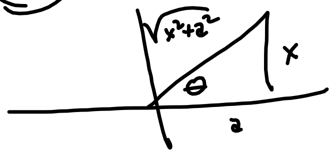
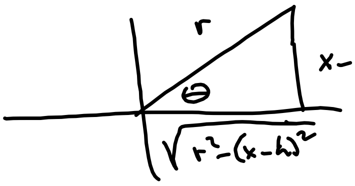
1420 WEEK 06 SOL'NS H. MILLS

① We write an algebraic expression equivalent to the given trigonometric expression.

ⓐ 5pts $\csc(\arctan(\frac{x}{a})) = \csc\theta = \boxed{\frac{\sqrt{x^2+a^2}}{x}}$



ⓑ 5pts $\cos(\arcsin(\frac{x-h}{r})) = \cos\theta = \boxed{\frac{\sqrt{r^2-(x-h)^2}}{r}}$



② Verify the identity algebraically.

ⓐ 5pts $\frac{\cos\theta\cot\theta}{1-\sin\theta} = 5\csc\theta$?! No, it doesn't!

Free 5pts: $\left(\frac{\cos\theta\cot\theta}{1-\sin\theta}\right)\left(\frac{1+\sin\theta}{1+\sin\theta}\right) = \frac{(\cos\theta)\left(\frac{\cos\theta}{\sin\theta}\right)(1+\sin\theta)}{1-\sin^2\theta}$

$= \frac{\left(\frac{\cos^2\theta}{\sin\theta}\right)(1+\sin\theta)}{\cos^2\theta} = \boxed{\frac{1+\sin\theta}{\sin\theta} = \csc\theta + 1}$ doesn't look like $5\csc\theta$!

I was looking for the $\left(\frac{1+\sin\theta}{1+\sin\theta}\right)$ move, but must have mis-typed the darn question!

ⓑ 5pts $\sqrt{\frac{1+\cos\theta}{1-\sin\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$?! I don't think so.

$\sin\theta \rightarrow \cos\theta$

$\sqrt{\left(\frac{1+\cos\theta}{1-\sin\theta}\right)\left(\frac{1+\sin\theta}{1+\sin\theta}\right)} = \sqrt{\frac{(1+\sin\theta)(1+\cos\theta)}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)(1+\cos\theta)}{\cos^2\theta}} = \frac{\sqrt{(1+\sin\theta)(1+\cos\theta)}}{|\cos\theta|}$, but

this can't be made into $\frac{1+\cos\theta}{|\sin\theta|}$.

I wanted this:

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$$

Not sure WHAT I was shooting for in part a!

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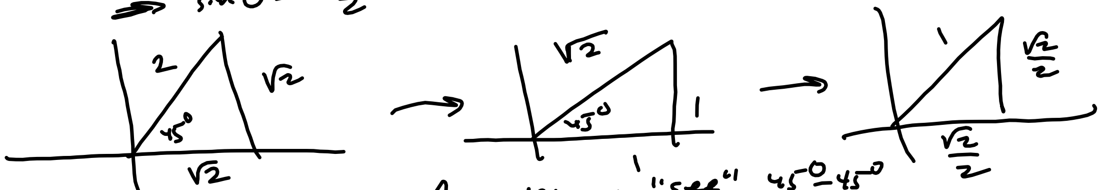
#3 Solve the trig equations:

a 5pts $\frac{\sqrt{2}}{2}\csc\theta - 1 = 0 \longrightarrow$

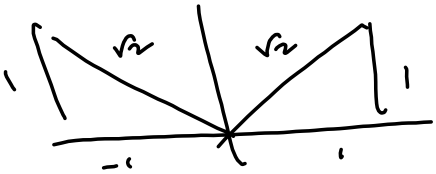
$\csc\theta = \frac{2}{\sqrt{2}}$

$\Rightarrow \sin\theta = \frac{\sqrt{2}}{2}$



Any way you "see" 45°–45° right triangle is OK.

