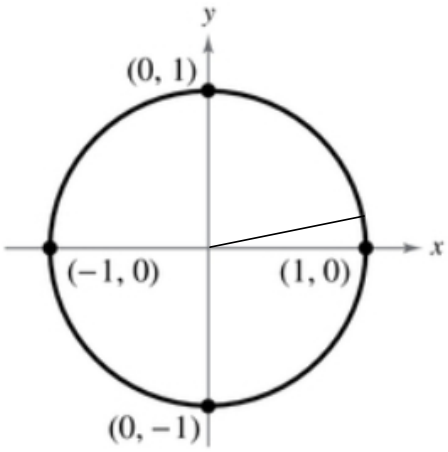
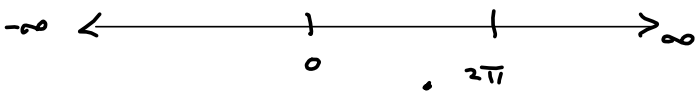
**S 1.2 - The Unit Circle.**

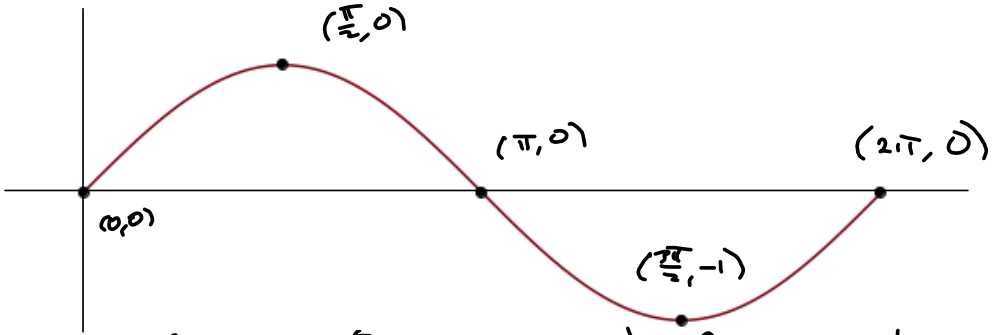



Map the y-values as you go around the circle, counter-clockwise. $y = \sin\theta$

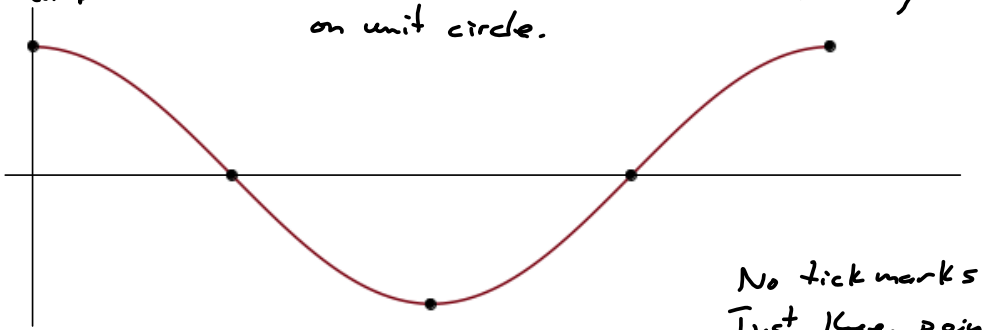



**From Desmos: Click here for the animation.**

Graph of $y = \sin\theta$

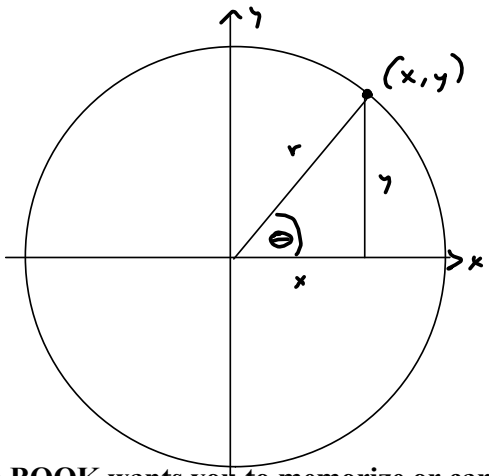



Graph of $\cos\theta$ gives x-coord. of corresponding (x,y) on unit circle.



