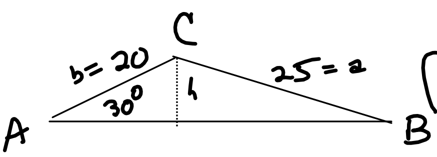
1. For each of the following, state whether there is no solution, one solution or there are two solutions. Then explain why, using one or two pictures, and some comparisons. Finally, solve the triangle using the Law of Sines. In the case of two solutions, B_1 and B_2, you will find B_1, using the Law of Sines, and then $B_2 = 180^{\circ} - B_1$, etc.

   a. (5 pts) The measure of angle A is 30°. The length of side b is 20 and the length of side a is 25.



$\frac{h}{20} = \sin 30^{\circ} = \frac{1}{2} \longrightarrow h = 10 < 20 < 25$

$h < b < a$

$\boxed{h < b < a \longrightarrow 1 \text{ solution.}}$

$\frac{\sin B}{b} = \frac{\sin A}{a} \longrightarrow \sin B = \frac{b \sin A}{a} = \frac{20\left(\frac{1}{2}\right)}{25} = \frac{10}{25} = \frac{2}{5}$

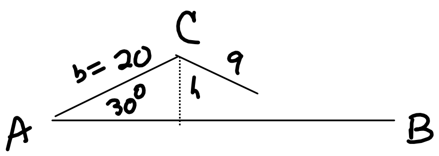
$\longrightarrow B = \sin^{-1}\left(\frac{2}{5}\right) = \arcsin\left(\frac{2}{5}\right) \approx \boxed{23.5781784782^{\circ} \approx B}$

$C = 180^{\circ} - A - B \approx 180^{\circ} - 30^{\circ} - 23.5781784782 \approx \boxed{126.421821522^{\circ} \approx C}$

$\frac{c}{\sin C} = \frac{a}{\sin A} \longrightarrow c = \frac{a \sin C}{\sin 30^{\circ}} \approx \frac{25 \sin(126.421821522)}{\frac{1}{2}} \approx$

$\approx \boxed{40.2333865504 \approx c \text{ (length)}}$

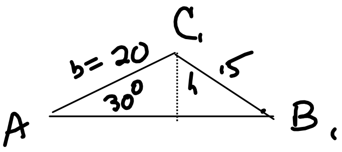
   b. (5 pts) The measure of angle A is 30°. The length of side b is 20 and the length of side a is 9.



$\left.\begin{matrix} h = 10 \\ a = 9 \end{matrix}\right\}$ $\boxed{a < h \longrightarrow \text{No Solution}}$

1420 MILLS

c. (5 pts) The measure of angle A is 30°. The length of side b is 20 and the length of side a is 15.

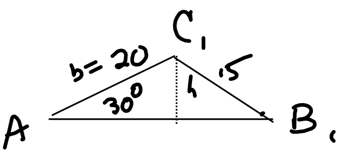


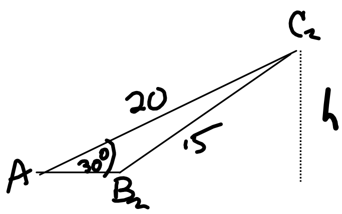
$h = 10$
$a = 15$
$b = \cancel{25}$

$h < a < b \Longrightarrow$
2 sol'ns:

① B_1 acute, C_1 obtuse ② B_2 obtuse, C_2 acute

I'm sort of breaking a rule, putting these side by side, but the right column is calculated directly from the right column, so it's an exception.





$\sin B_1 = \dfrac{b \sin A}{a} = \dfrac{20 \sin(30^{\circ})}{15} = \dfrac{4\left(\frac{1}{2}\right)}{3} = \dfrac{2}{3}$

$\Longrightarrow B_1 = \arcsin\left(\frac{2}{3}\right) \approx \boxed{41.8103148958^{\circ} \approx B_1}$ $\Longrightarrow B_2 = 180^{\circ} - B_1 \approx \boxed{138.189685104^{\circ} \approx B_2}$

$\Longrightarrow C_1 = 180^{\circ} - A - B_1 \approx \boxed{108.189685104^{\circ} \approx C_1}$ Angle

$\Longrightarrow C_2 = 180^{\circ} - 30^{\circ} - B_2 \approx \boxed{11.810314896^{\circ} \approx C_2}$ Angle

