1420 **Week 14** **Mills**

1 $x = 4\cos t,\ y = 3\sin t$

a (5pts)

$\Rightarrow \frac{x}{4} = \cos t,\ \frac{y}{3} = \sin t$

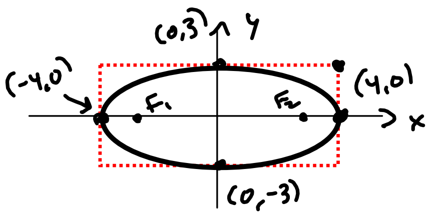
$\Rightarrow \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = \cos^2 t + \sin^2 t = 1$

$\Rightarrow \boxed{\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1}$ ellipse!

b (5pts)

sketch $a = 4,\ b = 3,$

Center $= (0,0)$



c (5pts) Find F_1, F_2

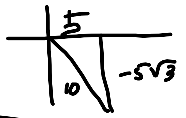
$c^2 = a^2 - b^2 = 4^2 - 3^2 = 7 \Rightarrow c = \sqrt{7} \approx 2.64575$

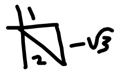
$$\boxed{\begin{aligned} F_1 &= (-\sqrt{7}, 0) \\ F_2 &= (\sqrt{7}, 0) \end{aligned}}$$

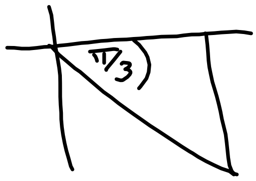
1420 **Week 14** **Mills**

(2) (5pts) Convert to polar coords. Express 4 ways with $\theta \in (-2\pi, 2\pi)$

$(5, -5\sqrt{3})$

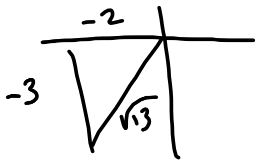






$= \boxed{\left(10, \frac{5\pi}{3}\right) \text{ or } \left(10, -\frac{\pi}{3}\right) \text{ or } \left(-10, \frac{2\pi}{3}\right) \text{ or } \left(-10, -\frac{4\pi}{3}\right)}$

(3) (5pts) Convert $(-2,-3)$ to polar coords. θ in radians. $0 < \theta < 2\pi$.



$\approx (3.605551275, 4.124386378)$

$\approx \boxed{(3.606, 4.124) \approx (r, \theta)}$

(4) (5pts) $\left(\sqrt{13}, \pi + \arctan\left(\frac{3}{2}\right)\right)$ **Exact Answer for #3.**

OR $\left(\sqrt{13}, \pi + \arccos\left(\frac{2}{\sqrt{13}}\right)\right)$

OR $\left(\sqrt{13}, \pi + \arcsin\left(\frac{3}{\sqrt{13}}\right)\right)$

(5) $r = 2\sin(2\theta) + 1$

ONE METHOD. 2^{nd} Method on Next PAGE.

Strategy: Find zeros ($r = 0$) and intersections with the midline ($r = 1$).

To find the zeros, we want to find all $\theta \in [0, 2\pi] \ni r = 0$

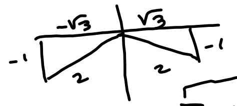
This means we want all $2\theta \in [0, 4\pi] \ni r = 0.$

$r = 0$

$2\sin(2\theta) + 1 = 0$

$2\sin(2\theta) = -1$

$\sin(2\theta) = -\frac{1}{2}$



$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6},$ $\boxed{\frac{31\pi}{6} \text{ is too far.}}$

+2π
+2π

$\Rightarrow \theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$\boxed{A = \left(\frac{7\pi}{12}, 0\right), B = \left(\frac{11\pi}{12}, 0\right), C = \left(\frac{19\pi}{12}, 0\right), D = \left(\frac{23\pi}{12}, 0\right)}$

Low Points are $\frac{1}{2}$-way between intercepts

$\frac{\frac{7\pi}{12} + \frac{11\pi}{12}}{2} = \frac{18\pi}{24} = \frac{3\pi}{4}, \quad \frac{\frac{19\pi}{12} + \frac{23\pi}{12}}{2} = \frac{42\pi}{24} = \frac{7\pi}{4}$

$\boxed{E = \left(\frac{3\pi}{4}, -1\right), F = \left(\frac{7\pi}{4}, -1\right)}$

Directly: $2\sin(2\theta) + 1 = -1$

$2\sin(2\theta) = -2$

$\sin(2\theta) = -1$

p+2π

$2\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

To find midline points:

