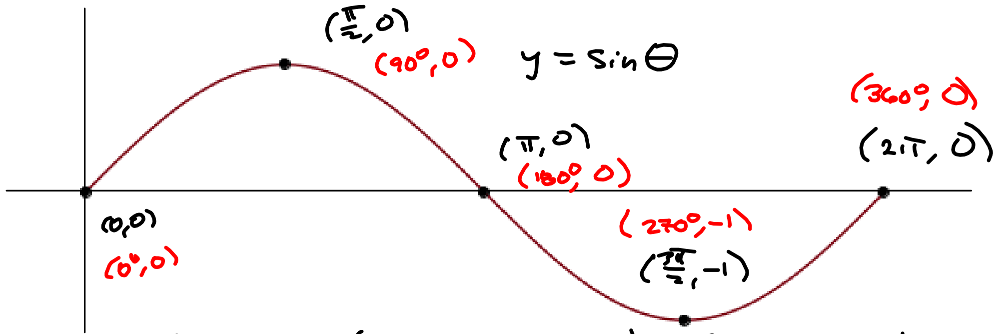


Map the y-values as you go around the circle, counter-clockwise. $y = \sin\theta$

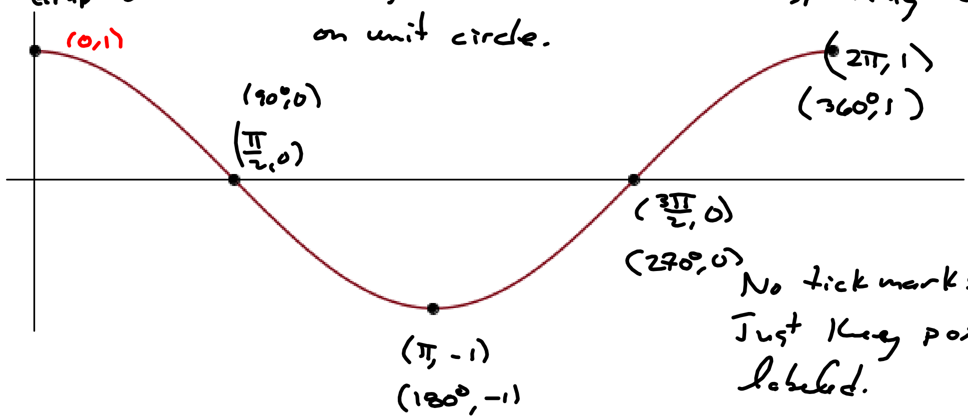


From Desmos: Click here for the animation.

Graph of $y = \sin\theta$

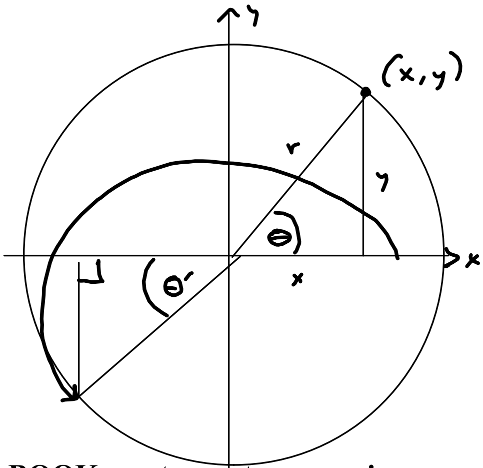


Graph of $\cos\theta$ gives x-coord. of corresponding (x, y) on unit circle.



No tick marks
Just key points labeled.

I'd rather think of sine and cosine as ratios of sides of a triangle to one another.



$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

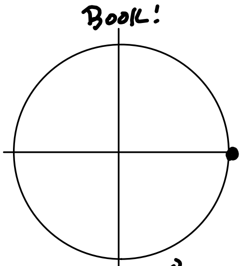
$\tan\theta = \frac{y}{x}$ = slope! $\cot\theta = \frac{x}{y}$

θ' = reference angle.

The BOOK wants you to memorize or carry around a 16-point unit circle. I want you to learn a few triangles in Quadrant I and then logic out the rest.

$270° \longrightarrow 4.71$

Book!