$c_1 = \dfrac{a \sin C_1}{\sin A} = \dfrac{15 \sin(108.189\ldots^{\circ})}{\sin(30^{\circ})}$

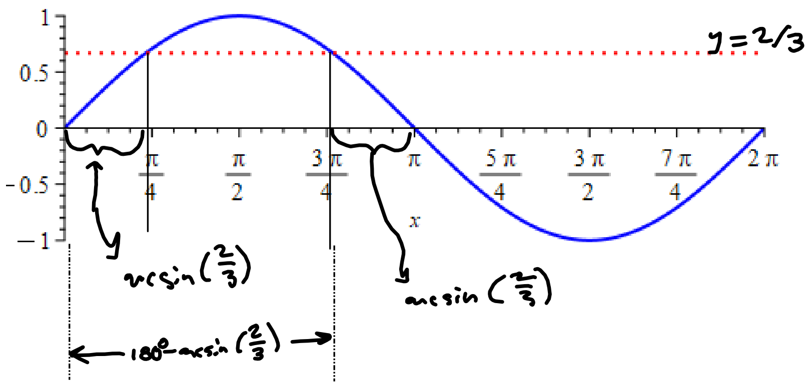
$\approx \boxed{28.5008479632 \approx c_1}$ length

$c_2 = \dfrac{a \sin C_2}{\sin A} \approx \boxed{6.1401681883 \approx c_2}$ length

1420 MILLS

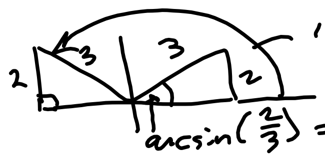
d) Bonus 5pts If $\sin B_1 = \frac{2}{3}$, why is $B_2 = 180^\circ - B_1$?

We show 2 sol'ns to $\sin(B) = \frac{2}{3}$ and they satisfy $B_2 = 180^\circ - B_1$



e. **(Bonus 5 pts)** Draw 2 triangles in the plane in standard position satisfying $\sin\theta = \frac{2}{3}$. Show that if θ_1 is a solution of the equation in Quadrant I, then the 2nd solution is $180^\circ - \theta_1$. Include the phrase "reference angle" in your discussion.

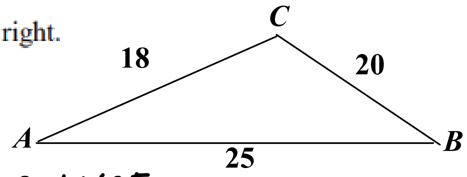
Two solutions to sin(B_1) = 2/3



$180^\circ - \arcsin\left(\frac{2}{3}\right) = B_2$

The reference angle for B_2 is $B_1 = \arcsin\left(\frac{2}{3}\right)$, and the angle B_2 is plainly $180^\circ - B_1$.

2. (5 pts) Solve the triangle given in the figure on the right.
Round answers to 3 places.



$$\cos A = \frac{b^2+c^2-a^2}{bc} = \frac{18^2+25^2-20^2}{2(18)(25)} = \frac{324+625-400}{900} = \frac{549}{900}$$

$$A = \arccos\left(61/100\right) \approx \boxed{52.4104970351^\circ \approx A} \approx \boxed{52.411^\circ \approx A}$$

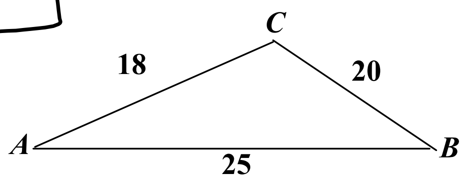
From here, you have 2 paths
① Law of sines ② Law of cosines
to get a 2nd angle. I usually do Sines,
so much easier, with the number I need
sitting in my calculator.