Logic: They divide 2π into 4 equal parts:

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\boxed{G = (0,1), H = \left(\frac{\pi}{2}, 1\right), I = (\pi, 1), J = \left(\frac{3\pi}{2}, 1\right), K = (2\pi, 1)}$

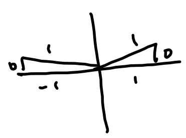
Directly: $2\sin(2\theta) + 1 = 1$

$2\sin(2\theta) = 0$

$\sin(2\theta) = 0$

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

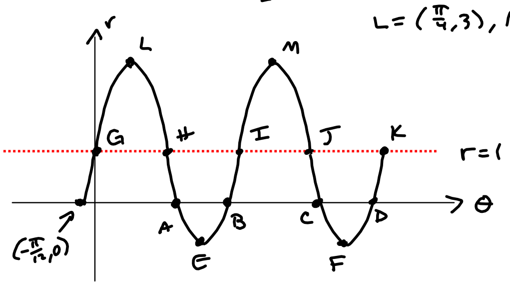
$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi.$



High points: Halfway between midline pts:

$\theta = \frac{0 + \frac{\pi}{2}}{2} = \frac{\pi}{4}, \quad \frac{\pi + \frac{3\pi}{2}}{2} = \frac{5\pi}{4}$

$L = \left(\frac{\pi}{4}, 3\right), M = \left(\frac{5\pi}{4}, 3\right)$



(5) $r = 2\sin(2\theta) + 1$

$r = 2\sin(2\theta) + 1$

Amplitude $a = 2$

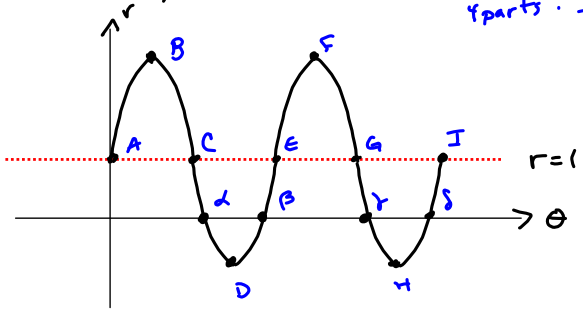
starts @ $\theta = 0$

$r = 1$ Midline

Period: $2\theta = 2\pi$ when $\theta = \pi$ = Period.

Graphing over $[0, 2\pi]$: 1st Graph 1 period $\to \pi$. then Repeat.

4 parts: $\frac{\pi}{4}$ = increment



$A = (0, 1)$

$B = (\frac{\pi}{4}, 3)$

$C = (\frac{\pi}{2}, 1)$

$D = (\frac{3\pi}{4}, -1)$

$E = (\pi, 1)$

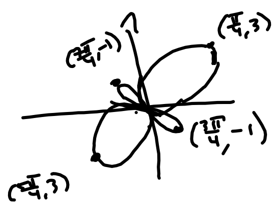
$F = (\frac{5\pi}{4}, 3)$

$G = (\frac{3\pi}{2}, 1)$

$H = (\frac{7\pi}{4}, -1)$

$I = (2\pi, 1)$

Now, fill in the zeros:

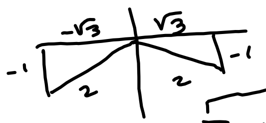


$r = 0$

$2\sin(2\theta) + 1 = 0$

$2\sin(2\theta) = -1$

$\sin(2\theta) = -\frac{1}{2}$



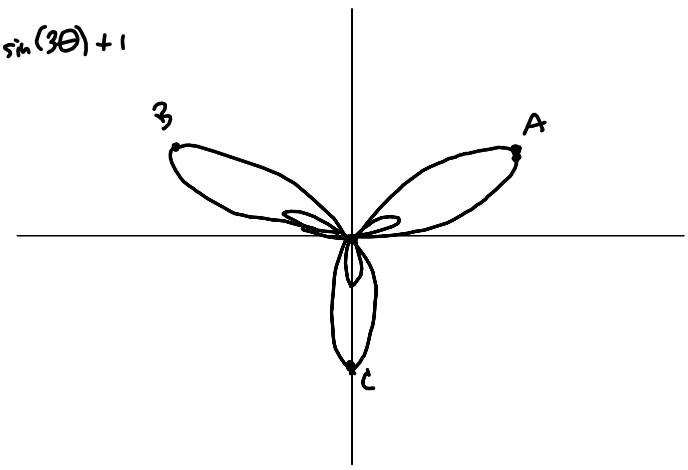
$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6},$ $\boxed{\frac{31\pi}{6} \text{ is too far.}}$