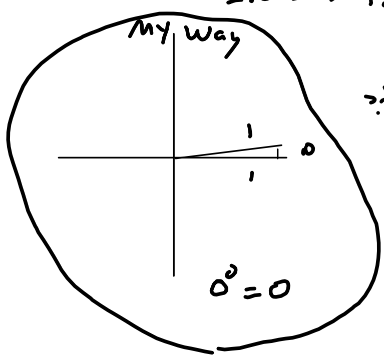


$(1,0)$

$\sin(0) = 0$

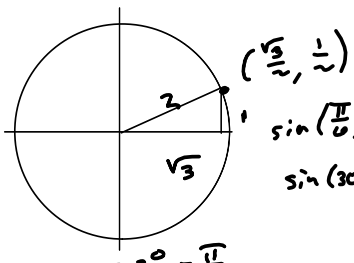
$\cos(0) = 1$

$0° = 0$



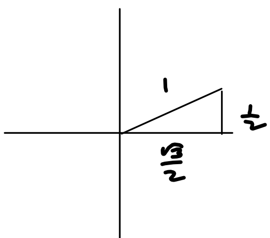
$\sin 0 = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 0 = \frac{x}{r} = \frac{1}{1} = 1$

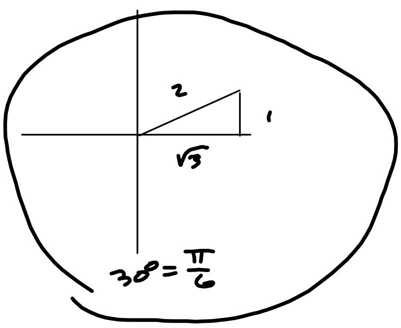


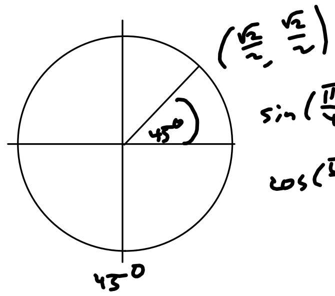
$\sin(\frac{\pi}{6}) = \frac{1}{2}$

$\sin(30°) = \frac{1}{2}$



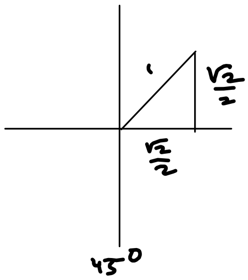
Better.

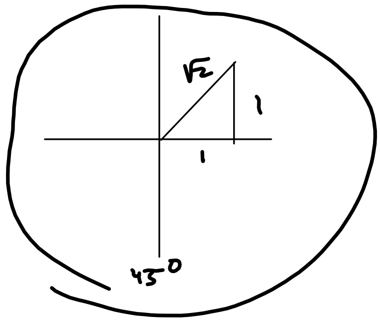


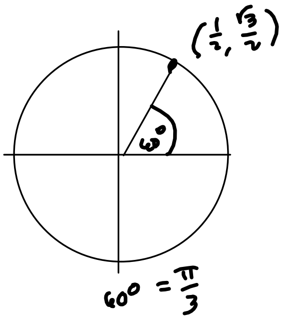


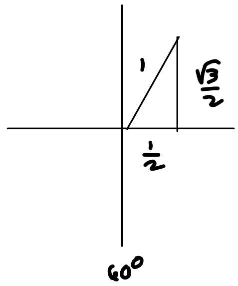
$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

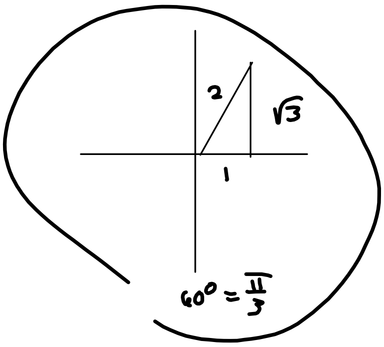
$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

