MAT 1420 Week 13 Written Assignment
45 pts Covers Sections 4.1 – 4.5

4.1 - Complex Numbers
4.2 - Complex Solutions of Equations
4.3 - The Complex Plane
4.4 - Trigonometric Form of Complex Numbers
4.5 - DeMoivre's Theorem

1. Simplify the products and quotients.

   a. (5 pts) $(3+2i)(5-7i)$

   b. (5 pts) $(3+2i)(3-2i)$

   c. (5 pts) $\dfrac{3+2i}{3-2i}$

   d. (5 pts) $\left(5\left(\cos\left(\dfrac{2\pi}{3}\right)+i\sin\left(\dfrac{2\pi}{3}\right)\right)\right)\left(7\left(\cos\left(\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{\pi}{4}\right)\right)\right)$

2. Let $f(x)=x^4+2x^3-18x^2+106x-91$

   a. (5 pts) Use the fact that $2-3i$ is a zero of $f(x)$ to find all the zeros of $f$ without using a graph.

   b. (5 pts) Graph $f(x)$. Include all intercepts in your graph. Do not graph with a graphing utility. Include only the features your work suggests. Assume $f$ is nice and smooth, and that's it.

3. (5 pts) Use DeMoivre's Theorem to generate the five zeros of $f(x)=x^5-32$ in trigonometric form. Then approximate your answers to 3 decimal places.

4. Use DeMoivre's Theorem to generate the 3rd roots of $z=1-\sqrt{3}\,i$ in two main steps:

   a. (5 pts) Write $z$ in trigonometric form.

   b. (5 pts) Write the 3 roots in trigonometric form. Give exact answers. Express all arguments in radians.

   c. (**Bonus 5 pts**) Draw a very neat sketch showing $z$ and its 3rd roots.

5. (**Bonus 5 pts**) Show that the parametric equations $x=4\cos(t), y=3\sin(t)$ describe an ellipse.