No tick marks
Just key points labeled.

**I'd rather think of sine and cosine as ratios of sides of a triangle to one another.**

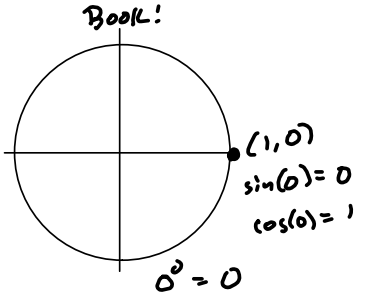



$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

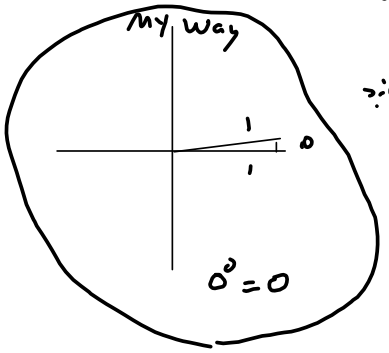
$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ = slope! $\quad \cot\theta = \frac{x}{y}$

**The BOOK wants you to memorize or carry around a 16-point unit circle. I want you to learn a few triangles in Quadrant I and then logic out the rest.**

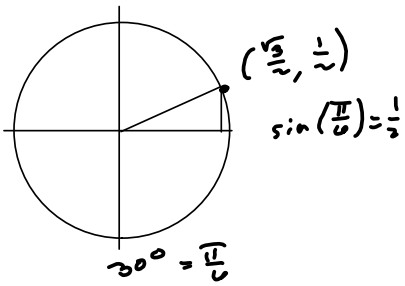


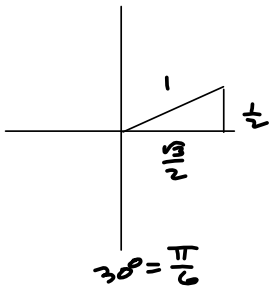

270° → 4:71



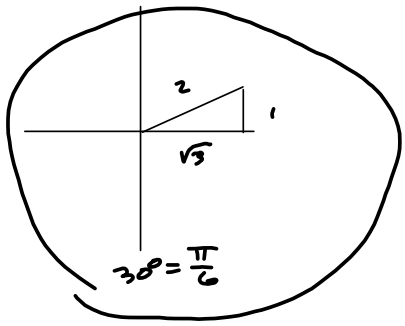

$\sin 0 = \frac{y}{r} = \frac{0}{1} = 0$

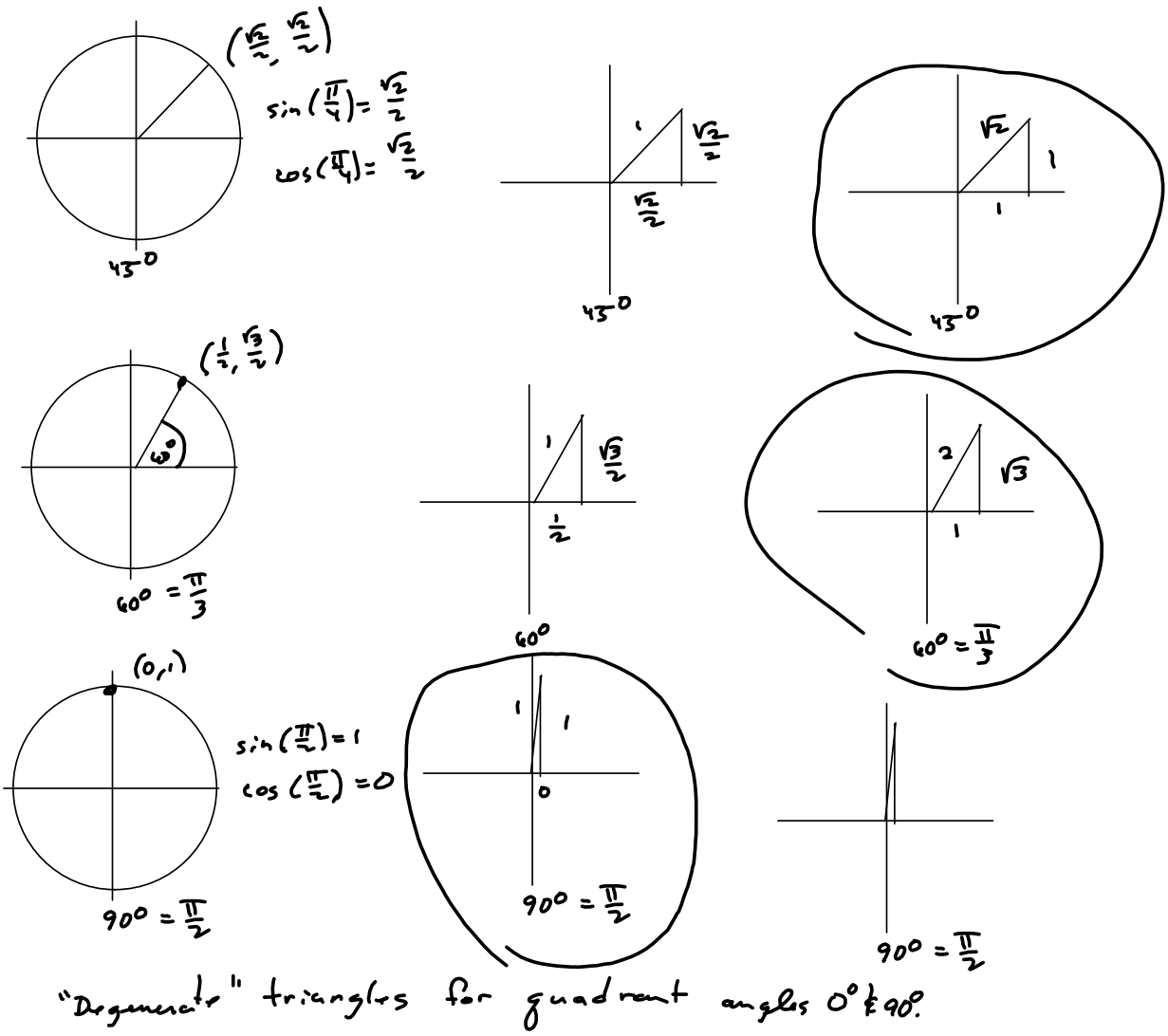
$\cos 0 = \frac{x}{r} = \frac{1}{1} = 1$




Better!