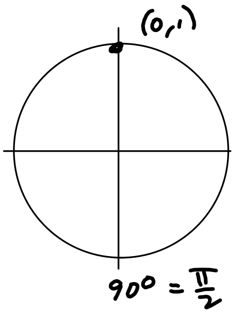






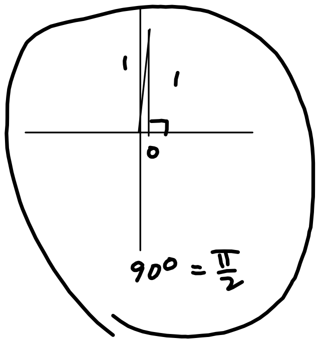


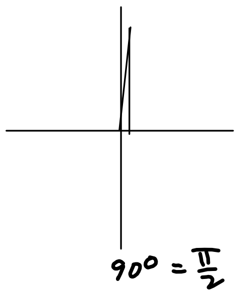




$\sin\left(\frac{\pi}{2}\right) = 1$

$\cos\left(\frac{\pi}{2}\right) = 0$

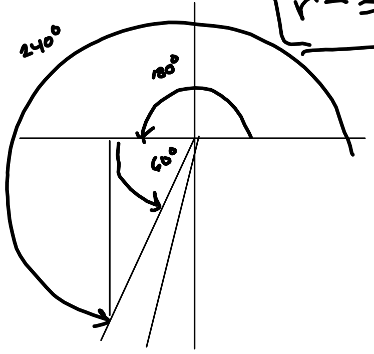




"Degenerate" triangles for quadrant angles 0° & 90°.

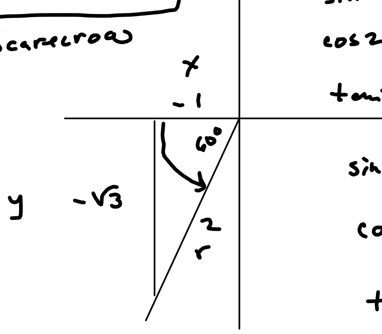
If you can learn these 5 triangles, you can generate the entire 16-point unit circle or any part of it you need.

Find 6 trigs for $\theta = 240°$



$\boxed{r^2 = x^2 + y^2}$

Scarecrow



$\sin 240° = -\frac{\sqrt{3}}{2}$ $\quad \csc 240° = -\frac{2}{\sqrt{3}}$

$\cos 240° = -\frac{1}{2}$ $\quad \sec 240° = -2$

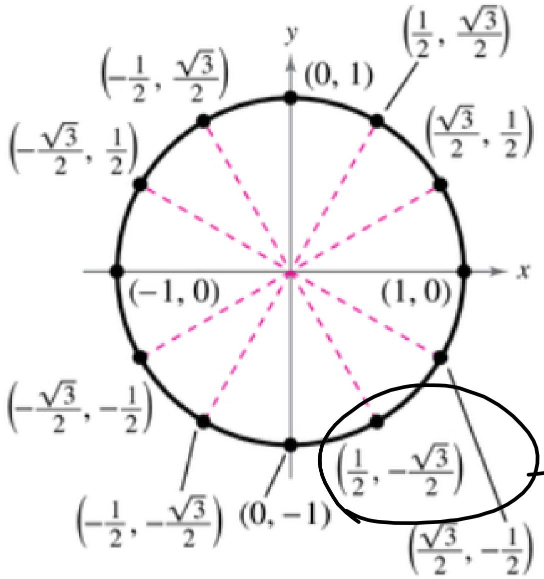
$\tan 240° = \frac{-\sqrt{3}}{-1} = \sqrt{3}$ $\quad \cot 240° = \frac{1}{\sqrt{3}}$

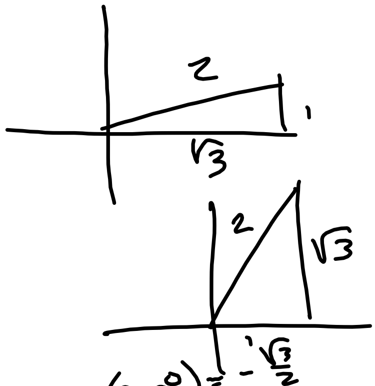
$\sin\theta = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$

12-point unit circle covers all the 30-60 triangles around the citcle.