$+2\pi = \frac{12\pi}{6}$

$\Rightarrow \theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$$\boxed{\alpha = (\tfrac{7\pi}{12}, 0), \beta = (\tfrac{11\pi}{12}, 0), \gamma = (\tfrac{19\pi}{12}, 0), \delta = (\tfrac{23\pi}{12}, 0)}$$

$r = 2\sin(3\theta) + 1$



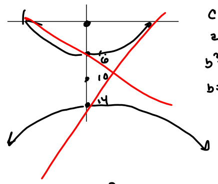
$A =$

(6) Identify & sketch.

(a) (5pts) $\dfrac{18}{1-2\sin\theta} = \dfrac{2(9)}{1-2\sin\theta}$

$r\left(\frac{\pi}{2}\right) = \frac{18}{1-2} = -18$

$r\left(\frac{3\pi}{2}\right) = \frac{18}{1+2} = \frac{18}{3} = 6$



$c = 10$

$a = 6$

$b^2 = c^2 - a^2 = 100 - 36 = 64 \Rightarrow b = 8$

$b = \sqrt{108}$

$100 - 36 = 64 = b^2 \Rightarrow b = 8$

$c^2 - a^2 = b^2 \Rightarrow$

$e = \frac{c}{a} = \frac{10}{4} = \frac{5}{2}$

$r = \dfrac{ep}{1-e\sin\theta}$

$= \dfrac{\frac{5}{2}p}{1-\frac{5}{2}\sin\theta} = \dfrac{5p}{2-5\sin\theta}$

$r\left(\frac{\pi}{2}\right) = \frac{5p}{-3} = -14 \Rightarrow 5p = 42 \Rightarrow p = \frac{42}{5}$

$r\left(\frac{3\pi}{2}\right) = \frac{5p}{2+5} = \frac{5p}{7} = 6 \Rightarrow$

$5p = 42$

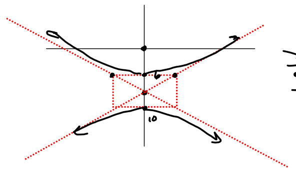
$p = \frac{42}{5}$ ✓

$r = \dfrac{\frac{5}{2}\left(\frac{42}{5}\right)}{1-\frac{5}{2}\sin\theta} = \dfrac{21}{1-\frac{5}{2}\sin\theta}$

$= \dfrac{42}{2-5\sin\theta}$

$c = 10,\ a = 6$

$c^2 - a^2 = 100 - 36 = 64 = b^2 \Rightarrow b = 8$



$\cancel{\dfrac{x^2}{8^2} - \dfrac{(y+10)^2}{6^2}} = 1$

$(y+10)^2$

$e = \frac{c}{a} =$

$\frac{x^2}{64} +$

$c = 10,\ a = 6,\ b = 8$

$e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

$r\left(\frac{\pi}{2}\right) = \dfrac{\frac{5}{3}p}{1-\frac{5}{3}\sin\theta}\quad -14$

$= \dfrac{\frac{5}{3}p}{-\frac{2}{3}} = -\frac{5}{2}p = -14 \Rightarrow p = \frac{-28}{-5} = \frac{28}{5}$

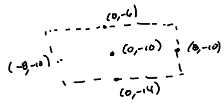
$\dfrac{(y+10)^2}{36} - \dfrac{x^2}{64} = 1$

$r = \dfrac{\frac{5}{3}\left(\frac{28}{5}\right)}{1-\frac{5}{3}\sin\theta} = \dfrac{\frac{28}{3}}{1-\frac{5}{3}\sin\theta}$

$= \dfrac{28}{3-5\sin\theta}$

$r\left(\frac{\pi}{2}\right) = \frac{28}{3-5} = \frac{28}{-2} = -14$

$r\left(\frac{3\pi}{2}\right) = \frac{28}{3+5} = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$

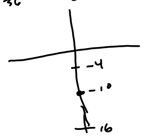


$\dfrac{(y+10)^2}{36} - \dfrac{x^2}{64} = 1$

$\dfrac{(y+10)^2}{36} = 1 + \dfrac{x^2}{64} = \dfrac{x^2+64}{}$

$y+10$

$a = 6$



$a^2 + b^2 = 10^2 = c^2$

$e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

$r,\ e = \frac{10}{6} = \frac{5}{3},\ \cancel{p = 10}$

$r = \dfrac{ep}{1-e\sin\theta} = \dfrac{\frac{5}{3}(\ \ )}{1-\frac{5}{3}\sin\theta} = \dfrac{5p}{3-5\sin\theta}$