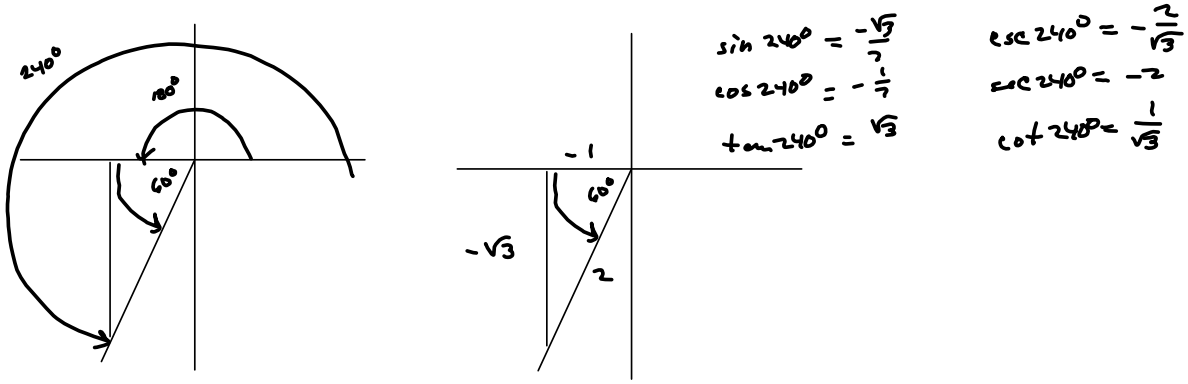




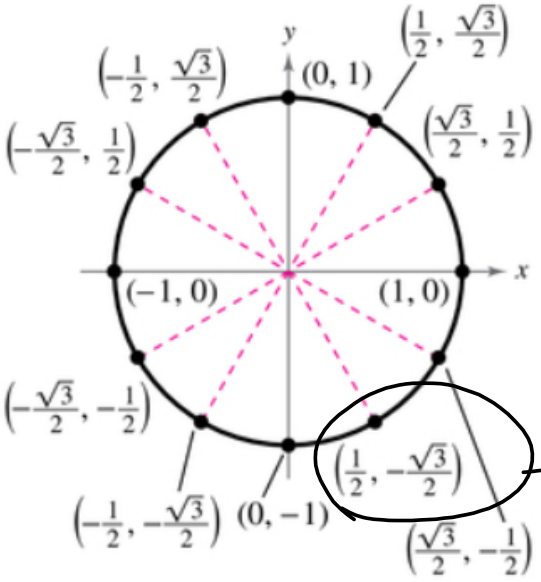

"Degenerate" triangles for quadrant angles 0° & 90°.

**If you can learn these 5 triangles, you can generate the entire 16-point unit circle or any part of it you need.**

Find 6 trigs for $\theta = 240°$



**12-point unit circle covers all the 30-60 triangles around the citcle.**



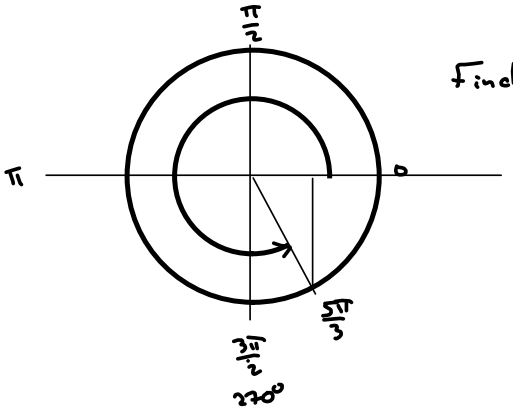

$\sin(300^\circ) = -\frac{\sqrt{3}}{2}$

$\cos(300^\circ) = \frac{1}{2}$

$\sin\left(\frac{5\pi}{3}\right) =$

$\cos\left(\frac{5\pi}{3}\right) =$

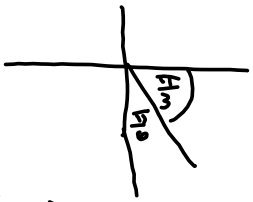
Find 6 trigs for $\theta = \frac{5\pi}{3}$




$\frac{5\pi}{3} = \frac{10\pi}{6}$

$\frac{3\pi}{2} = \frac{9\pi}{6}$

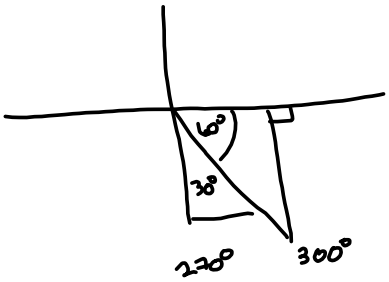
$\frac{5\pi}{3} - \frac{3\pi}{2} = \frac{10\pi}{6} - \frac{9\pi}{6} = \frac{\pi}{6}$




$\frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi - \pi}{6}$

$= \frac{2\pi}{6} = \frac{\pi}{3}$

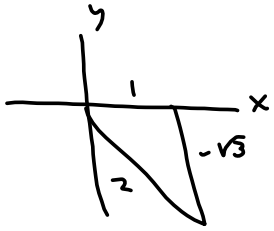
$\left(\frac{5\pi}{3}\right)\left(\frac{180^\circ}{\pi}\right) = (5)(60)^\circ = 300^\circ$




60-30 TRIANGLE

$1 - 2 - \sqrt{3}$


Quadrant IV.