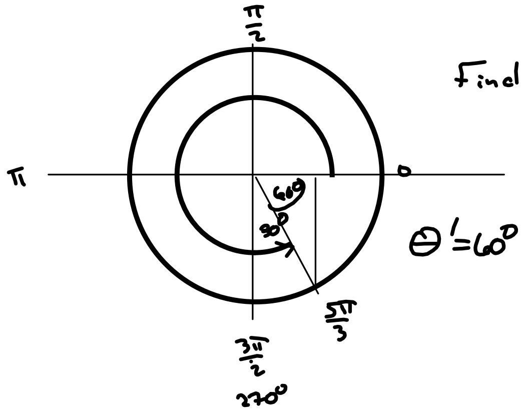


$\sin(300^\circ) = -\frac{\sqrt{3}}{2}$

$\cos(300^\circ) = \frac{1}{2}$

$\sin\left(\frac{5\pi}{3}\right) =$

$\cos\left(\frac{5\pi}{3}\right) =$



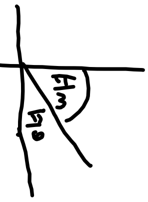
$\theta' = 60^\circ$

Find 6 trigs for $\theta = \frac{5\pi}{3}$

$\frac{5\pi}{3} = \frac{10\pi}{6}$

$\frac{3\pi}{2} = \frac{9\pi}{6}$

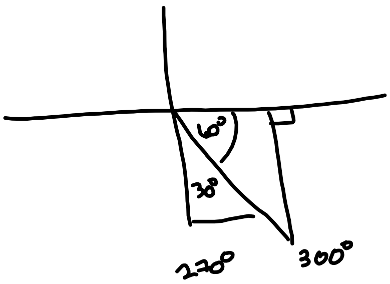
$\frac{5\pi}{3} - \frac{3\pi}{2} = \frac{10\pi}{6} - \frac{9\pi}{6} = \frac{\pi}{6}$



$\frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi - \pi}{6}$

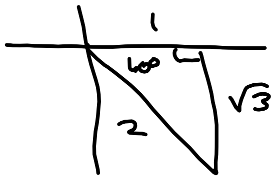
$= \frac{2\pi}{6} = \frac{\pi}{3}$

$\left(\frac{5\pi}{3}\right)\left(\frac{180^\circ}{\pi}\right) = (5)(60)^\circ = 300^\circ$



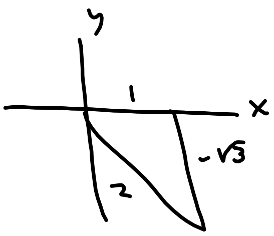
60–30 TRIANGLE

1–2–$\sqrt{3}$



$60^\circ = \theta' =$ reference angle.

Quadrant IV.



$\sin\theta = -\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2}$

$\tan\theta = -\sqrt{3}$

is OK w/ WebAssign.

$\csc\theta = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ simplified radical form.

$\sec\theta = 2$

$\cot\theta = \frac{1}{-\sqrt{3}}$

4. Find the point (x, y) on the unit circle that corresponds to the real number t.

$t = \frac{11\pi}{6}$

Book WAY: Look it up on the unit circle.

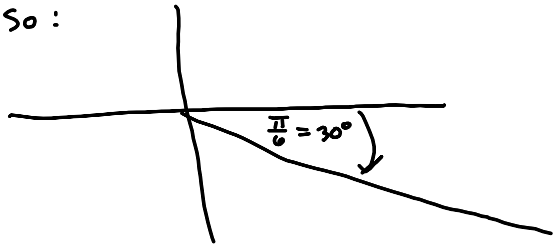
My way:

Find terminal ray

M1 $\frac{11\pi}{6}$ is $\frac{\pi}{6}$ less than $\frac{12\pi}{6} = 2\pi$

M2 $\left(\frac{11\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right) = 330^\circ$ That's 30° less than 360°.

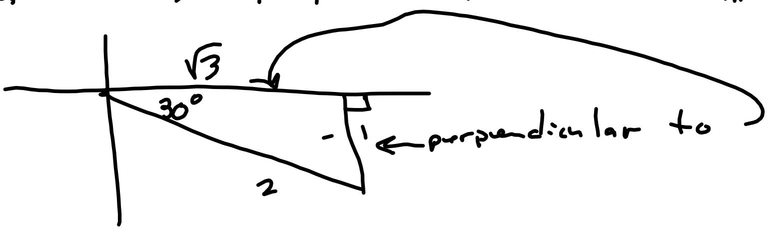
So:



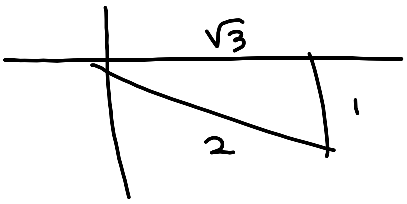
$\theta = 330^\circ$

$\theta' = 30^\circ$ $(360^\circ - 30^\circ)$

Drop (or lift) a perpendicular to the x-axis

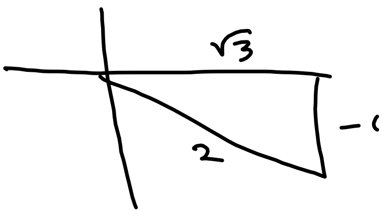


Supply the lengths for a 30-60 triangle:



Now overlay the signs, based on the quadrant.