So, $\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}$ on $[0, 2\pi)$. In general,

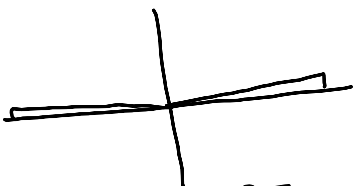
$$\boxed{x = \frac{\pi}{4} + 2n\pi \text{ or } \frac{3\pi}{4} + 2n\pi,\ n \in \mathbb{Z}}$$

b 5pts $\sin(2x)(2\sin(x)+1) = 0 \longrightarrow$

$\sin(2x) = 0$ or $2\sin(x) + 1 = 0$

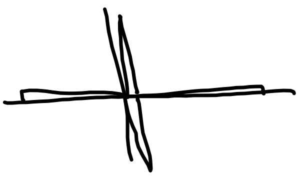
To find all $x \in [0, 2\pi)$, find all $2x \in [0, 4\pi)$

$\sin(2x) = 0$



$2x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

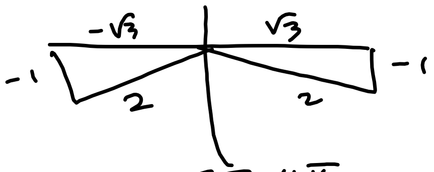


All $\frac{\pi}{2}$ apart, so

$x = \frac{n\pi}{2}$

$2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$



$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ on $[0, 2\pi)$

In general,

$$\boxed{x = \frac{7\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi,\ n \in \mathbb{Z}}$$

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5 $\sin(u) = -\frac{3}{5}$, $\cos(u) < 0$ and $\cos(v) = \frac{14}{17}$, $\tan(v) < 0$

5pts Find $\cos(u+v)$

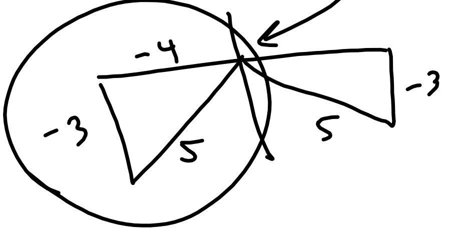
Let's find the quadrant for u & v

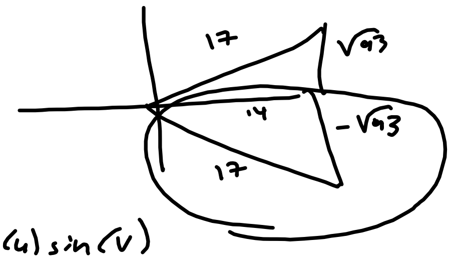
$\sqrt{17^2 - 14^2} = \sqrt{289 - 196}$

$\sqrt{93} = \sqrt{93}$

$\sin(u) = -\frac{3}{5}$, $\cos(u) < 0$

$\cos(v) = \frac{14}{17}$, $\tan(v) < 0$



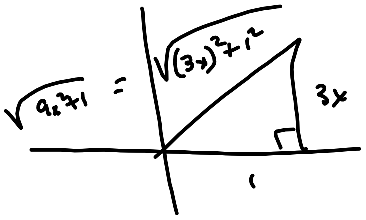


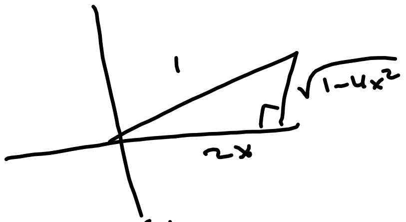
$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$
$$= \left(-\frac{4}{5}\right)\left(\frac{14}{17}\right) - \left(-\frac{3}{5}\right)\left(-\frac{\sqrt{93}}{17}\right)$$
$$= \boxed{\frac{-56 - 3\sqrt{93}}{85}}$$

6 5pts $\sin(\arctan(3x) - \arccos(2x)) = \sin(u - v)$.

$u = \arctan(3x)$

$v = \arccos(2x)$





$$\sin(u-v) = \sin(u)\cos(-v) + \sin(-v)\cos(u)$$
$$= \sin(u)\cos(v) - \sin(v)\cos(u)$$
$$= \left(\frac{3x}{\sqrt{9x^2+1}}\right)\left(\frac{2x}{1}\right) - \left(\frac{\sqrt{1-4x^2}}{1}\right)\left(\frac{1}{\sqrt{9x^2+1}}\right)$$
$$= \boxed{\frac{6x^2 - \sqrt{1-4x^2}}{\sqrt{9x^2+1}}}$$