$\sin\theta = -\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2}$

$\tan\theta = -\sqrt{3}$, etc.

**4.** Find the point $(x, y)$ on the unit circle that corresponds to the real number $t$.

$t = \frac{11\pi}{6}$

BOOK WAY: Look it up on the unit circle.

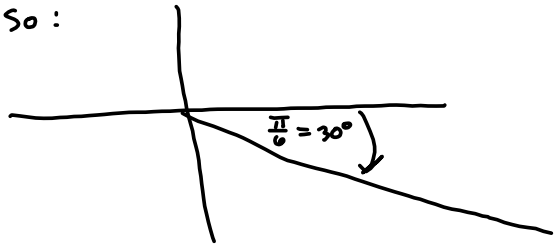
My way:

Find terminal ray

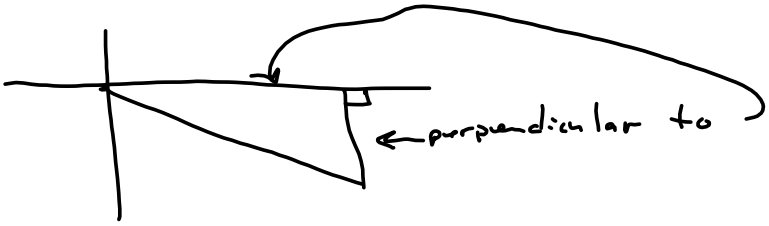
(M1) $\frac{11\pi}{6}$ is $\frac{\pi}{6}$ less than $\frac{12\pi}{6} = 2\pi$

(M2) $\left(\frac{11\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right) = 330^\circ$ That's $30^\circ$ less than $360^\circ$.

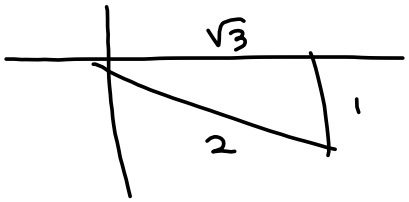
So:




Drop (or lift) a perpendicular to the x-axis



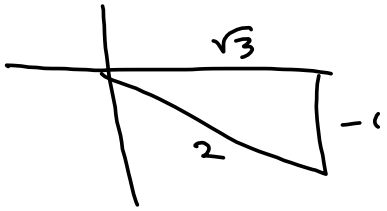

Supply the lengths for a 30-60 triangle:




---

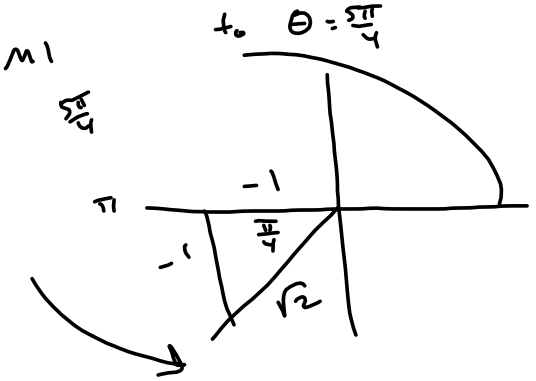
Now overlay the signs, based on the quadrant.