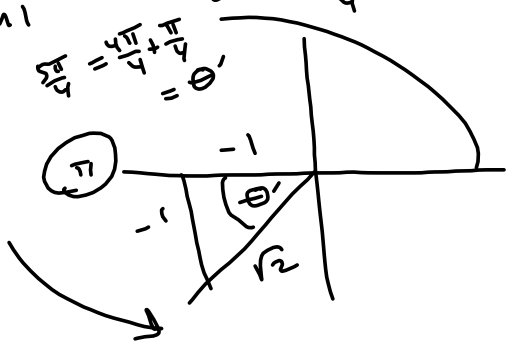
Here in QIV, $x > 0$ and $y < 0$



At speed. Find (x,y) on unit circle corresponding to $\theta = \frac{5\pi}{4}$

M1

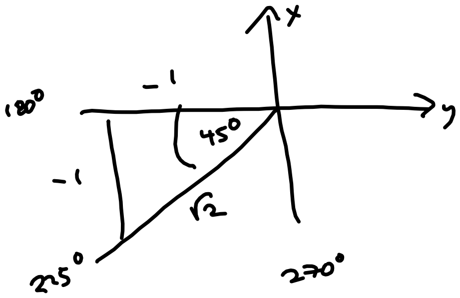
$\frac{5\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4}$
$= \theta'$



$\sin\theta = y = -\frac{1}{\sqrt{2}}$ OR $-\frac{\sqrt{2}}{2}$

$\cos\theta = x = -\frac{1}{\sqrt{2}}$ OR $-\frac{\sqrt{2}}{2}$

M2 $\frac{5\pi}{4} = \left(\frac{5\pi}{4}\right)\left(\frac{180^\circ}{\pi}\right) = 225^\circ$



$225^\circ - 180^\circ = 45^\circ =$ reference angle.

$x = -\frac{1}{\sqrt{2}}$

$y = -\frac{1}{\sqrt{2}}$

$\theta' = 45^\circ$

Hypotenuse always positive for Chapters 1-4.

What I will ask is something like:

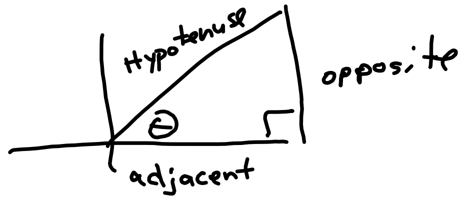
Find the values of the 6 trig functions at theta.

$\sin\theta = -\frac{1}{\sqrt{2}}$ $\csc\theta = -\sqrt{2}$

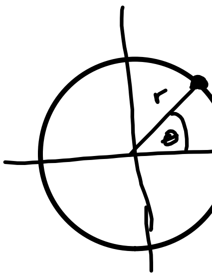
$\cos\theta = -\frac{1}{\sqrt{2}}$ $\sec\theta = -\sqrt{2}$

$\tan\theta = \frac{-1}{-1} = 1$ $\cot\theta = 1$

SOH CAH TOA



The only thing that will change when we move away from the unit circle to other circles of radius *r*, is:



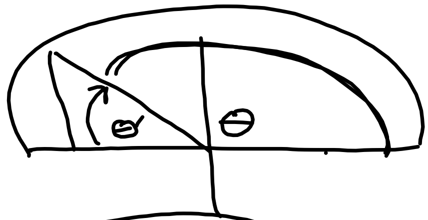
$(x,y) = (r\cos\theta, r\sin\theta)$,

The book only did $r=1$, so far.