$$\sin B = \frac{b \sin A}{a} \approx \frac{18 \sin A}{20} \approx 0.713161272083 \rightarrow \boxed{45.4927106983^\circ \approx B}$$

$$\boxed{B \approx 45.493^\circ}$$

$$\rightarrow C = 180^\circ - A - B \approx \boxed{82.0967922666^\circ \approx C}$$

$$\boxed{C \approx 82.097^\circ}$$

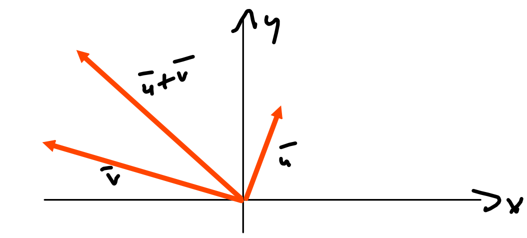


1420 MILLS

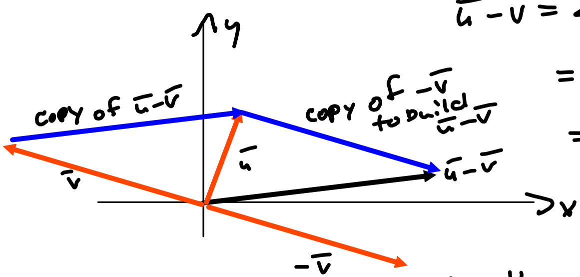
3. Let $\overline{u} = \langle 2,5\rangle, \overline{v} = \langle -10,3\rangle$.

a. (5 pts) Compute $\overline{u}+\overline{v}$. Sketch a graph of $\overline{u}, \overline{v}$, and $\overline{u}+\overline{v}$.

$$\overline{u}+\overline{v} = \langle 2-10, 5+3\rangle = \langle -8, 8\rangle$$



b. (5 pts) Compute $\overline{u}-\overline{v}$. Sketch a graph of $\overline{u}, \overline{v}$, and $\overline{u}-\overline{v}$. Observe that $\overline{u}-\overline{v}$ is equivalent to a vector from the tip of $\overline{v}$ to the tip of $\overline{u}$. Show this on your graph.



$$\overline{u}-\overline{v} = \langle 2,5\rangle - \langle -10,3\rangle$$
$$= \langle 2-(-10), 5-3\rangle$$
$$= \langle 12, 2\rangle = \overline{u}-\overline{v}$$

Indeed, $\overline{u}-\overline{v}$ may be thought of as the vector from the tip of $\overline{v}$ to the tip of $\overline{u}$. Remember this fact

c. (5 pts) Find a unit vector in the direction of $\overline{v}$. I want an exact answer, here, not a decimal approximation.

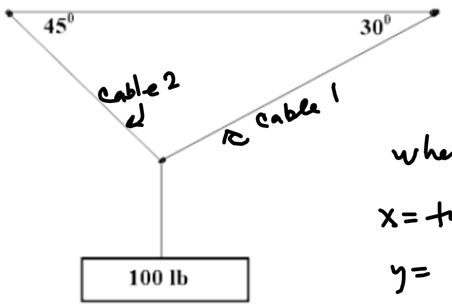
$\overline{v} = \langle -10, 3\rangle \implies \|\overline{v}\| = \sqrt{10^2+3^2} = \sqrt{109}$

$\implies$ a unit vector in the direction of $\overline{v}$ is $\frac{1}{\|\overline{v}\|}\overline{v}$

$= \boxed{\frac{1}{\sqrt{109}}\langle -10, 3\rangle = \frac{1}{\|\overline{v}\|}\overline{v}}$ or $\left\langle \frac{-10}{\sqrt{109}}, \frac{3}{\sqrt{109}} \right\rangle$

1420 MILLS

4. (10 pts) A 100-pound weight is suspended from cables attached to the ceiling as shown in the figure, below. Find the tension in each cable.



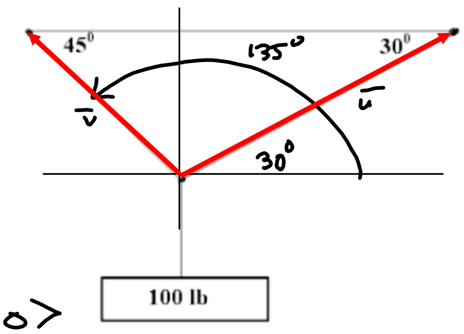
We represent the directed forces in the two cables by vectors $\bar{u} = \|\bar{u}\| \langle \cos 30°, \sin 30° \rangle = x \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

and $\bar{v} = \|\bar{v}\| \langle \cos 135°, \sin 135° \rangle = y \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$,

where $x = \|\bar{u}\|$ and $y = \|\bar{v}\|$.

x = the tension in cable 1

y = " " " " 2



Then, as there is a net downward force of 100 lbs, the resultant $\bar{u} + \bar{v} = \langle 0, 100 \rangle$, to balance all