Here in QIV, $x > 0$ and $y < 0$



At speed. Find (x,y) on unit circle corresponding to $\theta = \frac{5\pi}{4}$

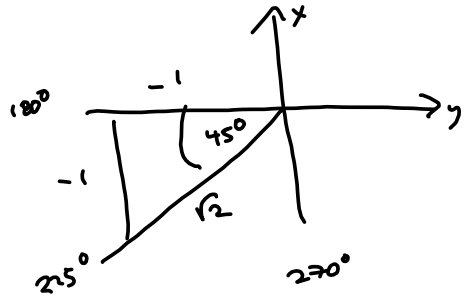
M1




$\sin\theta = y = -\frac{1}{\sqrt{2}}$ OR $-\frac{\sqrt{2}}{2}$

$\cos\theta = x = -\frac{1}{\sqrt{2}}$ OR $-\frac{\sqrt{2}}{2}$

M2 $\frac{5\pi}{4} = \left(\frac{5\pi}{4}\right)\left(\frac{180^\circ}{\pi}\right) = 225^\circ$




$225^\circ - 180^\circ = 45^\circ$ = reference angle.

$x = -\frac{1}{\sqrt{2}}$

$y = -\frac{1}{\sqrt{2}}$

**What I will ask is something like:**

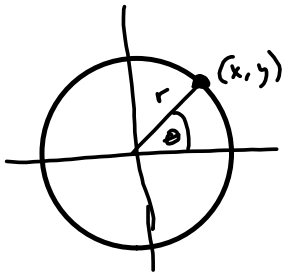
**Find the values of the 6 trig functions at theta.**

$\sin\theta = -\frac{1}{\sqrt{2}}$ $\quad \csc\theta = -\sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}}$ $\quad \sec\theta = -\sqrt{2}$

$\tan\theta = \frac{-1}{-1} = 1$ $\quad \cot\theta = 1$

**The only thing that will change when we move away from the unit circle to other circles of radius *r*, is:**




The book only did $r = 1$, so far.

$(x, y) = (\cos\theta, \sin\theta)$ on unit circle $(r = 1)$

$(x, y) = (r\cos\theta, r\sin\theta)$ on circle of radius $r$

**Now go back and re-label all your reference angles with primes, Steve.**

**Yes. Soon we will be using calculators to crank these out. But it's imporant that you *see* what we're doing.**

**The "I don't know what I'm doing" answer to #4 is:**

$\cos\left(\frac{\pi}{4}\right)$

$= 0.707106781187$

$\sin\left(\frac{\pi}{4}\right)$

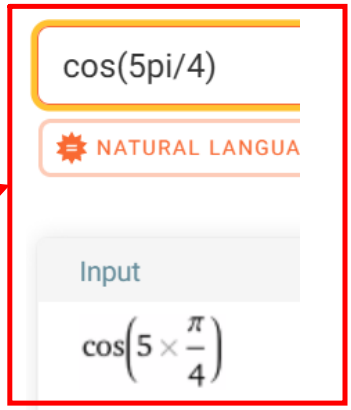
$= 0.707106781187$

**Click the above to open up Desmos.**

**But a decimal answer will not suffice.**

**A computer algebra system can give you an *exact* answer, without your understanding anything:**

**Click on the Wolfram Alpha session snapshot to open it up.**



**6.** Use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

$$\cot(-1.7)$$

**Desmos. Make sure it's in radians mode, by checking the wrench icon in the top right corner of the screen.**

Might not have a cotangent key, but you know $\cot\theta = \frac{1}{\tan\theta}$, i.e., cotangent & tangent are reciprocals.

So,

$\cot(-1.7)$ = 0.129927464338

$\frac{1}{\tan(-1.7)}$ = 0.129927464338

**Desmos can do a straight-up cotangent. This is something you need to make sure you can do on a scientific calculator, because that's all that's allowed on a test. You may need to do a**

**1/tan(-1.7)**

**on your scientific calculator.**

"Evaluate the trig function using its period as an aid."