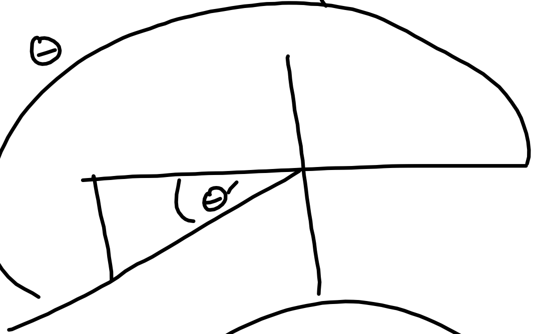
$(x,y) = (\cos\theta, \sin\theta)$ on unit circle $(r=1)$

$(x,y) = (r\cos\theta, r\sin\theta)$ on circle of radius r.

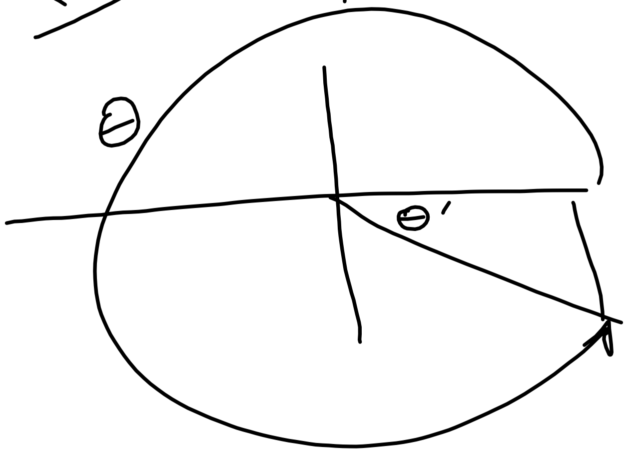
Now go back and re-label all your reference angles with primes, Steve.



In this pic,
$\theta' = \pi - \theta$

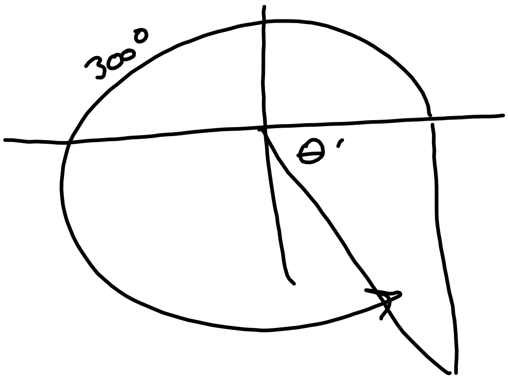


$\theta' = \theta - \pi$

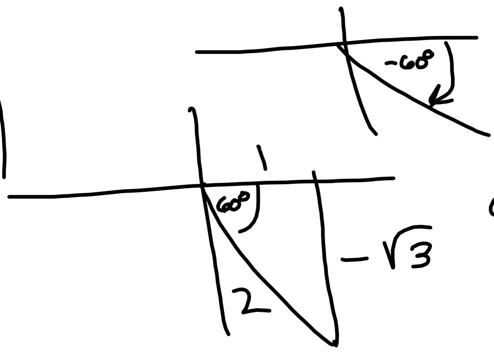


$\theta' = 2\pi - \theta$
$= 360^\circ - \theta$

$\theta = 300^\circ$



300° is coterminal with -60°



$\theta' = 60^\circ$

Yes. Soon we will be using calculators to crank these out. But it's imporant that you *see* what we're doing.

The "I don't know what I'm doing" answer to #4 is:

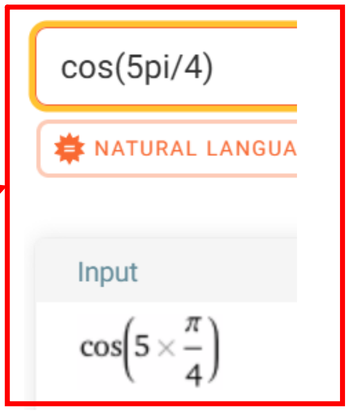
$$\cos\left(\frac{\pi}{4}\right) = 0.707106781187$$

$$\sin\left(\frac{\pi}{4}\right) = 0.707106781187$$

Click the above to open up Desmos.

But a decimal answer will not suffice.

A computer algebra system can give you an *exact* answer, without your understanding anything:



Click on the Wolfram Alpha session snapshot to open it up.

LarTrig10 1.2.048.

6. Use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

$\cot(-1.7)$

Desmos. Make sure it's in radians mode, by checking the wrench icon in the top right corner of the screen.