This means that

$$\bar{u} + \bar{v} = \langle \tfrac{\sqrt{3}}{2}x, \tfrac{1}{2}x \rangle + \langle \tfrac{-y}{\sqrt{2}}, \tfrac{y}{\sqrt{2}} \rangle = \langle 0, 100 \rangle$$

Then $\frac{\sqrt{3}}{2}x - \frac{1}{\sqrt{2}}y = 0$

$+ \left(\frac{1}{2}x + \frac{1}{\sqrt{2}}y = 100 \right)$ Add the 2 equations

$\left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) x + 0y = 100$ ⟶

$$x = \frac{100}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \boxed{\frac{200}{\sqrt{3}+1} = x} \approx \boxed{73.2050807569 \text{ lbs} \approx \|\bar{u}\|}$$

1340 Mills

$$\Rightarrow \frac{\sqrt{3}}{2}x - \frac{1}{\sqrt{2}}y = \frac{\sqrt{3}}{2}\left(\frac{200}{\sqrt{3}+1}\right) - \frac{1}{\sqrt{2}}y = 0$$

$$\Rightarrow \frac{100\sqrt{3}}{\sqrt{3}+1} = \frac{1}{\sqrt{2}}y \longrightarrow$$

$$\boxed{\frac{100\sqrt{6}}{\sqrt{3}+1} = y} \approx \boxed{89.6575472168 \text{ lbs} \approx \|\bar{v}\|}$$

Check: $\bar{u} + \bar{v} = \frac{200}{\sqrt{3}+1}\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle + \frac{100\sqrt{6}}{\sqrt{3}+1}\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$

$$= \left\langle \frac{100\sqrt{3}}{\sqrt{3}+1}, \frac{100}{\sqrt{3}+1}\right\rangle + \left\langle -\frac{50\sqrt{12}}{\sqrt{3}+1}, \frac{50\sqrt{12}}{\sqrt{3}+1}\right\rangle$$

$$= \left\langle \frac{100\sqrt{3}}{\sqrt{3}+1}, \frac{100}{\sqrt{3}+1}\right\rangle + \left\langle \frac{-100\sqrt{3}}{\sqrt{3}+1}, \frac{100\sqrt{3}}{\sqrt{3}+1}\right\rangle$$

$$= \left\langle \frac{100\sqrt{3}}{\sqrt{3}+1} - \frac{100\sqrt{3}}{\sqrt{3}+1}, \frac{100}{\sqrt{3}+1} + \frac{100\sqrt{3}}{\sqrt{3}+1}\right\rangle$$

$$= \left\langle 0, \frac{100+100\sqrt{3}}{\sqrt{3}+1}\right\rangle = \left\langle 0, \frac{100(1+\sqrt{3})}{\sqrt{3}+1}\right\rangle = \langle 0, 100\rangle \checkmark$$

1420 Mills

Substitution Method for this problem.

Then $\frac{\sqrt{3}}{2}x - \frac{1}{\sqrt{2}}y = 0$

$\frac{1}{2}x + \frac{1}{\sqrt{2}}y = 100$

$x = \frac{100}{\frac{\sqrt{3}}{2}+\frac{1}{2}} = \boxed{\frac{200}{\sqrt{3}+1} = x} \approx \boxed{73.2050807569\ \text{lbs} \approx \|\vec{u}\|}$

$\frac{\sqrt{3}}{2}x = \frac{1}{\sqrt{2}}y$

$x = \frac{2}{\sqrt{6}}y$

$\frac{1}{2}\left(\frac{2}{\sqrt{6}}y\right) + \frac{1}{\sqrt{2}}y = 100$

$\frac{y}{\sqrt{6}} + \frac{y}{\sqrt{2}} = 100$

$\frac{y+\sqrt{3}y}{\sqrt{6}} = \frac{1+\sqrt{3}}{\sqrt{6}}y = 100 \longrightarrow \boxed{y = \frac{100\sqrt{6}}{1+\sqrt{3}}}$

$\Rightarrow x = \frac{2}{\sqrt{6}}y = \frac{2}{\sqrt{6}}\left(\frac{100\sqrt{6}}{1+\sqrt{3}}\right) = \boxed{\frac{200}{1+\sqrt{3}} = x}$