Sine & cosine have period $2\pi$

(tangent & cotangent have period of $\pi$)

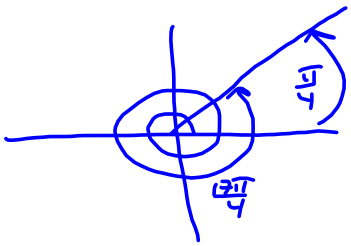
We call this "modding-out by a factor of $2\pi$:"

$\sin\left(\frac{17\pi}{4}\right)$

$\frac{17\pi}{4} \div 2\pi = \frac{17\pi}{4} \cdot \frac{1}{2\pi} = \frac{17}{8} = 2.125$

$2 \times 2\pi = 4\pi$

$\frac{17\pi}{4} - 4\pi = \frac{17\pi - 16\pi}{4} = \frac{1\pi}{4}$




$\frac{\pi}{4} = \frac{17\pi}{4}$ modulo $2\pi$

Another way of saying this

$\frac{17\pi}{4} = 4\pi + \frac{\pi}{4} = 2(2\pi) + \frac{\pi}{4}$

$\frac{\pi}{4}$ is the remainder, $2\pi$ is the divisor, 2 is the quotient

We're basically finding the remainder when $\frac{17\pi}{4}$ is divided by $2\pi$.

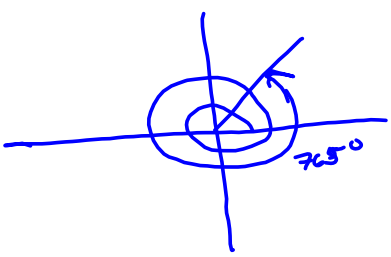
Degrees version:

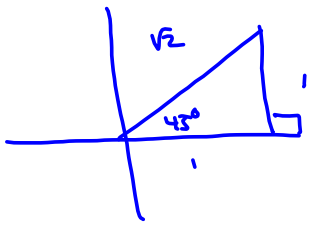
$\left(\frac{17\pi}{4}\right)\left(\frac{180^\circ}{\pi}\right) = 765^\circ$

$\frac{765^\circ}{360^\circ} = 2.125$

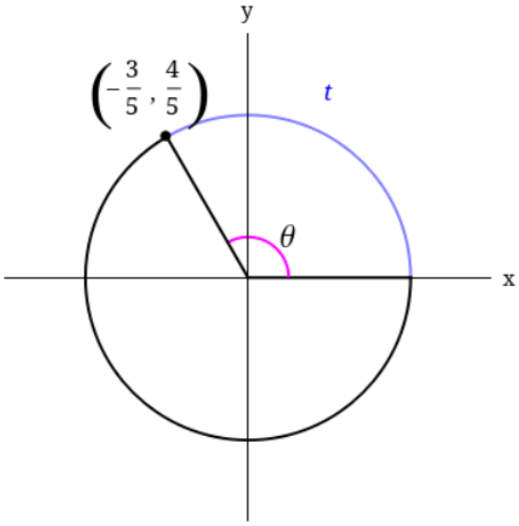
$(2)(360^\circ) = 720^\circ$

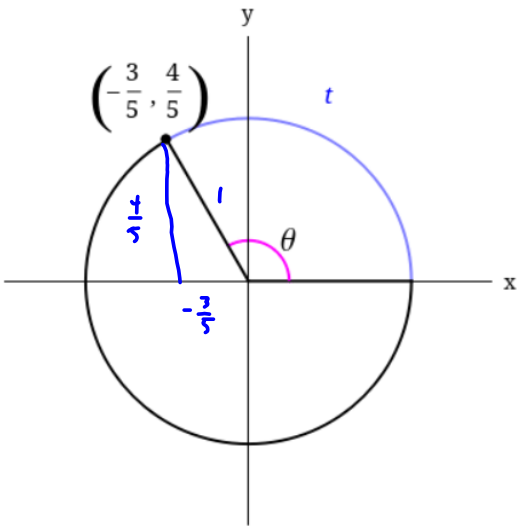
$765^\circ - 720^\circ = 45^\circ$



23 Find the exact values of the six trigonometric functions of the real number $t$.




$\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\sec\theta = -\frac{5}{3}$

$\tan\theta = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$