Might not have a cotangent key, but you know $\cot\theta = \dfrac{1}{\tan\theta}$, i.e., cotangent & tangent are reciprocals.

TI-36 is recommended.

I recommend TI-30 XS

Make sure you know your scientific calculator frontwards and backwards.

$\cot(-1.7)$

$= 0.129927464338$

$\dfrac{1}{\tan(-1.7)}$

$= 0.129927464338$

Desmos can do a straight-up cotangent. This is something you need to make sure you can do on a scientific calculator, because that's all that's allowed on a test. You may need to do a

1/tan(-1.7)

on your scientific calculator.

$\sin(90^\circ) = 1$

$\sin^{-1}(1) = 90^\circ$ or $\frac{\pi}{2} \approx 1.57$

arcsin

Solve:

$\sin(x) = \frac{1}{2}$

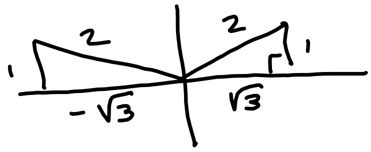
$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ Calculator

$\arcsin\left(\frac{1}{2}\right)$ = arc sine of $\frac{1}{2}$ = angle corresponding to a sine value of $\frac{1}{2}$.

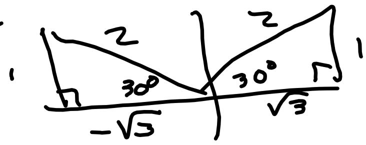
Find all solutions $\theta \in [0, 2\pi)$

$\theta \in [0^\circ, 360^\circ)$

to $\sin\theta = \frac{1}{2}$



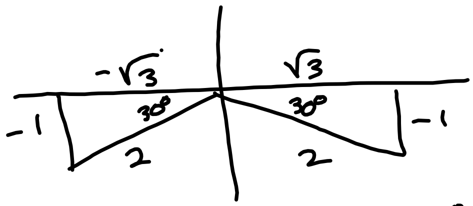
$\theta' = 30^\circ$



$\theta = 30^\circ,\ 180^\circ - 30^\circ = 150^\circ$

$\theta = 30^\circ, 150^\circ$

$\sin\theta = -\frac{1}{2}$

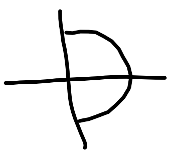


$\theta' = 30^\circ$

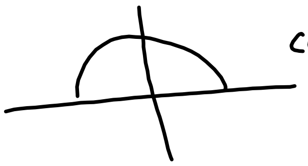
Calculator answer is -30°

$\theta = 210^\circ, 330^\circ$

$180^\circ + 30^\circ$ $\quad$ $360^\circ - 30^\circ =$

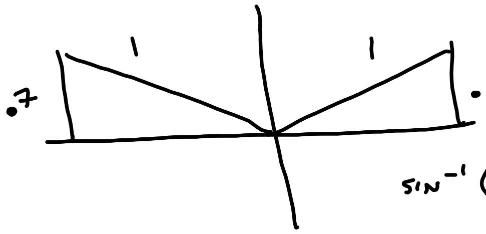


$\sin^{-1}$



$\cos^{-1}$ only spits out angles between 0° & 180°.

Solve $\sin\theta = .7$

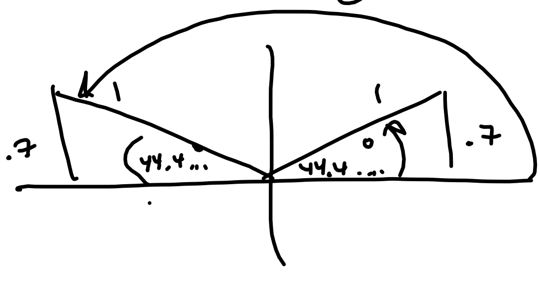


$\sin^{-1}(.7)$

$\sin\theta = .7$

$\sin^{-1}(\sin\theta) = \sin^{-1}(.7) \approx 44.4270040008^{\circ}$

$\theta =$



$\theta \approx 44.43^{\circ}$

OR $180^{\circ} - 44.427...$

$\approx 135.572995999^{\circ}$

"Evaluate the trig function using its period as an aid."