$r\left(\frac{\pi}{2}\right) = \frac{5p}{3-5} = \frac{5p}{-2} = -16 \Rightarrow$

$5p = 32$

$p = \frac{32}{5}$

$r = \dfrac{5\left(\frac{32}{5}\right)}{3-5\sin\theta}$

$= \dfrac{32}{3-5\sin\theta}$

$r\left(\frac{3\pi}{2}\right) = \frac{5p}{3+5} = \frac{5p}{8} = 4 \Rightarrow$

$5p = 32$

$p = \frac{32}{5}$

$\dfrac{(y+10)^2}{36} - \dfrac{x^2}{64} = 1$

$64(y+10)^2 - 36x^2 = (36)(64)$

$64(y+10)^2 = 36x^2 + 36(64) = 36(x^2+64)$

$(y+10)^2 = \dfrac{36(x^2+64)}{64}$

$y + 10 = \pm\dfrac{6\sqrt{x^2+64}}{8} \Rightarrow y = -10 \pm \dfrac{3\sqrt{x^2+64}}{4}$

1420 **Week 14** **Mills**

$$\frac{32}{3-5\sin\theta} = r$$

$$\frac{32}{3-5\frac{y}{r}} = r \qquad \frac{32r}{3r-5y} = \sqrt{x^2+y^2}$$

$$\frac{32\left(\sqrt{x^2+y^2}\right)}{3\sqrt{x^2+y^2}-5y} = \sqrt{x^2+y^2}$$

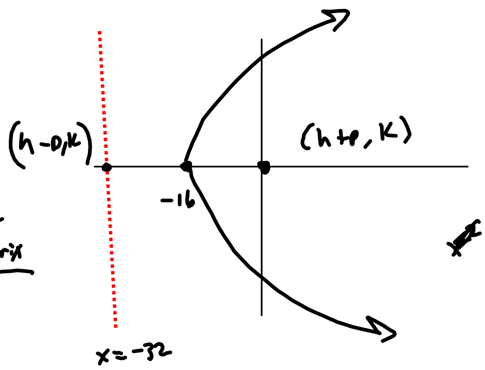
$$32\sqrt{x^2+y^2} = 3(x^2+y^2) - 5y\sqrt{x^2+y^2}$$

1420 **Week 14** **Mills**

$\frac{32}{1-\cos\theta}$ polar

$e = 1 = \frac{c}{a}$

p = dist. from focus to directrix $= 32$



rectangular

$y^2 = 4px$

$y^2 = 4 \cdot 16x = 64x$

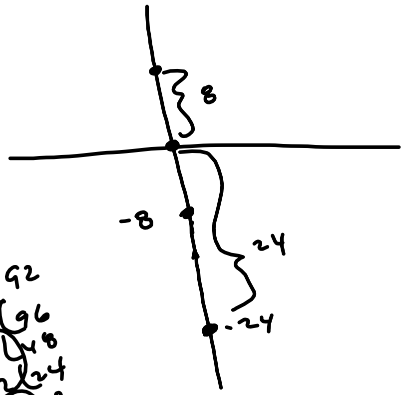
p = distance from focus to vertex

$r = \frac{48}{4+2\sin\theta} = \frac{48}{1+\frac{1}{2}\sin\theta}$

$e = \frac{1}{2} \Rightarrow$ ellipse , directrix above

$r(\frac{\pi}{2}) = \frac{48}{4+2} = \frac{48}{6} = 8$

$r(\frac{3\pi}{2}) = \frac{48}{4-2} = \frac{48}{2} = 24$



$c = 8$

$e = \frac{1}{2} \Rightarrow = \frac{c}{a} = \frac{8}{a} \Rightarrow a = 16$

$a^2 - c^2 = 16^2 - 8^2$

$= 2^2 \cdot 8^2 - 8^2$

$= 8^2(4-1) = 64 \cdot 3 = 192$

$= b^2 \Rightarrow$

$b = \sqrt{192}$

$=$

$192: 2 \cdot 96,\ 2 \cdot 48,\ 2 \cdot 24,\ 2 \cdot 12,\ 2 \cdot 6,\ 3$

$192 = 8\sqrt{3}$

$1.732 \times 8 = 13.856$

$128 + 64 = 192$

$\frac{(y+8)^2}{8^2} - \frac{x^2}{192} = 1$

1420 Week 14 Mills

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