Sine & cosine have period 2π

(tangent & cotangent have period of π)

we call this "modding-out by a factor of 2π:"

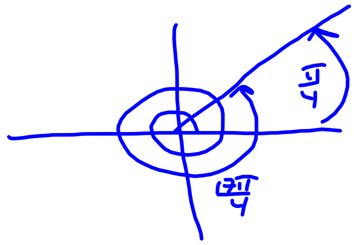
$\sin\left(\frac{17\pi}{4}\right)$

$\frac{17\pi}{4} \div 2\pi = \frac{17\pi}{4} \cdot \frac{1}{2\pi} = \frac{17}{8} = 2.125$

$2 \times 2\pi = 4\pi$

$\frac{17\pi}{4} - 4\pi = \frac{17\pi - 16\pi}{4} = \frac{1\pi}{4}$

$\frac{\pi}{4} = \frac{17\pi}{4}$ modulo 2π



Another way of saying this

$\frac{17\pi}{4} = 4\pi + \frac{\pi}{4} = 2(2\pi) + \frac{\pi}{4}$

$\frac{\pi}{4}$ is the remainder, 2π is the divisor, 2 is the quotient

we're basically finding the remainder when $\frac{17\pi}{4}$ is divided by 2π.

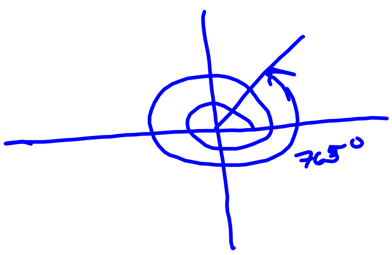
Degrees version:

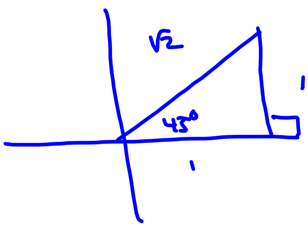
$\left(\frac{17\pi}{4}\right)\left(\frac{180^\circ}{\pi}\right) = 765^\circ$

$\frac{765^\circ}{360^\circ} = 2.125$

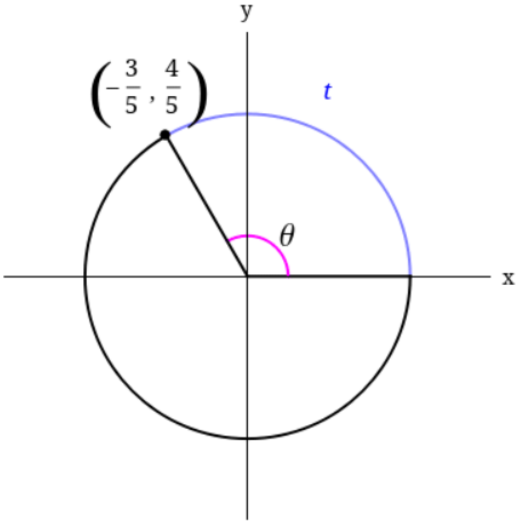
$(2)(360^\circ) = 720^\circ$

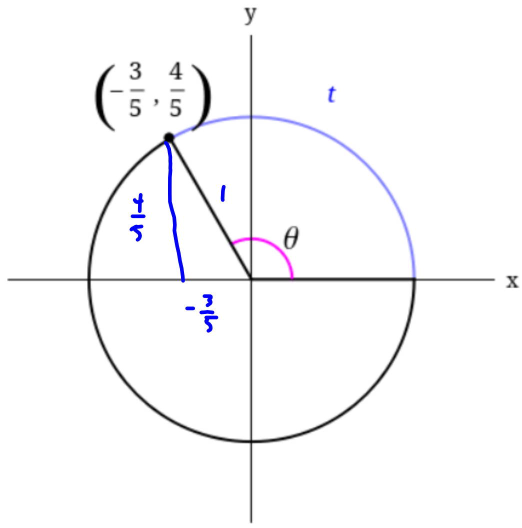
$765^\circ - 720^\circ = 45^\circ$





23 Find the exact values of the six trigonometric functions of the real number t.





$\sin\theta = \frac{4}{5}$ $\quad$ $\csc\theta = \frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\quad$ $\sec\theta = -\frac{5}{3}$

$\tan\theta = -\frac{4}{3}$ $\quad$ $\cot\theta = -\frac{3}{4}$